

Fuzzy Dynamic Model of Tungro Virus in Rice Plant Related to Pruning and Insecticide

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Abstract. Rice Tungro Disease (RTD) is a significant threat to global rice production, with planthoppers acting as the principal transmission vectors. While insecticide application is a common vector control method, many agricultural practices also incorporate pruning interventions to mitigate the spread of the Tungro virus within rice crops. This study develops a fuzzy dynamical model to capture the uncertainties associated with two key parameters: the mortality of rice plants induced by pruning and the mortality of planthoppers due to insecticide treatment. Specifically, the fuzzy membership function for pruning-induced mortality is formulated as a function of the Tungro virus load. In contrast, the insecticide-related planthopper mortality is defined as a function of insecticide dosage, accounting for the varying levels of resistance observed in planthopper populations. The resulting fuzzy dynamical system is numerically solved using the Non-Standard Finite Difference (NSFD) scheme, which preserves the qualitative properties of the underlying mode. Numerical simulations are conducted to analyze the effects of fuzzy parameter variations on the disease dynamics. Numerical results indicate that the integrated application of pruning and insecticide strategies significantly reduces infected populations compared to single-control strategies. This modeling framework provides a more realistic and flexible tool for analyzing plant disease control strategies under parameter uncertainty.

Key words and Phrases: Tungro virus, fuzzy parameter, NSFD scheme, sensitivity.

1. INTRODUCTION

Rice (*Oryza sativa* L.) is a food crop that is crucial to the Indonesian economy. This crop can provide employment, which automatically reduces poverty and

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increases income. According to the Food and Agriculture Organization (FAO), the agricultural sector contributes approximately 14% to Indonesia's gross domestic product and employs about one-third of the national workforce. However, it is undeniable that the agricultural sector faces very complex challenges, including supply fluctuations. Rice Tungro Disease (RTD) is one of the biggest threats to the global rice supply [1],[2],[3]. RTD is caused by a combined infection of two viruses: Rice Tungro Bacilliform Virus (RTBV) and Rice Tungro Spherical Virus (RTSV), which are transmitted by green leafhoppers as vectors. RTSV plays a key role in the transmission process, while RTBV plays a direct role in the appearance of disease symptoms. It can cause stunted rice growth, reduce the number of rice seedlings, and change the color of infected rice plants. Various strategies have been implemented to mitigate the impact of the Tungro virus on rice by preventing its spread and minimizing damage.

In rice plants, several vectors spread the Tungro virus; the main vector of this virus is the green leafhopper (a type of planthopper). Spread prevention can be done by using insecticides to suppress the planthopper population. Several experimental studies have demonstrated the effectiveness of specific insecticides, such as imidacloprid and buprofezin, in suppressing green leafhopper populations and reducing rice Tungro virus transmission [4]. However, using insecticides can harm the environment, human health, and the plants themselves [5],[6]. In addition, several studies have shown that excessive and continuous use of insecticides can make vectors resistant [7], so that the Tungro virus continues to spread. The optimization of rice production will be better if insecticides are combined with other methods, like natural predators, companion plants, pruning rice plants, etc. However, existing models largely assume constant control parameters, which limits their ability to capture real-world variability.

Mathematical modeling is a standard tool used to represent and analyze dynamic populations. A systematic review of the literature shows that the spread of the Tungro virus in rice plants is rarely performed by mathematical modeling [8]. The presence of a natural predator can reduce the number of vectors. It can minimize the spread of the virus. Some studies [9],[10], develop a mathematical model to describe the spread of the Tungro virus considering the presence of planthoppers, as their natural enemies, and the use of insecticides. Amelia et al. [11] construct a mathematical model similar to Ali, but add the planting of refugia as food for natural enemies. It shows that refugia plants can increase the natural predators so that the planthoppers can be significantly reduced as a virus vector. Conversely, Suandi et al. [12] have successfully developed the dynamical populations between planthoppers and rice plants without the natural enemies. However, their model was completed by pruning the rice plants and using insecticides to control the spread of the Tungro virus. Pruning can help break the Tungro virus cycle because infected plants can cause healthy planthoppers to become virus vectors.

Pruning and insecticides are the methods to control the spread of the Tungro virus, considering the situation and conditions in the field. Farmers can adjust the efficacy of pruning and insecticide use based on another parameter that influences

the dynamics of the Tungro virus. For illustration, some plants are infected with viruses but do not show symptoms of infection. In this case, rice plants become the resistant subpopulation, so pruning is unnecessary here. It means that the pruning intervention may not be helpful in some cases. What about insecticide use? The efficacy of insecticide use may decrease as the planthopper becomes resistant. Noviantri et al. [13] introduce the fuzzy parameter to describe the efficacy of pesticides in pest management. This research shows that the fuzzy parameter has a better interpretation for modeling the resistance.

Several previous studies have examined dynamic models of Tungro virus in rice plants and evaluated the effectiveness of insecticide-based control strategies and other agricultural methods. However, these studies still use control parameters, such as insecticide application and pruning, as constants. These models cannot represent the complexity of field conditions influenced by environmental factors such as insecticide dosage and virus load. This limitation highlights the need to develop more realistic and adaptive models. Therefore, the primary objective of this study is to develop and analyze a dynamic model of Tungro virus spread in rice plants, considering pruning and insecticide use as fuzzy parameters (which successful applied in some research [14], [15], [16]) dependent on the insecticide and virus load. The NSFD scheme will solve the fuzzy dynamical system, then some simulations will be carried out to get further analysis. Thus, this research is expected to provide methodological contributions to fuzzy-based mathematical modeling of plant diseases as well as practical insights into more flexible and sustainable control strategies.

2. THE DYNAMICAL SYSTEM WITH FUZZY PARAMETER

Suandi et al. [12] introduce the dynamical system model to represent the dynamics of Tungro virus spread in rice plants. This model consists of five populations: susceptible, infected, and virus-resistant rice plants (S_h, I_h, R_h), susceptible and infected planthoppers (S_v, I_v). In this paper, we consider the heterogeneity in the dynamical model by considering the pruning intervention rate δ and planthopper death rate γ as fuzzy membership functions of the virus load ω and insecticide load Ω , respectively. Thus, a new fuzzy dynamical system is:

$$\begin{aligned}
 dS_h/dt &= \Lambda - \beta_h S_h I_v - \mu_h S_h \\
 dI_h/dt &= \beta_h S_h I_v - (\delta(\omega) + \alpha + \mu_h) I_h \\
 dR_h/dt &= \alpha I_h - \mu_h R_h \\
 dS_v/dt &= \Pi - \beta_v S_v I_h - (\gamma(\Omega) + \mu_v) S_v \\
 dI_v/dt &= \beta_v S_v I_h - (\gamma(\Omega) + \mu_v) I_v
 \end{aligned} \tag{1}$$

where Λ and Π is the recruitment rate of plants and planthoppers, β_h and β_v infection rate due to interaction, μ_h and μ_v is the natural date rate of plants and planthoppers, and α is the proportion of plants resistant to Tungro virus. Here, ω represents the normalized virus load in the plant population, while Ω represents the

normalized insecticide concentration applied in the field. Hereinafter, the model (1) is called the fuzzy dynamical model.

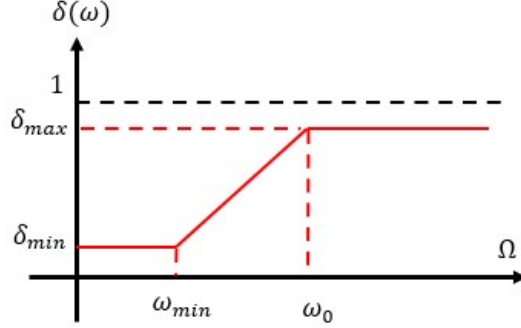


FIGURE 1. Fuzzy Membership Function $\delta(\Omega)$

2.1. Pruning Intervention. Pruning intervention effectiveness is affected by some parameters in the dynamic system [17]. If the rates of infected plants (β_h) increase, the pruning intervention must be increased because it can spread viruses into healthy planthopper. Furthermore, the facts show that some plants are infected with viruses but do not show infection symptoms. In this case, rice plants become the resistant subpopulation, so pruning is unnecessary here. The pruning intervention can be reduced when the proportion of plants resistant to the Tungro virus (α) and the natural death rate of the plants (μ_v) are increased. According to these analyses, the maximum plant reduction rate because of pruning intervention will be

$$\delta_{max} = \beta_h(1 - \alpha)(1 - \mu_h) + \delta_{min}. \quad (2)$$

This formulation assumes that pruning intensity increases proportionally with infection pressure and decreases with plant resistance and natural mortality.

Assume that if the number of Tungro virus loads is relatively low, the chance of pruning intervention is negligible. A minimum Tungro virus load is needed to perform the minimum pruning intervention (ω_{min}). In this condition, the rice plant reduction rate because of pruning becomes δ_{min} when the virus load does not meet the minimum criteria for pruning intervention ($\omega < \omega_{min}$). There is a certain amount of Tungro virus load, where the pruning rate is maximum or equal to one (ω_0). Furthermore, when $\omega_{min} < \omega < \omega_0$, the plant reduction because of pruning is proportional to virus load, which means that $\delta(\omega)$ will increase as virus load increases, up to δ_{max} . Mathematically, the plant reduction rate of based on the pruning intervention $\delta(\omega)$ can be interpreted in the following fuzzy membership function:

$$\delta(\omega) = \begin{cases} \delta_{min}, & \omega < \omega_{min}, \\ \delta_{min} + \frac{\omega - \omega_{min}}{\omega_0 - \omega_{min}} \delta^*, & \omega_{min} \leq \omega \leq \omega_0 \\ \delta_{max}, & \omega > \omega_0 \end{cases} \quad (3)$$

which can be described by Figure 1.

According to (3), we can determine the pruning load based on the desired maximum rice plant reduction rate using the basic concept of alpha-cut in fuzzy theory. Let P be the maximum expected rice plant reduction rate, then the pruning load used should be $\delta_P = \{\omega \in [\omega_{min}, \omega_0] | \delta(\omega) \leq P\}$, so that

$$\omega \leq \omega_{min} + \frac{1}{\delta^*}(P - \delta_{min})(\omega_0 - \omega_{min}). \quad (4)$$

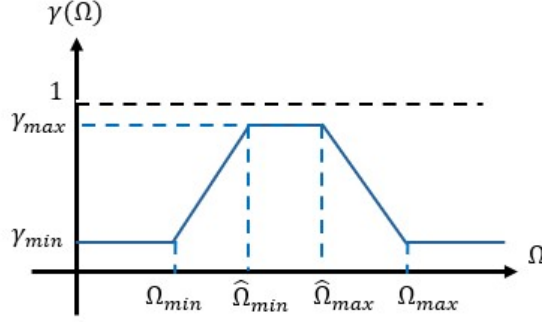
2.2. Insecticide. Insecticides are used to reduce the planthopper vector population that spreads viruses. Assume that the dynamic between the planthopper and the rice plant will influence the effectiveness of insecticides. Considering the virus load (proportional to insecticide load), the parameters γ can be treated as a function depending on the insecticide load Ω . In addition, the function $\gamma(\Omega)$ is developed by adjusting the insecticide load based on the dynamic behavior between the plant and the planthopper. If the recruitment rate (Π) and the infection rate (β_v) of the planthopper population increase, then the quantity of insecticide must be increased. Conversely, insecticide use can be reduced when the natural death rate of planthopper (μ_v) increases. Under these explanations, the maximum planthopper death rate because of insecticide use is

$$\gamma_{max} = \Pi\beta_v(1 - \mu_v) + \gamma_{min}. \quad (5)$$

Assume that the total virus load in the rice field is limited by Ω_{max} . Let Ω_{min} and Ω_{max} be a minimum and maximum insecticide load, respectively. When the minimum requirements for effective insecticide use ($\Omega < \Omega_{min}$) are not met by the insecticide load then the death rate of planthopper becomes γ_{min} . It is caused by the small chance of insecticide used, so it does not significantly affect the planthopper. In similar, it happens when the insecticide exceeds the maximum requirement for effective limit usage ($\Omega > \Omega_{max}$); this will cause the planthopper to be resistant to insecticide and develop immunity, making it unable to kill the planthopper [18], [19].

Furthermore, the planthopper is not resistant yet when $\Omega_{min} < \Omega < \hat{\Omega}_{min}$. In this case, the death rate of the planthopper will increase up to γ_{max} . When $\Omega = \hat{\Omega}_{min}$, the planthopper's biological system is at the end of its normal physiological response to the insecticide. Biologically, this condition indicates that almost all planthoppers susceptible to the insecticide have died from exposure to that dose. The remaining planthopper population becomes relatively homogeneous, dominated by those with higher physiological tolerance or initial potential for resistance. Thus, the number of planthopper deaths does not increase even if the insecticide dose is increased. By this reason, there is a range amount of insecticide load ($\hat{\Omega}_{min} < \Omega < \hat{\Omega}_{max}$), where the insecticide amount makes the planthopper death rate is maximum (γ_{max}) and equal to one.

On the other side, when $\Omega = \hat{\Omega}_{max}$, although the planthopper mortality rate remains at its maximum, the system is on the verge of a biological behavioral change. This condition indicates that the insecticide dose has exceeded the optimal level, causing the population to begin exhibiting adaptive responses, such

FIGURE 2. Fuzzy Membership Function $\gamma(\Omega)$

as increased physiological tolerance or initial resistance mechanisms. Further dose increases will no longer maintain this effectiveness but could potentially trigger the selection of more resistant individuals. This point marks the transition from optimal effectiveness to declining effectiveness due to resistance development. At this point, insecticide use begins to be counterproductive to the goal of pest control. By this explanation, the planthopper death rate decreases in $(\hat{\Omega}_{max} < \Omega < \Omega_{max})$. Mathematically, the planthopper death rate because of the insecticides usage $\gamma(\Omega)$ can be interpreted in the following fuzzy membership function:

$$\gamma(\Omega) = \begin{cases} \gamma_{min}, & \Omega < \Omega_{min}, \Omega > \Omega_{max} \\ \gamma_{min} + \frac{\Omega - \Omega_{min}}{\hat{\Omega}_{min} - \Omega_{min}} \gamma^*, & \Omega_{min} \leq \Omega \leq \hat{\Omega}_{min} \\ \gamma_{max}, & \hat{\Omega}_{min} < \Omega < \hat{\Omega}_{max} \\ \gamma_{max} - \frac{\Omega - \hat{\Omega}_{max}}{\Omega_{max} - \hat{\Omega}_{max}} \gamma^*, & \hat{\Omega}_{max} \leq \Omega \leq \Omega_{max} \end{cases} \quad (6)$$

which can be described by Figure 2.

Under the membership function (6), the insecticide load can be determined and adjusted to the desired minimum death rate of the planthopper using the fuzzy alpha cut concept. Let I be the minimum expected planthopper death rate, then the insecticide load used should be $\gamma_I = \{\Omega \in [\Omega_{min}, \Omega_{max}] | \gamma(\Omega) \geq I\}$, so that

$$\Omega_{min} + \frac{1}{\gamma^*} (I - \gamma_{min}) (\hat{\Omega}_{min} - \Omega_{min}) \leq \Omega \leq \hat{\Omega}_{max} + \frac{1}{\gamma^*} (\gamma_{max} - I) (\Omega_{max} - \hat{\Omega}_{max}). \quad (7)$$

2.3. Stability Criteria. Stability criteria for the fuzzy dynamical model (1) can be derived starting from its equilibrium point. Setting this dynamical system equal to zero, we can obtain equilibrium point $E = \{S_h^*, I_h^*, R_h^*, S_v^*, I_v^*\}$. There are two equilibrium points as follows:

$$E_1 = \left\{ \frac{\Lambda}{\mu_h}, 0, 0, \frac{\Pi}{\gamma(\Omega) + \mu_v}, 0 \right\}, \quad (8)$$

$$E_2 = \left\{ \frac{\Lambda}{c_1}, \frac{\beta_h \Lambda I_v^*}{c_1 c_2}, \frac{\alpha \beta_h \Lambda I_v^*}{c_1 c_2 \mu_h}, \frac{c_1 c_2 \Pi}{\beta_v \beta_h \Lambda I_v^* + c_1 c_2 (\gamma(\Omega) + \mu_v)}, I_v^* \right\}, \quad (9)$$

where

$$c_1 = \beta_h I_v^* + \mu_h, c_2 = \delta(\omega) + \alpha + \mu_h.$$

The equilibrium point E_1 describes the absence of infected plant and infected planthopper, while E_2 describes the endemic equilibrium point.

The stability criteria for each equilibrium point can be determined by solving the characteristic equation $\det(\lambda I - J) = 0$ in terms of λ as eigenvalues. The equilibrium point is stable when all eigenvalues are negative. By assessing the Jacobian matrix at E_1 , then the characteristic equation gives eigenvalues $\lambda_1 = \lambda_2 - \mu_h$ and $\lambda_3 = -\gamma(\Omega) - \mu_v$. The remaining eigenvalues, λ_4 and λ_5 can be determined by solving the following equation:

$$\lambda^2 + [c_2 + \mu_v + \gamma(\Omega)]\lambda + [\mu_v + \gamma(\Omega)]c_2 - \frac{\beta_h \beta_v \Lambda \Pi}{\mu_h (\gamma(\Omega) + \mu_v)} = 0. \quad (10)$$

The Equation (10) has negative eigenvalues when $c_2 \mu_h (\mu_v + \gamma(\Omega))^2 - \beta_h \beta_v \Lambda \Pi > 0$. By this result, the basic reproduction number becomes

$$\mathcal{R}_0 = \frac{\beta_h \beta_v \Lambda \Pi}{c_2 \mu_h (\mu_v + \gamma(\Omega))^2}. \quad (11)$$

This condition ensures that the basic reproduction mechanism is sufficiently suppressed by mortality effects. Moreover, the equilibrium point E_1 is stable when $\mathcal{R}_0 < 1$.

Next, assess the Jacobian matrix at E_2 , then the characteristic equation gives eigenvalues:

$$\lambda_1 = -\mu_h; \lambda_2 = -\mu_h - \alpha - \delta(\omega); \lambda_3 = \frac{-\beta_v \beta_h \Lambda I_v^*}{(c_1 c_2)} - \gamma(\Omega) - \mu_v, \quad (12)$$

and the remaining eigenvalues, λ_4 and λ_5 can be determined by solving the following equation:

$$\lambda^2 + \left(c_2 + \frac{\beta_h \beta_v \Pi I_h^*}{(c_1 + \gamma(\Omega))(\mu_v + \gamma(\Omega))} \right) \lambda + c_2 \mu_h - \frac{\beta_h^2 \beta_v \Lambda \Pi I_h^*}{c_1 (c_1 + \gamma(\Omega))(\mu_v + \gamma(\Omega))} = 0. \quad (13)$$

Since all parameters must be positive, then λ_1, λ_2 , and λ_3 are always negative. The last eigenvalues λ_4 and λ_5 will be negative if $c_1 c_2 \mu_h (\beta_v I_h^* + \mu_h + \gamma(\Omega))(\mu_v + \gamma(\Omega)) - \beta_h^2 \beta_v \Lambda \Pi I_h^* > 0$

3. NUMERICAL METHOD

This research applied the non-standard finite difference (NSFD) approximation to discretize the fuzzy dynamic model (1). It was chosen since NSFD is better than the standard finite difference method in accuracy and stability [20], [21]. Let the differential equation

$$df/dt = g(f(t), t, \varepsilon), \quad t \in [0, t_e], \quad (14)$$

where $f = \{S_h, I_h, R_h, S_v, I_v\}$, ε is a parameters set, t_e is the end of observation (time), and $g(f(t), t, \varepsilon)$ is a nonlinear term in the right hand side of (1). Then the discretization (14) by NSFD scheme gives us

$$\frac{f_{n+1} - f_n}{\phi(\Delta t)} = G_n(f_n, t_n, \varepsilon), \quad (15)$$

where f_n is the approximation of $f(t_n)$ and G_n is the discrete nonlinear term.

Considering to Gabbriellini [22], the nonlinear term can stated as

$$g(f(t), t, \varepsilon) = (-1)^p f^i h(f, \varepsilon), \quad (16)$$

where p and i is an integer and $h(f, \varepsilon)$ is an appropriate function, so that the discrete nonlinear term becomes

$$G(f_n, \dots, t_n, \varepsilon) = \frac{(-1)^p}{2} [(1 - (-1)^{p+1}) f_n^i + (1 - (-1)^p) f_n^{i-1} f_{n+1}] h_n(f_n, \varepsilon), \quad (17)$$

with $\phi(\Delta t)$ is a time step size as follows:

$$\phi(\Delta t) = \frac{1 - e^{-q\Delta t}}{q}; q \geq \max \left\{ \frac{\lambda^2}{2|Re(\lambda)|} \right\}, \quad (18)$$

where λ is the eigenvalue, which is obtained in Section 2.3 The parameter q is chosen to satisfy stability conditions of the NSFD scheme (see [20]).

Let $f_n = \{S_{h_n}, I_{h_n}, R_{h_n}, S_{v_n}, I_{v_n}\}$, then rewrite each nonlinear term fuzzy dynamical model (1) as (14) to determine p, i and function $h_n(f_n, \varepsilon)$ through Equation (16). Use these results to construct the discrete nonlinear term $G_n(f_n, t_n, \varepsilon)$ through (17). After completing these processes, then the NSFD scheme for (1) become

$$\begin{aligned} S_{h_{n+1}} &= [S_{h_n} + \Lambda\phi(\Delta t)] / [1 + (\beta_h I_{v_n} + \mu_h)\phi(\Delta t)] \\ I_{h_{n+1}} &= [I_{h_n} + \beta_h S_{h_n} I_{v_n} \phi(\Delta t)] / [1 + (\delta(\omega) + \alpha + \mu_h)\phi(\Delta t)] \\ R_{h_{n+1}} &= [R_{h_n} + \alpha I_{h_n} \phi(\Delta t)] / [1 + \mu_h \phi(\Delta t)] \\ S_{v_{n+1}} &= [S_{v_n} + \Pi\phi(\Delta t)] / [1 + (\beta_v I_{h_n} + \gamma(\Omega) + \mu_v)\psi(\Delta t)] \\ I_{v_{n+1}} &= [I_{v_n} + \beta_v S_{v_n} I_{h_n} \phi(\Delta t)] / [1 + (\gamma(\Omega) + \mu_v)\phi(\Delta t)]. \end{aligned} \quad (19)$$

4. RESULTS AND DISCUSSION

4.1. Numerical Results. The susceptible, infected, and resistant rice plants; susceptible and infected planthopper populations are approximated by the NSFD scheme (19) using input parameters taken from the previous research [23], [24]. Here $A = 1, \beta_h = 0.008, \mu_h = 0.033, \alpha = 0.6, \Pi = 1, \beta_v = 0.008, \mu_v = 0.12$, and initial condition $\{S_h(0), I_h(0), R_h(0), S_v(0), I_v(0)\} = \{75, 50, 0, 50, 10\}$. These parameter data satisfy the stability criteria of E_1 for equilibrium point E_1 . We perform three simulations to see the performance of each subpopulation related to pruning intervention and insecticide use.

The first simulation is conducted for $\delta(\omega) = 0$ and $\gamma(\Omega) = 0$, which is the population dynamic without any control. The simulation result is shown in Figure 3 (a). This figure only captures the number of each subpopulation over time until

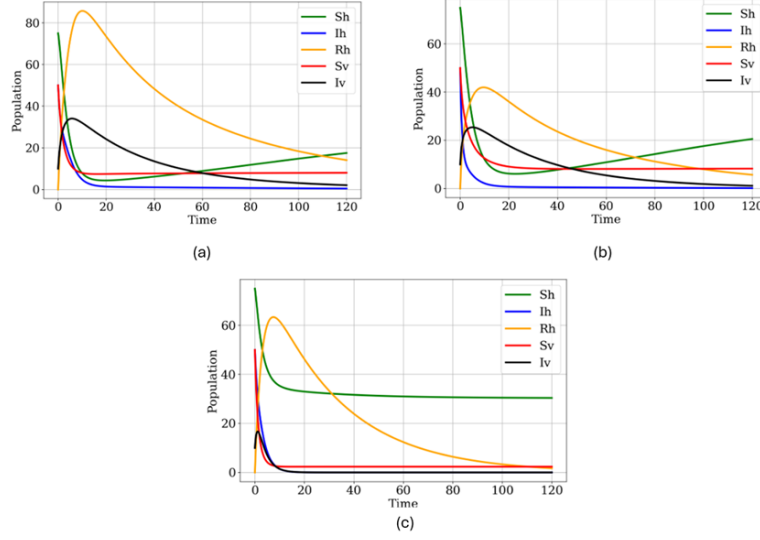


FIGURE 3. Time Evolution of Riceplant and Planthoppers under Different Control Strategies. (a) Without Pruning and Insecticide Intervention, (b) Pruning Control Only, (c) Insecticide use Only

$t = 120$ days. Without pruning and insecticide intervention: The resistant rice population (R_h) exhibits a pronounced outbreak, reaching a maximum peak of approximately 80–85 individuals around $t \approx 10$. The infected planthopper (I_v) also persists for a long duration with maximum value is 35 individuals, declining slowly and approaching near extinction after $t \approx 120$. This scenario represents sustained virus transmission with high peak infection and long persistence. Furthermore, the equilibrium point is $\{30.303; 0; 0; 8.334; 0\}$ which satisfy the steady-state Equations (8) and (10).

In the second simulation, we take into account pruning interventions without insecticide use $\gamma(\Omega) = 0$. Assume that $\delta_{min} = 0.2$, then by (2) we got $\delta(\omega) = \delta_{max} = 0.604$. In this situation, the dynamical population will shown in Figure 3 (b). Even though the population of infected plants only differs slightly, pruning is still able to reduce this population. The maximum R_h decreases to approximately 40–45 individuals, corresponding to a ≈ 45 –50% reduction compared with the no control case. The maximum infected planthopper (I_v) is 25 individuals and the extinction time reaches faster than no control case. Although pruning shortens the outbreak duration and lowers the infection peak, virus persistence is still observable over a moderate time horizon.

The last simulation was conducted to see how much the use of insecticides will affect the dynamical population due to the Tungro virus without pruning intervention $\delta(\omega) = 0$. Assume that $\gamma_{min} = 0.3$, then by (5) we get $\gamma(\Omega) = \gamma_{max} = 0.3071$. Figure 3 (c) plots the dynamical population over time. Based on this figure, we

can see that using insecticides can significantly reduce the number of susceptible and infected planthoppers than in the first and second simulations. The infected planthopper peak drops to approximately 15 individuals, yielding 60% reduction relative to the uncontrolled case. It declines rapidly and reaches near extinction by $t \approx 10$. This situation also can increase the number of healthy (S_h) and resistant (R_h) rice plants. This scenario demonstrates the most effective and rapid reduction of virus transmission among the three cases.

TABLE 1. Pruning Load and Fuzzy Parameter $\delta(\omega)$: Parameter Values are Selected to Represent Low, Medium, and High Intervention Scenarios

Simulation	ω_{min}	ω_0	ω	$\delta(\omega)$
A.1	0.3	0.7	0.1	0.2000
A.2	0.3	0.7	0.5	0.4020
A.3	0.3	0.7	0.9	0.6040
B.1	0.15	0.7	0.5	0.4571
B.2	0.3	0.7	0.5	0.4020
B.3	0.45	0.7	0.5	0.2808
C.1	0.3	0.55	0.5	0.5232
C.2	0.3	0.7	0.5	0.4020
C.3	0.3	0.85	0.5	0.3469

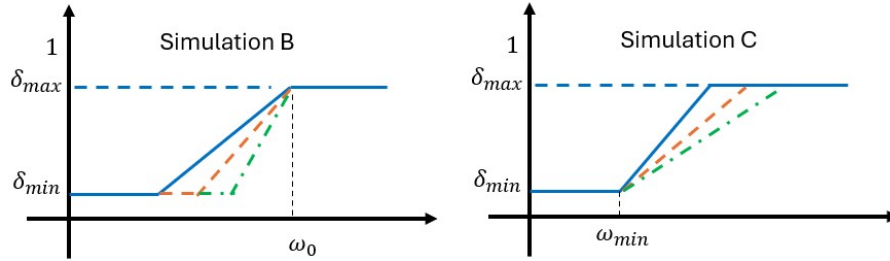


FIGURE 4. Fuzzy Parameter $\delta(\omega)$ for Simulation B and C

4.2. Pruning Intervention Load Simulation. To analyze the effect of the pruning intervention on the dynamical system, we conduct some simulations based on the parameter data in subsection 4.1. Assume that insecticide use is constant with $\gamma(\Omega) = 0.3$ and $\delta_{min} = 0.2$. There are three simulations to describe some variation of fuzzy parameter function $\delta(\omega)$, named simulations A, B, and C as in Table 1. Simulation A does not change the fuzzy membership function $\delta(\omega)$ since the

minimum and maximum pruning loads are fixed. Unlike simulations B and C, the fuzzy membership function $\delta(\omega)$ changes as in Figure 4 because the minimum and maximum pruning load change. In this figure, the blue, orange, and green lines related to Experiments 1,2 and 3 in Table 1, respectively. Next, the simulation results are shown in Figure 5. Each simulations comprises three subfigures describing the population of I_h , R_h , and I_v .

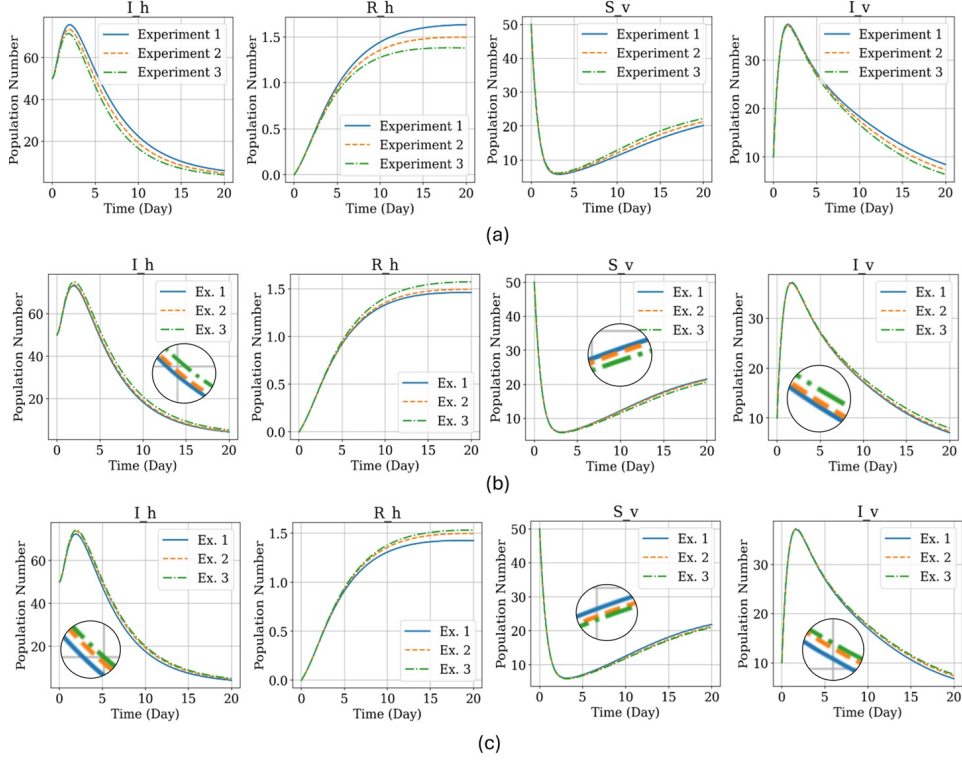


FIGURE 5. Pruning Load: (a) Simulation A Results, (b) Simulation B Results, (c) Simulation C Results

Simulation A consists of three experiments: A.1, A.2, and A.3 to represent different ω , taken from different fuzzy parameter subdomains (3). The population dynamic changes over time, as in Figure 5 (a). It can be seen that the number of infected and resistant plants decreases when the pruning load ω gets closer to ω_0 . When the number of infected plants decreases, the virus's chance of spreading from plants to the planthopper decreases, and the number of infected planthoppers also decreases. It can be concluded that more intensive pruning intervention can make the infected plant and infected planthopper extinct more quickly.

See Figure 4, Simulation B and C will change the parameter $\delta(\omega)$ so that the line slope in domain $[\omega_{min}, \omega_0]$ will be different. In this domain, a bigger ω_{min} leads

to a bigger line slope, but a bigger ω_0 leads to a smaller line gradient. However, these simulations give a smaller $\delta(\omega)$ for a bigger ω_{min} and ω_0 . The simulation results is shown in Figure 5 (b) and 5 (c). The number of infected subpopulation is slightly different for different experiments. However, Simulation B and C's results differ from Simulation A. Here, a bigger minimum pruning load (ω_{min}) leads to a larger population of infected and resistant plants and infected planthoppers.

In general, the simulations show that changes in the fuzzy pruning parameters affect the extinction rate of infected populations. This confirms that pruning is effective when implemented responsively to actual infection levels in the field. These biological implications emphasize that pruning is a control strategy that is highly dependent on timing and intensity.

4.3. Insecticide Load Simulation. As in Subsection 4.1, we perform some simulations to see how much the insecticide load affects the dynamical population model. The previous parameter data is still used here and assumes that the pruning load is a constant with $\delta(\Omega) = 0.1$. There are three simulations to describe some variation of parameter $\gamma(\Omega)$, named simulations D, E, and F which show the variation of Ω , Ω_{min} , and $\hat{\Omega}_{min}$, as in Table 2. When $\gamma_{min} = 0.3$, the maximum planthopper death rate due insecticide use is $\gamma_{max} = 0.604$. Table 2 shows that the value of $\gamma(\Omega)$ is more varied than $\delta(\Omega)$ in the previous subsection. Furthermore, simulation D does not change the fuzzy membership function $\gamma(\Omega)$ as only Ω is changed. Unlike simulations E and F, the fuzzy parameter $\gamma(\Omega)$ changes as in Figure 6. In this figure, the blue, orange, and green lines are related to Experiments 1, 2 and 3 in Table 2, respectively. Next, we will see how much the dynamical populations change by these simulations in Figure 7. These figures do not show the susceptible rice plant and planthopper population dynamic because insecticide use aims to reduce the Tungro virus spread because of the infected rice plant and planthopper

We performed five experiments in Simulation D to accommodate the insecticide load from different subdomains in (6). As in Table 2, Simulation D describes the fuzzy membership function, which has a symmetry graph. It is related to Figure 7(a), which shows that all subpopulations in Experiment 1 and 5 are the same. A similar explanation for Experiments 2 and 4. Furthermore, the number of I_v , I_h and R_h decreases when Ω gets closer to $[\hat{\Omega}_{min}, \hat{\Omega}_{max}] = [0.4, 0.6]$. In contrast, it caused the susceptible plant (S_h) to become larger. As in Figure 6, Simulations E and F, a bigger Ω_{min} and $\hat{\Omega}_{min}$ leads to a smaller line slope in the domain $[\Omega_{min}, \hat{\Omega}_{min}]$, so the value $u(\Omega)$ becomes smaller. It can be concluded that the number of infected plants and infected planthopper is proportional to Ω_{min} and $\hat{\Omega}_{min}$, but inversely proportional to susceptible plants as in Figures 7(b) and 7(c).

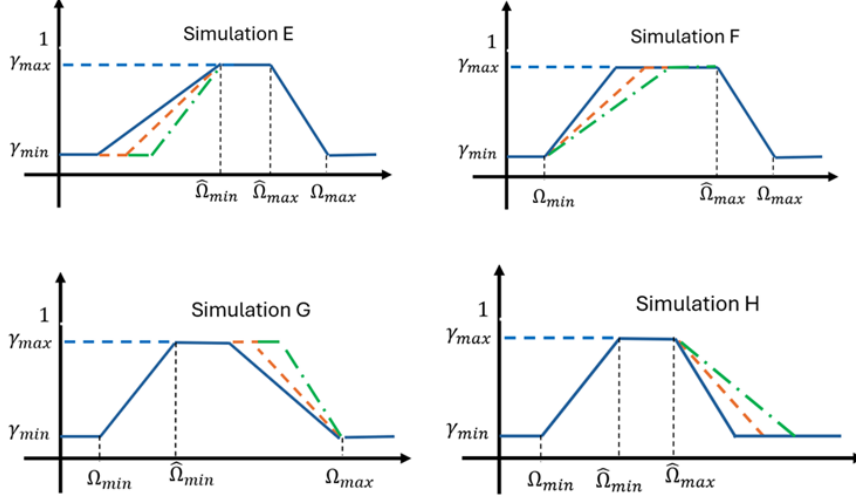


FIGURE 6. Fuzzy Membership Function $\gamma(\Omega)$ for Simulation E, F, G, and H

TABLE 2. Insecticide Load and Fuzzy Parameter $\gamma(\Omega)$: Parameter Values are Selected to Represent Low, Medium, and High Intervention Scenarios

Simulation	Experiment	Ω_{min}	$\hat{\Omega}_{min}$	$\hat{\Omega}_{max}$	Ω_{max}	Ω	$\gamma(\Omega)$
Simulation D (Different Ω)	1	0.2	0.4	0.6	0.8	0.1	0.3000
	2	0.2	0.4	0.6	0.8	0.3	0.4520
	3	0.2	0.4	0.6	0.8	0.5	0.6040
	4	0.2	0.4	0.6	0.8	0.7	0.4520
	5	0.2	0.4	0.6	0.8	0.9	0.3000
Simulation E (Different Ω_{min})	1	0.1	0.5	0.7	0.9	0.4	0.5376
	2	0.2	0.5	0.7	0.9	0.4	0.5112
	3	0.3	0.5	0.7	0.9	0.4	0.4584
Simulation F (Different $\hat{\Omega}_{min}$)	1	0.2	0.4	0.7	0.9	0.3	0.4584
	2	0.2	0.5	0.7	0.9	0.3	0.4056
	3	0.2	0.6	0.7	0.9	0.3	0.3792
Simulation G (Different $\hat{\Omega}_{max}$)	1	0.2	0.4	0.5	0.9	0.8	0.3792
	2	0.2	0.4	0.6	0.9	0.8	0.4056
	3	0.2	0.4	0.7	0.9	0.8	0.3792
Simulation H (Different Ω_{min})	1	0.2	0.4	0.5	0.7	0.6	0.4584
	2	0.2	0.4	0.5	0.8	0.6	0.5112
	3	0.2	0.4	0.5	0.9	0.6	0.5376

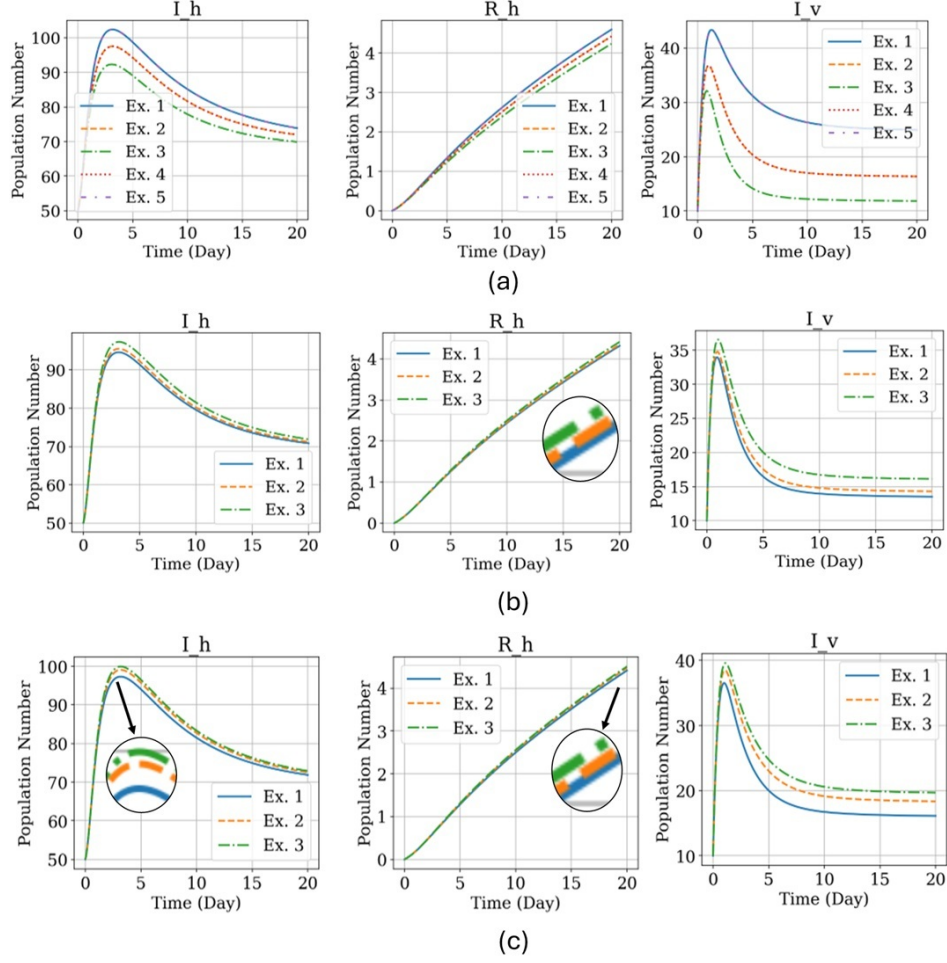


FIGURE 7. Insecticide Load: (a) Simulation D Results, (b) Simulation E Results, (c) Simulation F Results

Let Simulation G and H for variations of $\hat{\Omega}_{max}$ and Ω_{max} . Figure 6 shows that the function $\gamma(\Omega)$ in simulations E and H is like a reflection. In detail, the blue line in Simulation E reflects the green line in Simulation H, and the green line in Simulation E reflects the blue line in Simulation H. A similar explanation for Simulation F and G. For these reasons, the dynamic population results in Simulation G and H are not plotted.

Biologically, increasing the insecticide dose towards the optimal range caused a significant decline in the infected planthopper population. Consequently, virus transmission to rice plants also decreased. However, when the insecticide dose was below the minimum effective dose or exceeded the recommended maximum, the

planthopper mortality rate decreased, reflecting low exposure or the emergence of resistance. The decline in the infected planthopper population was accompanied by an indirect reduction in the number of infected plants, confirming the role of planthoppers as a key factor in the dynamics of the Tungro epidemic. These findings emphasize the importance of insecticide use at optimal and adaptive doses.

4.4. Sensitivity Analysis for Pruning Load. This section provides the sensitivity analysis for the parameters β_h , α , and μ_h . The parameter data in Subsection 4.1 is used here, but we add three different values of each β_h , α , and μ_h . Pruning load parameters taken from Table 1 for Simulation A - Experiment 2, $\delta_{min} = 0.2$, and $\gamma(\Omega) = 0$. By this analysis, we will see how much the infected rice plant and the planthopper change when these parameters are changed. The results are shown in Figures 8 - 10, which describe the dynamics of the infected subpopulation for various β_h , α , and μ_h , respectively. Here, the red and blue lines interpret the number population without pruning ($\delta(\omega) = 0$) and within pruning ($\delta(\omega) \neq 0$). The blue line is the absolute difference in the number population between red and green. Each figure consists of six subfigures: (a) - (c) shows the dynamic population of infected rice plant (I_h), and (d) - (f) shows the dynamic population of infected planthopper (I_v). In general, the variation in β_h , α , and μ_h gives the different value of $\delta(\omega)$ through Equation (3).

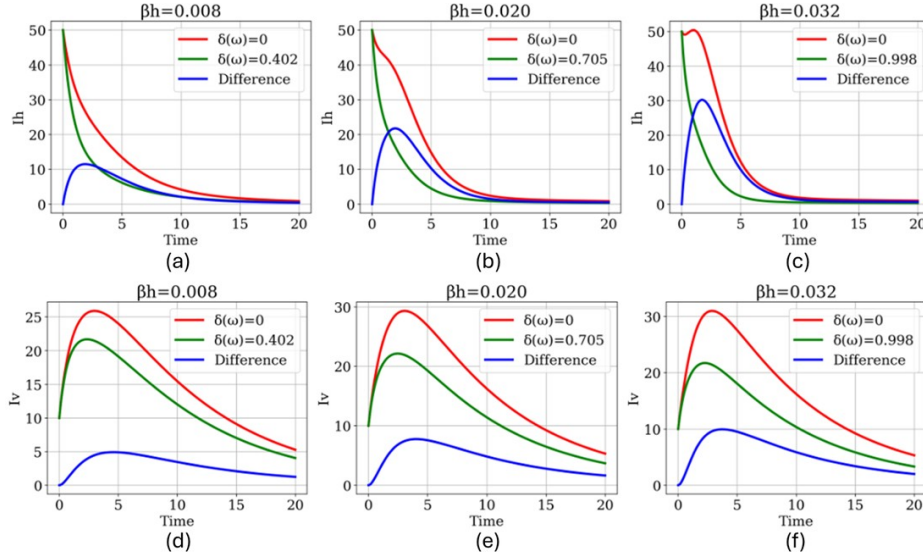


FIGURE 8. (a)-(c) Infected Plant, (d)-(f) Infected Planthopper in Various β_h

In Figure 8, the difference (blue line) in the infected population between the presence of pruning intervention and without pruning will increase as the β_h value increases. It occurs since the $\delta(\omega)$ increases too. Of course, the extinction of the

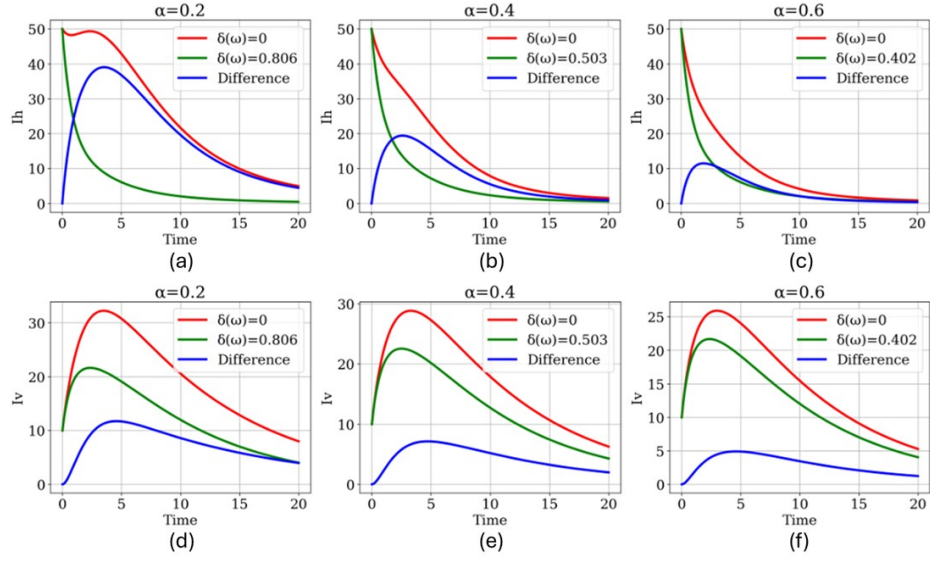


FIGURE 9. (a)-(c) Infected Plant, (d)-(f) Infected Planthopper in Various α

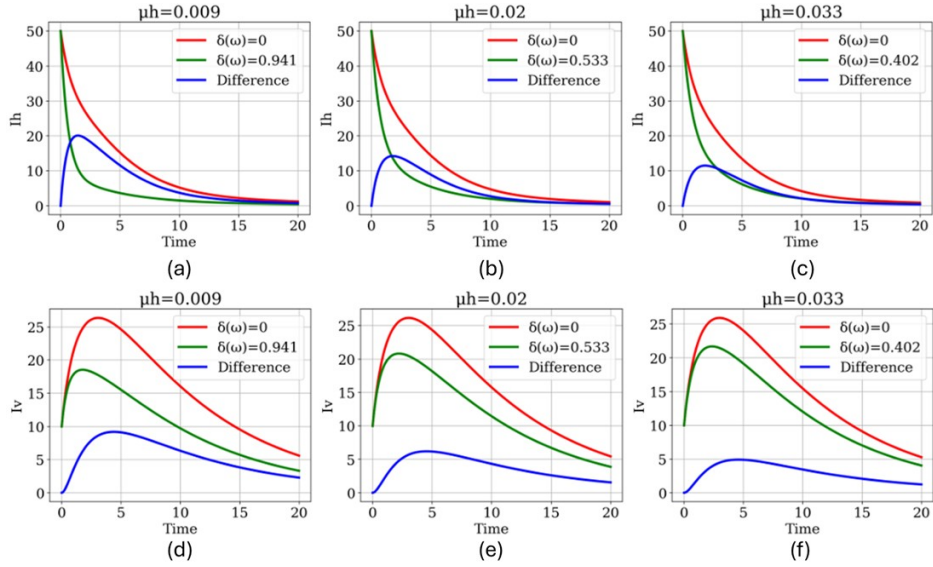


FIGURE 10. (a)-(c) Infected Plant, (d)-(f) Infected Planthopper in Various μ_h

infected population will happen more quickly than without pruning. The opposite situation will happen to the parameter α and μ_h , as in Figures 9 and 10. Since α and μ_h is inversely proportional to $\delta(\omega)$. The difference (blue line) in the infected population between the presence of pruning intervention and without pruning will decrease as the α and μ_h values increase. Furthermore, parameters β_h, α , and μ_h affected the number population of infected rice plants more significantly than the infected planthopper. This makes sense since the pruning intervention is an effort to reduce infected plants only. Based on Figure 8 - 10, the simulation results show that the parameters that influence the effectiveness of pruning intervention, from largest to smallest, are: β_h, μ_h, α .

Biologically, increasing β_h accelerates the accumulation of infected plants. This makes pruning more effective in breaking the cycle of virus transmission from plants to planthoppers. Conversely, increasing α and μ_h reduces the number of infected plants actively transmitting the virus, thus reducing the contribution of pruning to reducing the infected population. Furthermore, the impact of pruning is more dominant on the dynamics of infected plants than on planthoppers, which are indirectly affected by the reduction in virus reservoirs. These findings confirm that pruning is most effective when applied to conditions of high infection and low plant resistance. Therefore, this strategy needs to be adapted to actual field biological conditions

4.5. Sensitivity Analysis for Insecticide Load. Sensitivity analysis for insecticide load based on parameter Π, β_v , and μ_v are given here for three different values. Take $\gamma_{min} = 0.3, \delta(\omega) = 0$, while other parameters are same with the dataset in section 5.4. The results are in Figures 11 - 13, which the red and blue lines interpret the number population without insecticide ($\gamma(\Omega) = 0$) and within insecticide ($\gamma(\Omega) \neq 0$). The blue line is the absolute difference in the number population between red and green. All figures show that the use of insecticides in different scenarios can reduce the number of infected planthoppers and rice plants. However, the use of insecticides is more significant in infected planthopper than infected rice plant.

A higher planthopper recruitment rate (Π) leads to a higher insecticide load, so the extinction of the infected planthopper becomes quickly as in Figure 11 (d) - (f). However, this simulation shows that Π only makes small changes in infected rice plants as in Figure 11 (a) - (c). Similar to Π , a higher β_v leads to a higher insecticide load. In Figure 12 (a) - (c), the changes in the infected rice plant are more significant than in Figure 11 (a) - (c). Now, compare Figure 11 (d) - (f) and Figure 12 (d) - (f), which show that the changes in the infected planthopper is more significant by β_v than Π . This means that the parameter β_v has more impact than Π . For this reason, the amount of insecticide load is much better determined by β_v than Π .

In the opposite situation, the insecticide load should be reduced as the natural death of the planthopper (μ_v) increases. In Figure 13 (a)-(c), the difference between insecticide use and without insecticide are almost the same for the different μ_v . It means that the insecticide load based on parameter μ_v does not change the

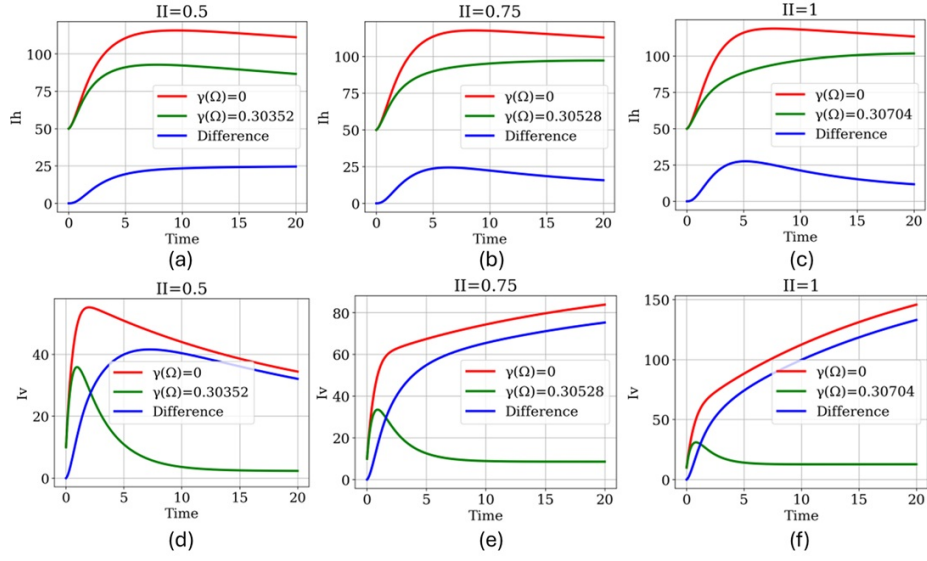


FIGURE 11. (a)-(c) Infected Plant in Various II , (d)-(f) Infected Planthopper in Various II

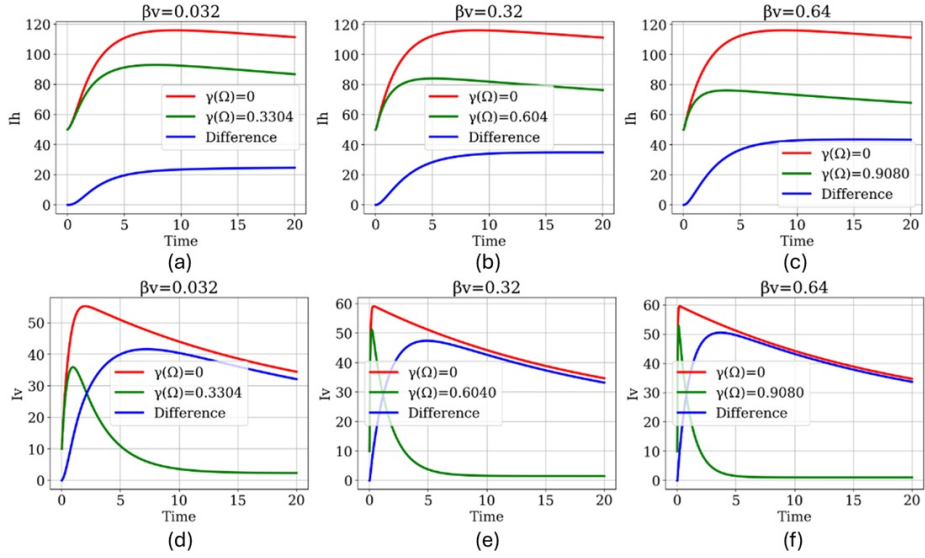


FIGURE 12. (a)-(c) Infected Plant in Various β_v , (d)-(f) Infected Planthopper in Various β_v

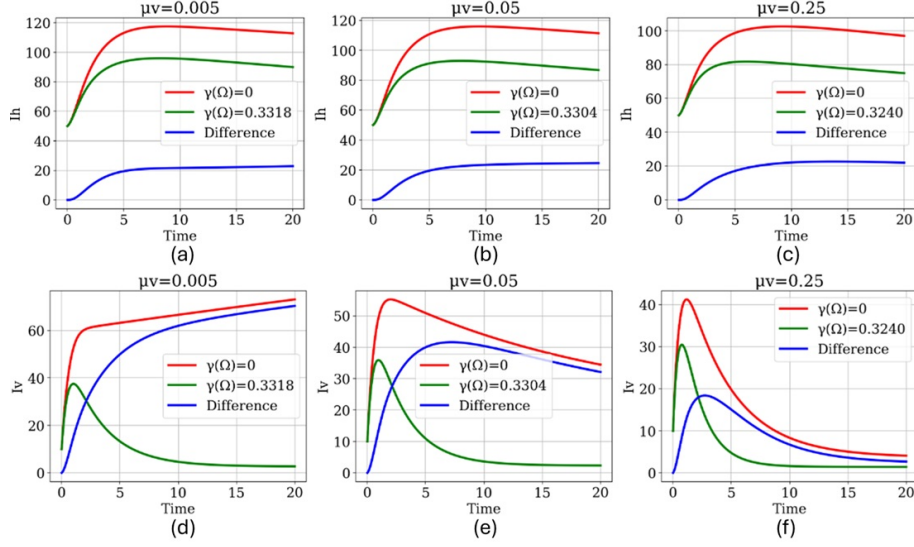


FIGURE 13. (a)-(c) Infected Plant, (d)-(f) Infected Planthopper in Various μ_v

infected rice plant significantly. However, this parameter give significant different for infected planthopper population as in Figure 13 (d) - (f). Based on Figure 14 - 16, the parameters that influence the effectiveness of insecticide use, from largest to smallest, are: β_v, μ_v, Π .

Biologically, the increase in β_v and Π reflects high activity and population growth of planthoppers. This makes insecticide use more extensive and effective in reducing infected planthopper populations and accelerating their extinction. Conversely, the increase in μ_v naturally suppresses planthopper populations, so the additional effect of insecticides on infected plants is relatively small. Simulation results also show that insecticides have a more direct impact on infected planthopper populations than on infected plants. These findings confirm that insecticide strategies are vector-focused control, most effectively applied when planthopper population dynamics are high, and should be tailored to the biological characteristics of planthoppers to avoid overuse that could potentially trigger resistance.

5. CONCLUSION

This paper successfully describes the heterogeneity of the dynamical system by considering the pruning intervention rate and planthopper death rate as fuzzy membership functions of the virus load and insecticide load, respectively. The pruning membership function is depicted right trapezoidal since the resistance plant does not show infection symptoms, so pruning is unnecessary here. The membership function is depicted as regular trapezoidal for the insecticide to describe the possibility of resistant planthoppers when the insecticide load pass over the effective

insecticide load. By the fuzzy parameter, we may examine the requirement of virus and insecticide load to produce the desired minimum planthopper death rate or the expected time to achieve the extinction of planthopper. The simulation results show that the combination of pruning and the usage of insecticides can reduce the number of infected plants and planthoppers up to 60%, also speed up planthopper extinction up to 20 times faster. This reduction is computed by comparing peak infected populations across simulations. Furthermore, the dynamical parameters that influence the pruning effectiveness, from largest to smallest, are infection rate due to interaction with the infected plant hopper, natural date rate of the plant, and the proportion of plants resistant to the Tungro virus. The dynamic parameters that influence the insecticide effectiveness from largest to smallest are infection rate due to interaction with infected plant, natural date rate of planthopper, and the recruitment rate of planthopper. Our results can help farmers determine the quantity of insecticide and pruning that is adjusted to field conditions so that it is optimal. However, it would be great if this model is complemented with further analysis of the basic reproduction number for deeper epidemiological interpretation.

Data Availability Statement The authors confirm that the data supporting the findings of this study are available in the article. The Python source code utilized in this study is publicly accessible. The code is accessible via GitHub at <https://github.com/viska-noviantri/fuzzy-tungro/blob/main/fuzzy%20insecticide>, and may be downloaded free of charge.

Author Contributions. **Viska Noviantri:** Writing - original draft, review and editing, methodology, investigation, formal analysis, conceptualization, validation, visualization. **Alvin Harjadi:** method, investigation, software, data curation. **Dani Suandi:** Writing - review and editing, conceptualization, validation, and visualization. All authors discussed the results and contributed to the final manuscript.

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