

The Complement of Cubic Power Graph of Dihedral Group

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Abstract. The complement of cubic power graph of dihedral group D_n with identity element e , $CCPG(D_n)$ is a simple, undirected and finite graph having vertices pair $x, y \in D_n$ adjacent if and only if there exists no element $z \in D_n$ such that $xy = z^3$ or $yx = z^3$ with $z^3 \neq e$. This research explores the representation of the complement of cubic power graph of D_n structure by giving the explicit formula to derive its structure and structural characteristics in particular the matching number, independence number, chromatic number, girth, clique, vertex degree, domination number, etc.

Key words and Phrases: Cubic power graph, Dihedral group, Matching number, Degree of vertex, Chromatic number.

1. INTRODUCTION

Graphs of groups are useful tools for demonstrating and assessing group structures, particularly in geometric group theory. They simplify complex groups by breaking them up into smaller components. They are helpful in both theoretical research and practical applications, from solving computing problems to understanding groups.

Recently, various graphs have been studied in association with finite groups [1, 2, 3, 4, 5]. Spectral properties such as signless Laplacian and normalized Laplacian eigen-values of the power graph and some of its supergraphs are computed in [6]. Connectivity Indices such as Harary index, Wiener index, hyper-Wiener index, first Zagreb index, second Zagreb index, etc of coprime graphs of generalized quaternion group are calculated in [7]. The energy and spectrum of the non-commuting graph for U_{6n} in association with the adjacency, degree sum and degree sum adjacency matrices are explored in [8]. The square power graphs of finite abelian groups and their complements were studied in [9, 10, 11, 12, 13]. The cubic power graphs of

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the finite abelian groups and their complements were introduced and studied in [14], whereas for dihedral group in [15, 16]. The structure of the complement of cubic power graphs of dihedral groups remained unexplored.

In this research the representation of the complement of cubic power graphs of D_n structure is done by giving the explicit formula to calculate its structure. Further structural properties such as the matching number, independence number, chromatic number, girth, clique, vertex degree, domination number, etc are calculated.

Since, degree of vertices and their distances have been calculated in this research, various spectral properties and topological indices can be studied in future. Also, the study of the structure of cubic power graphs of generalized quaternion groups may be an interesting topic for future research.

2. PRELIMINARIES

The complement of cubic power graph of dihedral group D_n with identity element e , $CCPG(D_n)$ is a simple, undirected and finite graph having vertices pair $x, y \in D_n$ adjacent if and only if there exists no element $z \in D_n$ such that $xy = z^3$ or $yx = z^3$ with $z^3 \neq e$.

The dihedral group of order $2n$ with identity element e is $D_n = \{e, r, r^2, r^3, \dots, r^{n-1}, s, rs, r^2s, r^3s, \dots, r^{n-1}s : s^2 = e, r^n = e, r^i s = sr^{n-i}\}$. Let $R = \{r^i : 1 \leq i \leq n\}$, $S = \{r^i s : 1 \leq i \leq n\}$ and $X = \{v : v \in D_n, v = v^{-1}\}$.

Let $D'_n = \{x^3 : \forall x \in D_n\}$ then $D'_n = D_n$ if $\gcd(n, 3) = 1$ and $D'_n = \{e, r^3, r^6, \dots, r^{n-3}, s, rs, r^2s, r^3s, \dots, r^{n-1}s\} = R' \cup S$ when $\gcd(3, n) = 3$ where $R' = \{e, r^3, r^6, \dots, r^{n-3}\}$. The pair of different vertices $x, y \in V(CCPG(D_n))$ has an edge if and only if $xy \notin D'_n \setminus \{e\}$.

K_n is the complete graph having every pair of vertices adjacent. $m - K_n$ is a regular graph having n vertices with each vertex having m degree. When graph G has n disjoint components $G_1, G_2, \dots, G_n$ then G is represented as $G = G_1 \cup G_2 \cup \dots \cup G_n$. When all vertices of G_1 adjacent with all vertices of G_2 , then it is represented as $G_1 + G_2$.

3. STRUCTURAL PROPERTIES OF $CCPG(D_n)$

Theorem 3.1. *The complement of cubic power graph of the dihedral group is always a disconnected graph.*

Proof. Let $CCPG(D_n)$ be the complement of the cubic power graph of the dihedral group D_n having identity element e .

Case 1. When $3 \nmid n$.

In this case, $D'_n = D_n$. So, for all non-identity elements $v \in D_n$, we have $ve = v \in D'_n \setminus \{e\}$. Thus, vertex e is not adjacent to any other vertex $v \in V(CCPG(D_n))$. Thus, there are atleast two disjoint components of $CCPG(D_n)$. Hence, for $3 \nmid n$, $CCPG(D_n)$ is a disconnected graph.

Case 2. When $3 \mid n$.

In this case, $D'_n = \{e, r^3, r^6, \dots, r^{n-3}, s, rs, r^2s, \dots, r^{n-1}s\}$. So, for any pair of elements $r^i, r^i s \in D_n$, we have $r^i r^i s = r^{2i \bmod n} s \in D'_n \setminus \{e\}$. Thus, r^i type vertices are not adjacent to any of the vertices of $r^i s$ type in $CCPG(D_n)$. Thus, there are at least two disjoint components of $CCPG(D_n)$. Hence, $CCPG(D_n)$ is a disconnected graph when $3 \mid n$. $\square$

Theorem 3.2. Let $CCPG(D_n)$ be the complement of the cubic power graph of D_n of order $2n$ such that $3 \nmid n$. Then,

$$CCPG(D_n) = \begin{cases} (n+2)K_1 \cup \left(\frac{n-2}{2}\right)K_2 & \text{when } n \text{ is even number,} \\ (n+1)K_1 \cup \left(\frac{n-1}{2}\right)K_2 & \text{when } n \text{ is odd number.} \end{cases}$$

Proof. For $3 \nmid n$, $D'_n = D_n$.

Case 1. n is even.

In this, $|X| = n + 2$. Now there are three ways of choosing pairs of vertices from $V(CCPG(D_n))$.

Sub-case (i) $u \in D_n \setminus X$ and $v \in X$.

Since $v \in X$, we have $v = v^{-1} \implies uv \neq e \implies uv \in D'_n \setminus \{e\}$. So, no such pair of vertices is adjacent in $CCPG(D_n)$.

Sub-case (ii) $u, v \in X$.

Since $u, v \in X$, we have $u = u^{-1}$ and $v = v^{-1}$. Thus, $uv \neq e$ and so $uv \in D'_n \setminus \{e\}$. So, no such pair of vertices is adjacent in $CCPG(D_n)$.

Sub-case (iii) $u, v \in D_n \setminus X$.

Since $u, v \in D_n \setminus X$, we have $u \neq u^{-1}$ and $v \neq v^{-1}$. Now, if $v^{-1} \neq u$ then $uv \neq e$ and so $uv \in D'_n \setminus \{e\}$. And if $v^{-1} = u$ then $uv = e$ and so $uv \notin D'_n \setminus \{e\}$. So, pair of vertices $u, v \in D_n \setminus X$ are adjacent in $CCPG(D_n)$ if and only if $v^{-1} = u$.

As, $|D_n \setminus X| = n - 2$, so there are $\frac{n-2}{2}$ pairs of vertices $u, v \in D_n \setminus X$ such that $v^{-1} = u$. So, there are $n + 2$ vertices in $CCPG(D_n)$ adjacent with no other vertex and $\frac{n-2}{2}$ distinct adjacent vertices pairs in $CCPG(D_n)$. Hence, $CCPG(D_n) = (n+2)K_1 \cup \left(\frac{n-2}{2}\right)K_2$.

Case 2. When n is odd.

In this, $|X| = n + 1$. Now there are three ways of choosing pairs of vertices from $V(CCPG(D_n))$.

Sub-case (i) $u \in D_n \setminus X$ and $v \in X$.

Since $v \in X$, we have $v = v^{-1} \implies uv \neq e \implies uv \in D'_n \setminus \{e\}$. So, no such pair of vertices is adjacent in $CCPG(D_n)$.

Sub-case (ii) $u, v \in X$.

Since $u, v \in X$, we have $u = u^{-1}$ and $v = v^{-1}$. Thus, $uv \neq e$ and so $uv \in D'_n \setminus \{e\}$. So, no such pair of vertices is adjacent in $CCPG(D_n)$.

Sub-case (iii) $u, v \in D_n \setminus X$.

Since $u, v \in D_n \setminus X$, we have $u \neq u^{-1}$ and $v \neq v^{-1}$. Now, if $v^{-1} \neq u$ then $uv \neq e$ and so $uv \in D'_n \setminus \{e\}$. And if $v^{-1} = u$ then $uv = e$ and so $uv \notin D'_n \setminus \{e\}$. So, pair of vertices $u, v \in D_n \setminus X$ are adjacent in $CCPG(D_n)$ if and only if $v^{-1} = u$.

As, $|D_n \setminus X| = n - 1$, so there are $\frac{n-1}{2}$ pairs of vertices $u, v \in D_n \setminus X$ such that $v^{-1} = u$. So, there are $n + 1$ vertices in $CCPG(D_n)$ adjacent with no other vertex and $\frac{n-1}{2}$ distinct adjacent vertices pairs in $CCPG(D_n)$. Hence, $CCPG(D_n) = (n + 1)K_1 \cup \left(\frac{n-1}{2}\right) K_2$. $\square$

Using theorem above, structures of the complement of cubic power graph of D_5 and D_8 are $CCPG(D_5) = 6K_1 \cup 2K_2$ and $CCPG(D_8) = 10K_1 \cup 3K_2$ as shown in Figures 1 and 2 respectively.

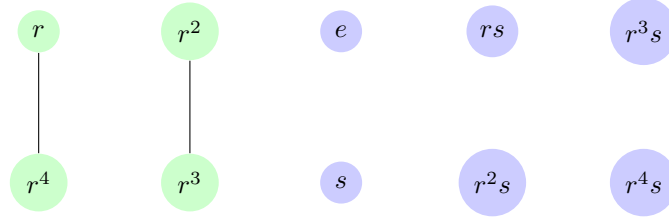


FIGURE 1. $CCPG(D_5)$

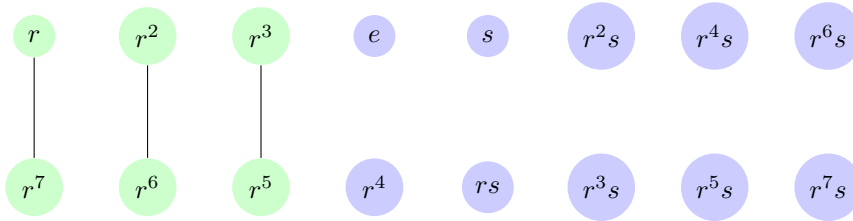


FIGURE 2. $CCPG(D_8)$

Corollary 3.3. Let $x, y \in V(CCPG(D_n))$ and $3 \nmid n$. Then,

- (i) $\deg_{CCPG(D_n)}(x) = \begin{cases} 0 & \text{when } x \in X, \\ 1 & \text{when } x \in D_n \setminus X. \end{cases}$
- (ii) $d(x, y) = \begin{cases} 1 & \text{if } x^{-1} = y \neq x, \\ \infty & \text{if } x^{-1} \neq y. \end{cases}$
- (iii) $|E(CCPG(D_n))| = \begin{cases} \frac{n-2}{2} & \text{if } n \text{ is even number,} \\ \frac{n-1}{2} & \text{if } n \text{ is odd number.} \end{cases}$
- (iv) $CCPG(D_n)$ is a prime graph.
- (v) $\text{Girth}, \text{gr}(CCPG(D_n)) = \infty$.
- (vi) $\text{Cliques}, \omega(CCPG(D_n)) = \begin{cases} 1 & \text{if } n \in \{1, 2\}, \\ 2 & \text{otherwise.} \end{cases}$
- (vii) $\text{Chromatic number}, \chi(CCPG(D_n)) = \omega(CCPG(D_n))$.
- (viii) $\text{Independence number}, \beta(CCPG(D_n)) = \begin{cases} \frac{3n+2}{2} & \text{if } n \text{ is even number,} \\ \frac{3n+1}{2} & \text{if } n \text{ is odd number.} \end{cases}$
- (ix) $\text{Domination number}, \gamma(CCPG(D_n)) = \beta(CCPG(D_n))$.
- (x) $\text{Matching number}, \mu(CCPG(D_n)) = |E(CCPG(D_n))|$.

Theorem 3.4. Let $CCPG(D_n)$ be the complement of cubic power graph of D_n of order $2n$ such that $3 \mid n$. Then,

$$CCPG(D_n) = \begin{cases} \frac{n}{3} - K_{\frac{2n}{3}} + \left\{ K_1 \cup \left(\frac{n-3}{6} \right) K_2 \right\} \cup K_{\frac{n}{3}, \frac{n}{3}, \frac{n}{3}} & \text{when } n \text{ is odd,} \\ \frac{n}{3} - K_{\frac{2n}{3}} + \left\{ 2K_1 \cup \left(\frac{n-6}{6} \right) K_2 \right\} \cup K_{\frac{n}{3}, \frac{n}{3}, \frac{n}{3}} & \text{when } n \text{ is even.} \end{cases}$$

Proof. For $3 \mid n$, $D'_n = R' \cup S$.

Now for adjacency of pairs of elements from D_n there are following three possibilities.

Case 1. When in pair of vertices (u, v) , vertices are of type $u = r^i \in R$ and $v = r^j s \in S$. For r^i and $r^j s$, we have $r^i r^j s = r^{(i+j) \bmod n} s \in S \subset D'_n \setminus \{e\}$. Thus, no such pairs of vertices are adjacent in $CCPG(D_n)$.

Case 2. When in pairs of vertices (u, v) , vertices are of type $u = r^i s \in S$ and $v = r^j s \in S$ for $i \neq j$. In this case, $r^i s r^j s = r^i s r^{n-j} = r^{(i+n-j) \bmod n}$. Then we have the following two sub-cases.

Sub-case (i) When $i \equiv 0(\bmod 3)$ & $j \equiv 0(\bmod 3)$ or $i \equiv 1(\bmod 3)$ & $j \equiv 1(\bmod 3)$ or $i \equiv 2(\bmod 3)$ & $j \equiv 2(\bmod 3)$. As $(i-j) \equiv 0(\bmod 3)$ and $(n+i-j) \bmod n \notin \{0, n\}$. So, $(i+n-j) \bmod n \in \{3, 6, \dots, n-3\}$. Thus, $r^{(i+n-j) \bmod n} \in R' \setminus \{e\} \subset D'_n \setminus \{e\}$. So, vertices in such pairs are not adjacent.

Sub-case (ii) When $i \equiv 0(\bmod 3)$ & $j \equiv 1(\bmod 3)$ or $i \equiv 0(\bmod 3)$ & $j \equiv 2(\bmod 3)$ or $i \equiv 1(\bmod 3)$ & $j \equiv 2(\bmod 3)$. As $(i-j) \equiv 1(\bmod 3)$ or $2(\bmod 3)$.

So, $(i + n - j) \bmod n \notin \{3, 6, \dots, n - 3\}$. Thus, $r^{(i+n-j) \bmod n} \notin D'_n \setminus \{e\}$. So, vertices in such pairs are adjacent.

As $|S| = n$, so there are total n such elements in D_n . So, there are $\frac{n}{3}$ number of reflection elements r^k s such that $k \equiv 0 \pmod{3}$, $\frac{n}{3}$ number of reflection elements r^k s such that $k \equiv 1 \pmod{3}$ and $\frac{n}{3}$ number of reflection elements r^k s such that $k \equiv 2 \pmod{3}$.

Also from case 1, it is clear that there is no edge between the vertices from S and vertices from R . So, vertices from S form the $K_{\frac{n}{3}, \frac{n}{3}, \frac{n}{3}}$ (complete 3-partite graph with three independent sets, each having $\frac{n}{3}$ vertices) disjoint component of $CCPG(D_n)$.

Case 3. When in pairs of vertices (u, v) , vertices are of type $u = r^i \in R$ and $v = r^j \in R$ for $i \neq j$. In this case $r^i r^j = r^{(i+j) \bmod n}$. We have the following three sub-cases.

Sub-case (i) Both r^i and r^j elements in pair belongs to R' . When both r^i and r^j belongs to R' and $i + j \neq n$ then $(i + j) \bmod n \in \{3, 6, \dots, n - 3\}$ so $r^i r^j = r^{(i+j) \bmod n} \in R' \setminus \{e\} \subset D'_n \setminus \{e\}$. So, vertices in such pairs are not adjacent in $CCPG(D_n)$. But when both r^i and r^j belongs to R' and $i + j = n$ then $r^i r^j = r^{(i+j) \bmod n} = r^{n \bmod n} = e \notin D'_n \setminus \{e\}$. So, such pairs of vertices are adjacent in $CCPG(D_n)$.

For even n , there are $\frac{n-6}{6}$ disjoint pairs of vertices (r^i, r^j) in R' for which $i + j = n$ and for odd n , there are $\frac{n-3}{6}$ disjoint pairs of vertices (r^i, r^j) in R' for which $i + j = n$. Thus, vertices from R' form the $K_1 \cup \left(\frac{n-6}{6}\right) K_2$ component of $CCPG(D_n)$ when n is even and $K_1 \cup \left(\frac{n-3}{6}\right) K_2$ component of $CCPG(D_n)$ when n is odd.

Sub-case 2. Both r^i and r^j elements in pair belongs to $R \setminus R'$. For every $r^k \in R \setminus R'$, either $k \equiv 2 \pmod{3}$ or $k \equiv 1 \pmod{3}$. There are $\frac{n}{3}$ vertices in $R \setminus R'$ having $k \equiv 1 \pmod{3}$ and $\frac{n}{3}$ vertices in $R \setminus R'$ having $k \equiv 2 \pmod{3}$. So, for vertices (r^i, r^j) in pair we get $j + i \equiv 2 \pmod{3}$ when $i \equiv 1 \pmod{3}$ & $j \equiv 1 \pmod{3}$, $j + i \equiv 1 \pmod{3}$ when $i \equiv 2 \pmod{3}$ & $j \equiv 2 \pmod{3}$ and $i + j \equiv 0 \pmod{3}$ when $i \equiv 1 \pmod{3}$ & $j \equiv 2 \pmod{3}$ or $i \equiv 2 \pmod{3}$ & $j \equiv 1 \pmod{3}$.

For vertices pair (r^i, r^j) , in case $j + i \equiv 1 \pmod{3}$ or $j + i \equiv 2 \pmod{3}$ we have $r^i r^j = r^{(i+j) \bmod n} \notin D'_n \setminus \{e\}$. So, such pairs of vertices are adjacent. Whereas for vertices pair (r^i, r^j) , in case $i + j \equiv 0 \pmod{3}$ & $j + i = n$ we have $r^i r^j = r^{(i+j) \bmod n} \notin D'_n \setminus \{e\}$ and in case $i + j \equiv 0 \pmod{3}$ & $j + i \neq n$ we have $r^i r^j = r^{(i+j) \bmod n} \in R' \setminus \{e\} \subset D'_n \setminus \{e\}$.

Thus, vertices from $R \setminus R'$ form the $\frac{n}{3}$ -regular component of $CCPG(D_n)$ i.e $\frac{n}{3} - K_{\frac{2n}{3}}$.

Sub-case 3. One rotation element belongs to R' and another rotation element belongs to $R \setminus R'$. For $r^i \in R'$ and $r^j \in R \setminus R'$, we have $i \equiv 0 \pmod 3$ and $j \equiv 2 \pmod 3$ or $j \equiv 1 \pmod 3$. So, $j + i \equiv 1 \pmod 3$ or $j + i \equiv 2 \pmod 3$. Thus, $r^i r^j = r^{(j+i) \pmod n} \notin D'_n \setminus \{e\}$. So, every such pair of vertices is adjacent in $CCPG(D_n)$.

Thus, these reflection elements of D_n forms the disjoint component $\frac{n}{3} - K_{\frac{2n}{3}} + \left\{ K_1 \cup \left(\frac{n-3}{6} \right) K_2 \right\}$ of $CCPG(D_n)$ when n is odd and $\frac{n}{3} - K_{\frac{2n}{3}} + \left\{ 2K_1 \cup \left(\frac{n-6}{6} \right) K_2 \right\}$ when n is even.

Hence, $CCPG(D_n) = \frac{n}{3} - K_{\frac{2n}{3}} + \left\{ K_1 \cup \left(\frac{n-3}{6} \right) K_2 \right\} \cup K_{\frac{n}{3}, \frac{n}{3}, \frac{n}{3}}$ when n is odd and $\frac{n}{3} - K_{\frac{2n}{3}} + \left\{ 2K_1 \cup \left(\frac{n-6}{6} \right) K_2 \right\} \cup K_{\frac{n}{3}, \frac{n}{3}, \frac{n}{3}}$ when n is even. $\square$

Thus, $\frac{n}{3} - K_{\frac{2n}{3}} + \left\{ K_1 \cup \left(\frac{n-3}{6} \right) K_2 \right\} \cup K_{\frac{n}{3}, \frac{n}{3}, \frac{n}{3}}$ when n is odd, and $\frac{n}{3} - K_{\frac{2n}{3}} + \left\{ 2K_1 \cup \left(\frac{n-6}{6} \right) K_2 \right\} \cup K_{\frac{n}{3}, \frac{n}{3}, \frac{n}{3}}$ when n is even.

Using above theorem, structures of the complement of the cubic power graph of D_3 , D_6 , D_9 and D_{12} are $CCPG(D_3) = 2K_3$, $CCPG(D_6) = 2 - K_4 + 2K_1 \cup K_{2,2,2}$, $CCPG(D_9) = 3 - K_6 + \{K_1 \cup K_2\} \cup K_{3,3,3}$ and $CCPG(D_{12}) = 4 - K_8 + \{2K_1 \cup K_2\} \cup K_{4,4,4}$ as shown in Figures 3, 4, 5 and 6 respectively.

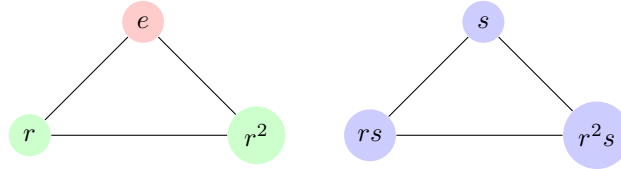
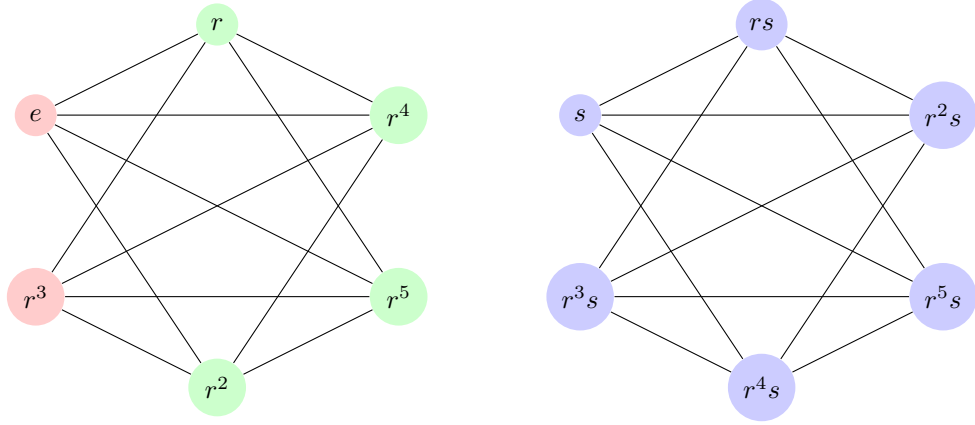
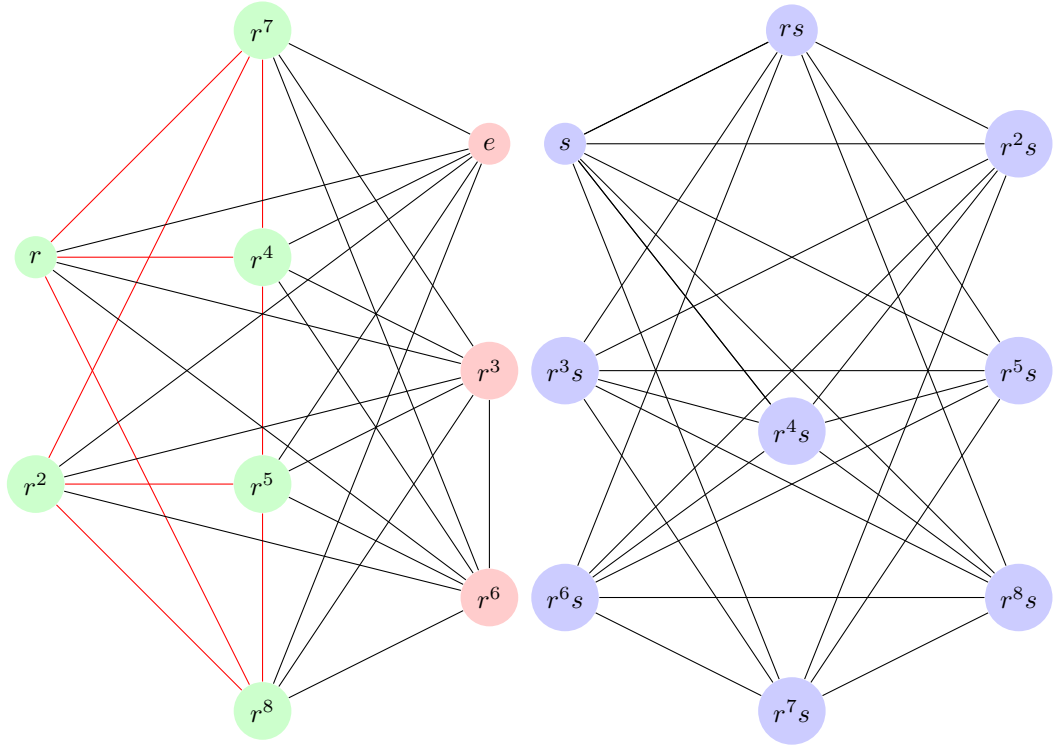
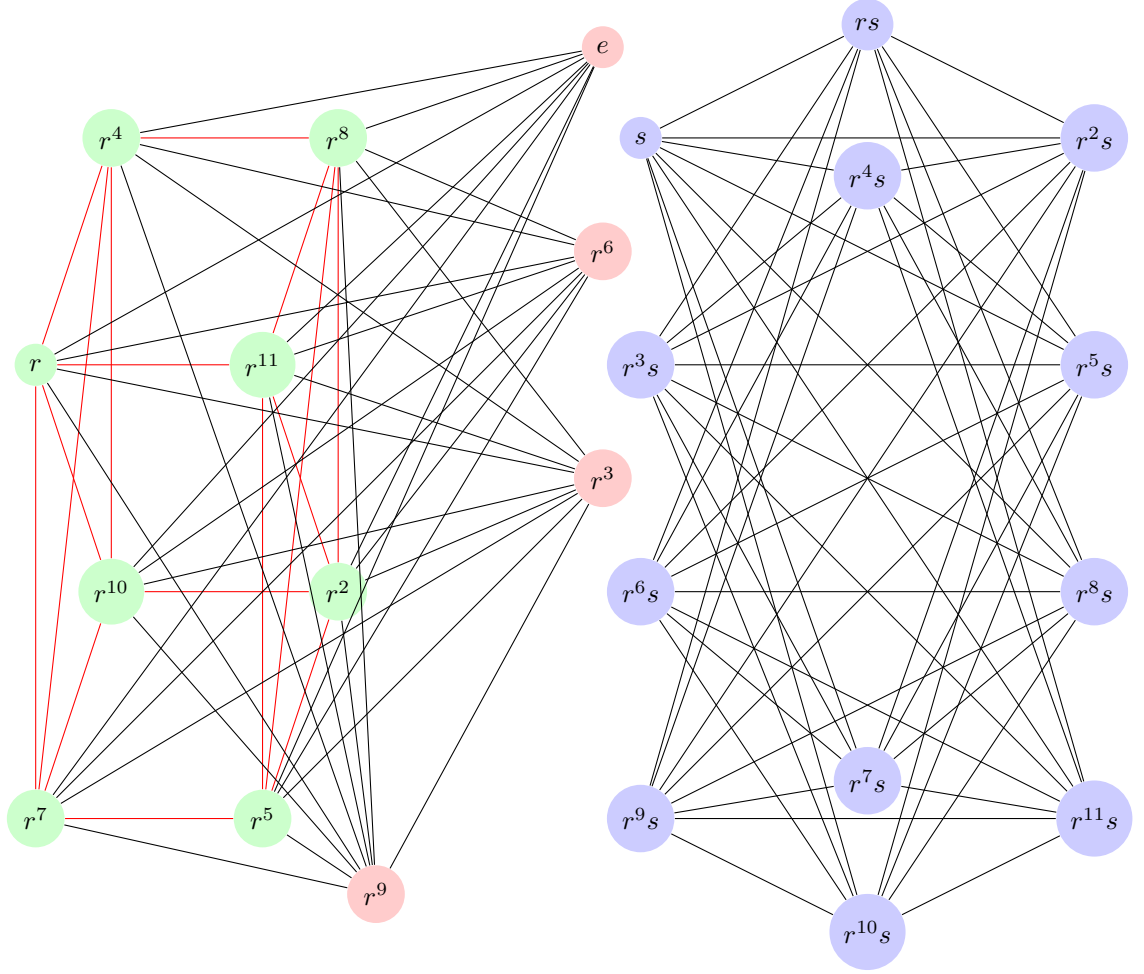


FIGURE 3. $CCPG(D_3)$

FIGURE 4. $CCPG(D_6)$ FIGURE 5. $CCPG(D_9)$

Corollary 3.5. *Let $x, y \in V(CCPG(D_n))$ and $3 \mid n$. Then,*

(i) $\deg_{CCPG(D_n)}(x) = \begin{cases} \frac{2n}{3} & \text{when } x \in \{D_n \setminus R'\} \cup \{R' \cap X\}, \\ \frac{2n}{3} + 1 & \text{when } x \in R' \setminus X. \end{cases}$

FIGURE 6. $CCPG(D_{12})$

- (ii) $|E(CCPG(D_n))| = \begin{cases} \frac{4n^2 + n - 6}{6} & \text{if } n \text{ is even number,} \\ \frac{4n^2 + n - 3}{6} & \text{if } n \text{ is odd number.} \end{cases}$
- (iii) Number of vertices with degree $\frac{2n}{3} = \begin{cases} \frac{5n+3}{3} & \text{when } n \text{ is odd,} \\ \frac{5n+6}{3} & \text{when } n \text{ is even.} \end{cases}$
- (iv) Number of vertices with degree $\left(\frac{2n}{3} + 1\right) = \begin{cases} \frac{n-3}{3} & \text{when } n \text{ is odd,} \\ \frac{n-6}{3} & \text{when } n \text{ is even.} \end{cases}$
- (v) Girth, $gr(CCPG(D_n)) = 3$.

- (vi) *Clique number*, $\omega(CCPG(D_n)) = \begin{cases} 3 & \text{if } n \in \{3, 6\}, \\ \frac{n+6}{3} & \text{otherwise.} \end{cases}$
- (vii) *Chromatic number*, $\chi(CCPG(D_n)) = \omega(CCPG(D_n))$.
- (viii) *Independence number*, $\beta(CCPG(D_n)) = \begin{cases} \frac{n+2}{2} & \text{if } n \text{ is even number,} \\ \frac{n+1}{2} & \text{if } n \text{ is odd number.} \end{cases}$
- (ix) *Domination number*, $\gamma(CCPG(D_n)) = \begin{cases} 2 & \text{if } n = 3, \\ 4 & \text{otherwise.} \end{cases}$
- (x) *Matching number*, $\mu(CCPG(D_n)) = \begin{cases} n & \text{if } n \text{ is even number,} \\ n-1 & \text{if } n \text{ is odd number.} \end{cases}$

4. CONCLUDING REMARKS

In this research, we have given explicit formula for the structures of the complement of cubic power graphs of dihedral groups and shown that these graphs are always disconnected. Further, we have calculated their structural properties such as matching number, independence number, chromatic number, girth, clique, vertex degree, domination number, etc.

Spectral properties such as Laplacian & signless Laplacian characteristic polynomials and topological indices based on degree of vertices & their distances may be studied in future.

Data Availability Statement. No new data were created or analyzed in this study.

Declarations. This research work is part of the thesis to be submitted by the first author (Pankaj Rana) to Baba Mastnath University for the award of Doctor of Philosophy.

Conflicts of Interest. The authors declare that there are no conflicts of interest.

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