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Soft S-Paracompact Space

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Abstract. This paper introduces a new topological class called soft S-paracompact spaces. These spaces generalize the concept of soft paracompact spaces. A soft topological space is considered soft S-paracompact if every soft open cover has a locally finite soft semi-open refinement. We explore the key properties of soft S-paracompact spaces and investigate their relationships with other well-established soft topological spaces. We depict an application of soft S-paracompactness in the decision-making problem.

Key words and phrases: soft set, soft semi-open set, soft semi-closed set, soft topology, soft paracompactness.

1. INTRODUCTION

In 1965, Zadeh [1] presented the idea of fuzzy set theory, which has since grown to be a useful mathematical tool for expressing uncertainty and has been essential in decision-making-related problem-solving. Molodtsov[2] introduced the novel concept of soft set theory in 1999. Soft set theory extends fuzzy set theory designed to address uncertainty issues parametrically. The "soft set" refers to a parameterized family of sets where the parameter defines the soft set's boundary. Despite the limitations of existing theories such as probability theory, rough sets, vague sets, and fuzzy sets, Molodtsov pointed out that these can be effectively utilized to manage uncertainty problems. Furthermore, Molodtsov presented the initial findings of soft set theory, which is not restricted by the limitations above and can handle uncertainty-related problems. Maji[3] and colleagues also applied soft set theory using rough techniques in decision-making scenarios. Later, in 2011, Shabir and Naz [4] defined soft topology and introduced a set of parameters on the initial universe. The concepts of soft open sets, soft closed sets, soft closure, soft interior points, soft neighborhoods of a point, and soft separation axioms were

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established, and their fundamental characteristics were examined. Subsequently, in the same year, Camgma and colleagues [5, 6] defined soft topology within the context of a soft set and explored its associated properties, thereby laying the groundwork for the theory of soft topological spaces.

Levin [7] introduced the concept of the semi-open set in topological space. Al-Zoubi [8] delved into the features of the S-paracompact space in the topological space, characterized the S-paracompact spaces with extremely disconnected [9] and S-closed space[10, 11], and also studied basic properties of S-paracompact space. In 1980, Ergun [12], introduced nearly paracompactness in topological space. In 2010, Baiju and Sunil [13] studied the behavior of various types of noncompact covering properties such as paracompactness, metacompactness, subparacompactness, submetacompactness under various types of fuzzy mappings such as open maps, closed maps, and perfect maps in L-topological spaces. In 2012, Chen [14] defined soft semi-open sets, soft semi-closed sets, soft semi-interior, and soft semi-closure in soft topological space and examined the characterizations of soft semi-open sets, soft semi-closed sets, soft semi-interior, and soft semi-closure in soft topological space. In 2013, Lin [15] defined soft paracompact and explored its characteristics in soft topological space.

In the present paper, we studied soft S-locally finite, soft S-refinement, soft extremely disconnected space, soft S-expandable space, and soft S-closed spaces in soft topological space. In Section 3, we introduce and study the concept of soft S-paracompact spaces within soft topological spaces. Soft S-paracompact defines every soft open cover of a soft topological space as having a locally finite semi-open refinement. We prove some characterizations of soft S-paracompact space and investigate the relation between soft S-paracompact space and some well-known spaces. In Section 4, we define soft αS-paracompact space, soft sg-closed space, soft θ-open set, soft θ-closed set, soft θ_s-open set, and soft θ_s-closed set in soft topological space. Section 5 studies some basic properties of soft S-paracompact space, i.e., subspace, sum, and product. Section 6 discusses the benefits and drawbacks of soft S-paracompactness in relation to different soft topological spaces. In the last section, we estimate the application of soft S-paracompactness in decision-making problems.

2. PRELIMINARY

This section provides a foundation for our work by reviewing relevant existing research. We discuss key concepts and findings related to a fixed set of parameters, which will serve as a basis for exploring our contributions.

Let $\tilde{U}$ represent the universal set and E denote the collection of parameters concerning $\tilde{U}$, where parameters represent the properties or characteristics of objectives $\tilde{U}$. Consider $\tilde{A} \subseteq E$ and $\mathcal{P}(\tilde{U})$ to be the power set of $\tilde{U}$. (F, E) denotes the soft set. Let $(\tilde{U}, \tilde{\tau}, E)$ be the soft topological space, where $\tilde{\tau}$ be the collection of soft sets. The soft closure, soft interior, and relative soft topological space are denoted by $Cl(F,\tilde{A})$, $Int(F,\tilde{A})$ and $\tilde{\tau}_{\tilde{A}}$, respectively. The soft semi-closure and soft semi-interior are denoted by $Cl_s(F,\tilde{A})$ and $Int_s(F,\tilde{A})$ respectively.

Definition 2.1. [2] *A soft set over $\tilde{U}$ is a pair (F,E), where F is a function defined as $F : E \to \mathcal{P}(\tilde{U})$. In other words, a soft set over $\tilde{U}$ can also be described as a parameterized family of subsets of the universe $\tilde{U}$. For any $e \in E$, $F(e)$ represents the set of e-approximate elements of the soft set (F,E).*

Example 2.2. *The collection $\tilde{U} = \{a,b,c,d,e\}$ represents a group of five motorcycles being analyzed. In contrast, the set $E = \{e_1,e_2,e_3\}$ encompasses the parameters of interest, which are top speed (e_1), fuel efficiency (e_2), and weight (e_3) of each motorcycle. To characterize these motorcycles, a soft set (F,E) is established. This soft set $(F,\tilde{A})$, where $\tilde{A} = E$, is defined by $F(e_1) = \{a,d,e\}$, $F(e_2) = \{b,c,e\}$, and $F(e_3) = \{b,c,d\}$. This formulation provides a representation of the motorcycle's characteristics. The parameterized family $\{F(e_i) : i = 1,2,3\}$ associated with the motorcycles in the set $\tilde{U}$ is identified as the soft set $(F,\tilde{A})$, which offers various approximations for describing the objects.*

Definition 2.3. [3] *Let $(F,\tilde{A})$ and $(G,\tilde{B})$ represent two soft sets defined over a common universal set $\tilde{U}$, we say that $(F,\tilde{A})$ is a subset of $(G,\tilde{B})$, If $\tilde{A} \subseteq \tilde{B}$ and for every element $e \in \tilde{A}$, the approximations provided by $F(e)$ and $G(e)$ are the same.*

Definition 2.4. [5] *Let (F,E) represent a soft set defined over $\tilde{U}$. The collection of all subsets of the soft set (F,E) is known as the soft power set of (F,E).*

$$|\mathcal{P}(F,E)| = 2^{\sum_{e\in E}|F(e)|}$$

where $|F(e)|$ is a cardinal of $F(e)$.

Example 2.5. *Let us consider the $\tilde{U} = \{a_1,a_2,a_3,a_4\}$ universal set, and $E = \{e_1,e_2\}$ is the collection of parameters and $\tilde{A} = E$. Then the soft set $(F,\tilde{A})$ is defined as, $(F,\tilde{A}) = \{(e_1,\{a_1\}),(e_2,\{a_1,a_2\})\}$ and the power set of a soft set $(F,\tilde{A})$ are given as follows:*
$(F,\tilde{A}_1) = \{(e_1,\{a_1\})\}$
$(F,\tilde{A}_2) = \{(e_1,\{a_1\}),(e_2,\{a_1\})\}$
$(F,\tilde{A}_3) = \{(e_1,\{a_1\}),(e_2,\{a_2\})\}$
$(F,\tilde{A}_4) = \{(e_2,\{a_1\})\}$
$(F,\tilde{A}_5) = \{(e_2,\{a_2\})\}$
$(F,\tilde{A}_6) = \{(e_2,\{a_1,a_2\})\}$
$(F,\tilde{A}_7) = \{(e_1,\{a_1\}),(e_2,\{a_1,a_2\})\} = (F,\tilde{A})$
$(F,\tilde{A}_8) = \phi = (F,\tilde{A}_\phi)$

Definition 2.6. [3] *Let us consider the soft set $(F,\tilde{A})$ defined over the universe $\tilde{U}$. The complement of this soft set is denoted as $(F,\tilde{A})^c$, which is expressed as $(F,\tilde{A})^c = (F^c,\neg\tilde{A})$. Here $F^c : \neg\tilde{A} \longrightarrow \mathcal{P}(\tilde{\mathcal{U}})$ represents a mapping defined by $F^c(\neg e) = \tilde{U} - F(e)$ for every $\neg e \in \neg\tilde{A}$.*

Definition 2.7. [3] *Let us consider two soft sets $(F,\tilde{A})$ and $(G,\tilde{B})$ over the universe $\tilde{U}$. The union of these two soft sets over the common universe $\tilde{U}$ is a soft set*

$(H,\tilde{C})$, where $\tilde{C}=\tilde{A}\cup\tilde{B}$ and every element $e\in\tilde{C}$.

$$H(e)=\begin{cases}F(e), & \text{if } e\in A-B\\ G(e), & \text{if } e\in B-A\\ F(e)\cup G(e), & \text{if } e\in A\cap B\end{cases}$$

then $(F,\tilde{A})\cup(G,\tilde{B})=(H,\tilde{C})$.

Definition 2.8. [3] *Let us consider two soft sets $(F,\tilde{A})$ and $(G,\tilde{B})$ over the universe $\tilde{U}$. The intersection of these two soft sets over common universe $\tilde{U}$ is a soft set $(H,\tilde{C})$, then $\tilde{C}=\tilde{A}\cap\tilde{B}$ and $F(e)\cap G(e)=H(e)$ for every element $e\in\tilde{C}$, we have $(F,\tilde{A})\cap(G,\tilde{B})=(H,\tilde{C})$.*

Definition 2.9. [15] *Let $(F,\tilde{A})$ represent a soft set defined over the universe $\tilde{U}$. Then $(F,\tilde{A})$ is said to be an absolute soft set if $e_i\in\tilde{A}$ and $F(e_i)=\tilde{U}$. The notation used for absolute soft set is $\tilde{U}_{\tilde{A}}$.*

Note 2.1. *To facilitate our discussion, let us consider a soft set (F,E) defined on a universe $\tilde{U}$. The family of soft set defined over $\tilde{U}$ is denoted by $SS(\tilde{U})_E$.*

Definition 2.10. [4] *Let $(F,\tilde{A})$ be the soft sets over $\tilde{U}$. A soft topology on $(F,\tilde{A})$ is represented by $\tilde{\tau}$, which consists collection of subsets of $(F,\tilde{A})$ satisfying the following conditions.*

i. $(F,\tilde{A}_\phi),(F,\tilde{A})\in\tilde{\tau}$.
ii. *The arbitrary union of elements from $\tilde{\tau}$ belongs to $\tilde{\tau}$.*
iii. *The intersection of a finite number of soft sets belonging to $\tilde{\tau}$ is also contained within $\tilde{\tau}$.*

Then $(\tilde{U},\tilde{\tau},E)$ is called a soft topological space.

Example 2.11. [5] *Let us consider $\tilde{U}=\{a,b,c,d\}$ is the universal set and $E=\{e_1,e_2\}$ is the collection of parameters and $\tilde{A}=E$. Then the soft set $(F,\tilde{A})$ is defined as, $(F,\tilde{A})=\{(e_1,\{a,b\}),(e_2,\{b,c\})\}$. Then the power set of a soft set $(F,\tilde{A})$ is given as,*
$(F,\tilde{A}_1)=\{(e_1,\{a\})\}$,
$(F,\tilde{A}_2)=\{(e_1,\{b\})\}$,
$(F,\tilde{A}_3)=\{(e_1,\{a,b\})\}$,
$(F,\tilde{A}_4)=\{(e_2,\{b\})\}$,
$(F,\tilde{A}_5)=\{(e_2,\{c\})\}$,
$(F,\tilde{A}_6)=\{(e_2,\{b,c\})\}$,
$(F,\tilde{A}_7)=\{(e_1,\{a,b\}),(e_2,\{c\})\}$,
$(F,\tilde{A}_8)=\{(e_1,\{b\}),(e_2,\{b,c\})\}$,
$(F,\tilde{A}_9)=\{(e_1,\{b\}),(e_2,\{b\})\}$,
$(F,\tilde{A}_{10})=\{(e_1,\{a\}),(e_2,\{b,c\})\}$,
$(F,\tilde{A}_{11})=\{(e_1,\{b\}),(e_2,\{c\})\}$,
$(F,\tilde{A}_{12})=\{(e_1,\{a\}),(e_2,\{c\})\}$,
$(F,\tilde{A}_{13})=\{(e_1,\{a,b\}),(e_2,\{b\})\}$,

$(F, \tilde{A}_{14}) = \{(e_1, \{a\}), (e_2, \{b\})\}$,
$(F, \tilde{A}_{15}) = \{(e_1, \{a, b\}), (e_2, \{b, c\})\} = (F, \tilde{A})$,
$(F, \tilde{A}_{16}) = \{\phi\} = (F, \tilde{A}_{\phi})$ *(which is called null soft set).*
Let us consider the collection of soft subsets of $(F, \tilde{A})$ *is denoted by* $\tilde{\tau}$ *and it is defined as* $\tilde{\tau} = \{(F, \tilde{A}_{\phi}), (F, \tilde{A}), (F, \tilde{A}_2), (F, \tilde{A}_{11}), (F, \tilde{A}_{13})\}$. *Consequently, the structure* $(\tilde{U}, \tilde{\tau}, E)$ *is identified as a soft topological space.*

Definition 2.12. [5] *Let* $(\tilde{U}, \tilde{\tau}, E)$ *is a soft topological space. The elements of* $\tilde{\tau}$ *are called soft open sets. If the complement of a soft set is a soft open set, it is called a soft closed set.*

Definition 2.13. [14] *The soft set* $(F, E) \in SS(\tilde{U})_E$ *is called a soft point in* $\tilde{U}_E$ *if there exist elements* $x \in \tilde{U}$ *and* $e \in E$ *that satisfy the following conditions:* $F(e) = \{x\}$*, and for every* $e' = E - \{e\}$, $F(e') = \phi$. *The notation* e_x *is used to represent the soft point* (F, E).

Definition 2.14. [14] *Let* $(\tilde{U}, \tilde{\tau}, E)$ *is the soft topological space and* $(F, \tilde{A})$ *is the soft set in* $(\tilde{U}, \tilde{\tau}, E)$. *If* $(F, \tilde{A})$ *is said to be soft semi-open set if and only if there exists a soft open set* $(F, \tilde{O})$ *, such that* $(F, \tilde{O}) \subset (F, \tilde{A}) \subset Cl(F, \tilde{O})$.

Definition 2.15. [14] *Let* $(\tilde{U}, \tilde{\tau}, E)$ *is the soft topological space and* $(F, \tilde{A})$ *is the soft set in* $(\tilde{U}, \tilde{\tau}, \tilde{\mathcal{E}})$. *If* $(F, \tilde{A})$ *is said to be soft semi-closed set if and only if there exists a soft closed set* $(F, \tilde{D})$*, such that* $Int(F, \tilde{D}) \subset (F, \tilde{A}) \subset (F, \tilde{D})$.

The relative complement of soft semi-open set is called as soft semi-closed set.

Definition 2.16. [16] *Consider the soft topological space* $(\tilde{U}, \tilde{\tau}, E)$. *Let* $(F, \tilde{A})$ *is a soft set over* $\tilde{U}$. *Then the soft semi-interior of a soft set* $(F, \tilde{A})$ *on* $\tilde{U}_E$ *is defined as the union of all soft semi-open sets contained within* $(F, \tilde{A})$.

Definition 2.17. [16] *Consider the soft topological space* $(\tilde{U}, \tilde{\tau}, E)$. *Let* $(F, \tilde{A})$ *is a soft set over* $\tilde{U}$. *Then the soft semi-closure of* $(F, \tilde{A})$ *over* $\tilde{U}_E$ *is defined as the intersection of all soft semi-closed set containing* $(F, \tilde{A})$.

Definition 2.18. [17] *If the pair* $(F, \tilde{A})$ *is classified as soft regular open and soft regular closed, it is defined by the conditions* $(F, \tilde{A}) = Int(Cl(F, \tilde{A}))$ *for soft regular open and* $(F, \tilde{A}) = Cl(Int((F, \tilde{A}))$ *for soft regular closed.*

Definition 2.19. [18] *A soft topological space* $(\tilde{U}, \tilde{\tau}, E)$ *is considered soft almost regular if, for any soft regular closed set* $(F, \tilde{A})$ *and any soft point* e_x *not contained in* $(F, \tilde{A})$*, there exist separate soft open sets that contain* $(F, \tilde{A})$ *and* e_x *respectively.*

In a soft topological space $(\tilde{U}, \tilde{\tau}, E)$, the collections of all subsets that are soft semi-open, soft regular open, soft semi-closed, and soft regular closed are represented by $SSO(\tilde{U}, \tilde{\tau}, E)$, $SRO(\tilde{U}, \tilde{\tau}, E)$, $SSC(\tilde{U}, \tilde{\tau}, E)$, and $SRC(\tilde{U}, \tilde{\tau}, E)$, respectively. It is established that for such a space, the set $SRO(\tilde{U}, \tilde{\tau}, E)$ forms a base for a soft topology $\tilde{\tau}_S$ on $\tilde{U}$, where $\tilde{\tau}_S \subseteq \tilde{\tau}$. The resulting soft topological space

$(\tilde{U}, \tilde{\tau_S}, E)$ is referred to as the soft semi-regularization of $(\tilde{U}, \tilde{\tau}, E)$[18].

Definition 2.20. [19] *Consider the soft topological space $(\tilde{U}, \tilde{\tau}, E)$ and let $(F, \tilde{A})$ is said to be the soft neighborhood of the soft set $(F, \tilde{B})$ if there exists a soft open set $(F, \tilde{O})$ such that $(F, \tilde{B}) \subseteq (F, \tilde{O}) \subseteq (F, \tilde{A})$.*

Definition 2.21. [16] *Let $(\tilde{U}, \tilde{\tau}, E)$ is the soft topological space. $(F, \tilde{A})$ is the soft set over $\tilde{U}$ and $e_x \in \tilde{U}_E$. Consequently, $(F, \tilde{A})$ is termed as a soft semi neighborhood of e_x if there exist soft semi-open set $(F, \tilde{O})$ such that $e_x \in (F, \tilde{O}) \subseteq (F, \tilde{A})$.*

Definition 2.22. [20] *Let $(\tilde{U}, \tilde{\tau}, E)$ is the soft topological space. If it is called soft expandable if for each soft locally finite collection $(F, \tilde{C}) = \{C_\alpha; \alpha \in \mathbb{I}\}$ of subset of $\tilde{U}_E$, there is a soft locally finite collection $(F, \tilde{K}) = \{K_\alpha; \alpha \in \mathbb{I}\}$ of soft open subsets of $\tilde{U}_E$ satisfying for each $\alpha \in \mathbb{I}$ that $K_\alpha \subseteq C_\alpha$.*

Definition 2.23. [21] *Let $(\tilde{U}, \tilde{\tau}, E)$ is the soft topological space and $(F, \tilde{A}) \subseteq \tilde{U_E}$. Then $(F, \tilde{A})$ is said to be the soft preopen set if $(F, \tilde{A}) \subset Int(Cl(F, \tilde{A}))$. Soft preopen set is denoted by $SPO(\tilde{U}, \tilde{\tau}, E)$ and if $(F, \tilde{A})$ is said to be soft semi preopen set if $(F, \tilde{A}) \subset Cl(Int(F, \tilde{A}))$, is denoted as $SPO_s(\tilde{U}, \tilde{\tau}, E)$.*

Definition 2.24. [22] *Let $(\tilde{U}, \tilde{\tau}, E)$ is the soft topological space and $(F, \tilde{A})$ be the soft set is called soft α-open set if $(F, \tilde{A}) \subset Int(Cl(Int(F, \tilde{A})))$. The complement of soft α-open set is called the soft α-closed set.*

Definition 2.25. [15, 23] *Consider a soft topological space $(\tilde{U}, \tilde{\tau}, E)$. A collection $(F, \tilde{C}) = \{C_\alpha; \alpha \in \mathbb{I}\}$ of subset of $(\tilde{U}, \tilde{\tau}, E)$ is said to be soft locally finite (or soft S-locally finite) if for every $e_x \in \tilde{U}_E$, there exists $(F, \tilde{O}) \in \tilde{\tau}$ (or $(F, \tilde{O}) \in SSO(\tilde{U}, \tilde{\tau}, E)$) that contains e_x and intersects with only a finite number of elements from $(F, \tilde{C})$.*

Lemma 2.26. [20] *Let $(\tilde{U}, \tilde{\tau}, E)$ is the soft topological space. Consider the collection $(F, \tilde{C}) = \{C_\alpha; \alpha \in \mathbb{I}\}$ of soft subsets of the soft topological space $(\tilde{U}, \tilde{\tau}, E)$. Then $(F, \tilde{C})$ is soft S-locally finite if and only if $\{Cl_s(C_\alpha); \alpha \in \mathbb{I}\}$ is soft S-locally finite.*

Lemma 2.27. [20] *Let $(\tilde{U}, \tilde{\tau}, E)$ is the soft topological space. Consider the collection $(F, \tilde{C}) = \{C_\alpha; \alpha \in \mathbb{I}\}$ of soft subsets of the soft topological space $(\tilde{U}, \tilde{\tau}, E)$. If $(F, \tilde{C})$ is soft locally finite, then $\bigcup_{\alpha \in \mathbb{I}} Cl_s(C_\alpha) = Cl_s(\bigcup_{\alpha \in \mathbb{I}} C_\alpha)$.*

It is clear that a family $(F, \tilde{C}) = \{C_\alpha; \alpha \in \mathbb{I}\}$ of subsets within a soft topological space $(\tilde{U}, \tilde{\tau}, E)$ is classified as soft locally finite if and only if the family $\{Cl_s(C_\alpha); \alpha \in \mathbb{I}\}$ is also soft locally finite.

Definition 2.28. [20] *Let $(\tilde{U}, \tilde{\tau}, E)$ is the soft topological space is said to be soft S-expandable if, for every soft S-locally finite collection $(F, \tilde{C}) = \{C_\alpha; \alpha \in \mathbb{I}\}$ of subsets of $(\tilde{U}, \tilde{\tau}, E)$, there is a soft S-locally finite collection $(F, \tilde{K}) = \{K_\alpha; \alpha \in \mathbb{I}\}$ of soft open subsets of $(\tilde{U}, \tilde{\tau}, E)$ such that for each $\alpha \in \mathbb{I}$, $K_\alpha \subseteq C_\alpha$.*

Definition 2.29. [23] *Let* $(\tilde{U}, \tilde{\tau}, E)$ *is the soft topological space. It is called soft extremely disconnected if the soft closure of each soft open set in soft topological space* $(\tilde{U}, \tilde{\tau}, E)$ *is soft open or if the soft interior of each soft closed set in soft topological space* $(\tilde{U}, \tilde{\tau}, E)$ *is soft closed.*

Lemma 2.30. [23] *Let* $(\tilde{U}, \tilde{\tau}, E)$ *is the soft extremely disconnected space, then* $Cl_s(M) = Cl(M)$ *for every* $M \in SSO(\tilde{U}, \tilde{\tau}, E)$.

Proposition 2.31. [23] *A soft topological space* $(\tilde{U}, \tilde{\tau}, E)$ *is soft extremely disconnected if and only if* $SSO(\tilde{U}, \tilde{\tau}, E) \subset SPO(\tilde{U}, \tilde{\tau}, E)$

Definition 2.32. [15] *Let* $(\tilde{U}, \tilde{\tau}, E)$ *is the soft topological space. Soft paracompact is called if every soft open cover has a locally finite soft open refinement.*

Definition 2.33. [18] *In a soft topological space* $(\tilde{U}, \tilde{\tau}, E)$*, the characteristic of being soft nearly paracompact is established when every soft regularly open cover has a locally finite soft open refinement. A subset* $(F, \tilde{A})$ *is deemed soft near paracompact if the topology it inherits demonstrates this same property of soft near paracompactness.*

3. SOFT S-PARACOMPACT SPACE

In this section, we investigate some characteristics of soft semi-open refinement, soft S-locally finite, and soft S-expandable in soft topological space, and we present the notion of a soft S-paracompact space within the context of soft topological spaces. Furthermore, we explored the relation between soft S-paracompact spaces and various soft separation axioms, soft S-closed spaces, soft α-open sets, and soft extremely disconnected spaces.

Definition 3.1. *Consider a soft topological space* $(\tilde{U}, \tilde{\tau}, E)$*, where* $(F, \tilde{A})$ *and* $(F, \tilde{B})$ *represent collections of soft sets within* $\tilde{U}_E$*. We define* $(F, \tilde{B})$ *as a soft refinement (or soft S-refinement) of* $(F, \tilde{A})$ *if for each element* B *in* $(F, \tilde{B})$ *(or* $SSO(F, \tilde{B})$ *respectively), there exists an element* A *in* $(F, \tilde{A})$ *(or* $SSO(F, \tilde{A})$ *respectively) such that* A *encompasses* B*. In cases where the elements of* $(F, \tilde{B})$ *are soft open sets (or soft semi-open sets), we may refer to* $(F, \tilde{B})$ *as a soft open refinement (or soft semi-open refinement) of* $(F, \tilde{A})$*. When the elements of* $(F, \tilde{B})$ *are soft closed sets (or soft semi-closed sets), we may refer to* $(F, \tilde{B})$ *as a soft closed refinement (or soft semi-closed refinement) of* $(F, \tilde{A})$.

Definition 3.2. *Let* $(\tilde{U}, \tilde{\tau}, E)$ *denote the soft topological space and collection* $(F, \tilde{C}) = \{C_\alpha; \alpha \in \mathbb{I}\}$ *of subsets of* $(\tilde{U}, \tilde{\tau}, E)$ *is said to be:*

1. *Soft discrete if every soft point* $e_x \in \tilde{U}_E$ *has a soft neighborhood that intersects at most one of the soft sets in* $(F, \tilde{C})$.
2. *Soft point finite if every soft point* $e_x \in \tilde{U}_E$ *is contained in at most a finite number of the soft sets in* $(F, \tilde{C})$.

Lemma 3.3. *Let $(\tilde{U}, \tilde{\tau}, E)$ is the soft topological space and $(F, \tilde{C})$ be the soft open covering of $\tilde{U}_E$,*

(1) *If $(F, \tilde{C})$ has a soft point finite soft semi-open refinement, then it also has a precise soft point finite soft semi-open refinement.*
(2) *If $(F, \tilde{C})$ possesses a soft s-locally finite soft semi-open (or closed) refinement, it necessarily follows that it also has a precise soft s-locally finite soft semi-open (or closed) refinement.*

Proposition 3.4. *Let $(\tilde{U}, \tilde{\tau}, E)$ is soft extremely disconnected semi-regular space. Then $(\tilde{U}, \tilde{\tau}, E)$ is soft expandable if and only if it is soft S-expandable.*

Proof. The proof follows from theorem 4.4 of [20] □

Proposition 3.5. *Let $(\tilde{U}, \tilde{\tau}, E)$ is the soft extremely disconnected semi-regular space. Then*

a) $SSO(\tilde{U}, \tilde{\tau}, E) = \tilde{\tau}$.
b) $(\tilde{U}, \tilde{\tau}, E)$ *is soft regular.*

Lemma 3.6. *Let $(\tilde{U}, \tilde{\tau}, E)$ be the soft topological space.*

(1) *If $(F, \tilde{A})$ is the soft open subset in a soft topological space $(\tilde{U}, \tilde{\tau}, E)$ and $(F, \tilde{V}) \in SSO(\tilde{U}, \tilde{\tau}, E)$ then $(F, \tilde{A}) \bigcap (F, \tilde{V}) \in SSO(\tilde{U}, \tilde{\tau}, E)$.*
(2) *$(\tilde{U_S}, \tilde{\tau}_y, E)$ represents the subspace of the soft topological space $(\tilde{U}, \tilde{\tau}, E)$. Consider the subset $(F, \tilde{B}) \subseteq \tilde{U_S}$. If it holds that $(F, \tilde{B}) \in SSO(\tilde{U}, \tilde{\tau}, E)$, then it follows that $(F, \tilde{B}) \in SSO(\tilde{U_S}, \tilde{\tau}_y, E)$.*
(3) *$(\tilde{U_S}, \tilde{\tau}_y, E)$ represents the subspace of a soft topological space $(\tilde{U}, \tilde{\tau}, E)$. Furthermore, if we consider a subset $(F, \tilde{B}) \subseteq \tilde{U_S}$, and it holds that $\tilde{U_S} \in \tilde{\tau}$, along with the condition that $(F, \tilde{B}) \in SSO(\tilde{U_S}, \tilde{\tau}_y, E)$, then it follows that $(F, \tilde{B}) \in SSO(\tilde{U}, \tilde{\tau}, E)$.*

Lemma 3.7. *Let $(\tilde{U}, \tilde{\tau}, E)$ is the soft topological space and $(F, \tilde{A})$ be the subset in a soft topological space $(\tilde{U}, \tilde{\tau}, E)$. Then $(F, \tilde{A}) \in SPO(\tilde{U}, \tilde{\tau}, E)$ if and only if $Cl_s(F, \tilde{A}) = Int(Cl(F, \tilde{A}))$.*

Definition 3.8. *Let $(\tilde{U}, \tilde{\tau}, E)$ be the soft topological space. It is said to be a soft S-paracompact space if every soft open cover has a locally finite soft semi-open refinement.*

Example 3.9. *Consider $\tilde{U} = \{a, b, c\}$ is the universal set and $E = \{e_1, e_2\}$ is the collection of parameters and $\tilde{A} = E$. Then the soft set $(F, \tilde{A})$ is defined as $(F, \tilde{A}) = \{(e_1, \{a, b, c\}), (e_2, \{a, b, c\})\}$, and the soft subset of $(F, \tilde{A})$ are:*
$(F, \tilde{A}_1) = \{(e_1, \{a, b\}), (e_2, \{a, b\})\}$,
$(F, \tilde{A}_2) = \{(e_1, \{b\}), (e_2, \{a, c\})\}$,
$(F, \tilde{A}_3) = \{(e_1, \{b, c\}), (e_2, \{a\})\}$,
$(F, \tilde{A}_4) = \{(e_1, \{b\}), (e_2, \{a\})\}$,
$(F, \tilde{A}_5) = \{(e_1, \{a, b\}), (e_2, \{a, b, c\})\}$,
$(F, \tilde{A}_6) = \{(e_1, \{a, b, c\}), (e_2, \{a, b\})\}$,

$(F,\tilde{A_7})=\{(e_1,\{b,c\}),(e_2,\{a,c\})\}$,
Consider a soft topology $\tilde{\tau}$ *is defeined as* $\tilde{\tau}=\{(F,\tilde{A_\phi}),(F,\tilde{A}),(F,\tilde{A_1}),(F,\tilde{A_2}),(F,\tilde{A_3}),$ $(F,\tilde{A_4}),(F,\tilde{A_5}),(F,\tilde{A_6}),(F,\tilde{A_7})\}$. *Suppose* $(F,\tilde{C})=\{(F,\tilde{A_5}),(F,\tilde{A_6}),(F,\tilde{A_7})\}$ *is a soft open cover of* $(F,\tilde{A})$ *and* $(F,\tilde{D})=\{(F,\tilde{A_1}),(F,\tilde{A_2}),(F,\tilde{A_3}),(F,\tilde{A_4}),(F,\tilde{A_5}),$ $(F,\tilde{A_6}),(F,\tilde{A_7})\}$ *is a soft semi-open refinement set of* $(F,\tilde{C})$*, as each element in* $(F,\tilde{D})$ *is a subset of a soft open cover* $(F,\tilde{C})$. $(F,\tilde{D})$ *includes soft semi-open sets, such as* $(F,\tilde{A_7})$*, which satisfies* $(F,\tilde{A_3})\subseteq(F,\tilde{A_7})\subseteq Cl(F,\tilde{A_3})$. *For a soft point* $(e_2\{a\})\in(F,\tilde{A})$*, there exists a soft open set* $(F,\tilde{A_3})$ *containing* $(e_2\{a\})$*, and* $(F,\tilde{A_7})$ *serves as a soft neighborhood of* $(e_2\{a\})$ *since* $(e_2\{a\})\in(F,\tilde{A_3})\subseteq(F,\tilde{A_7})$ *and* $(F,\tilde{A_7})\cap(F,\tilde{A_6})=(F,\tilde{A_3})\in(F,\tilde{D})$*, hence* $(F,\tilde{D})$ *encompasses locally finite soft semi-open sets.*

Theorem 3.10. *Every soft S-paracompact* T_2 *space is soft regular.*

Proof. Consider the soft S-paracompact T_2 space $(\tilde{U},\tilde{\tau},E)$. Let $e_x\in\tilde{U}_E$ and let $(F,\tilde{D})$ is the soft closed set of $\tilde{U}_E$, which is disjoint from e_x. By the soft T_2 condition, for every soft point e_y of $(F,\tilde{D})$, there exists a soft open set $(F,\tilde{B_{e_y}})$ about e_y, such that its closure is disjoint from e_x. Let $\mathcal{A}=\{(F,\tilde{B_{e_y}});e_y\in(F,\tilde{D})\}\bigcup(F,\tilde{D})^c$. $\mathcal{A}$ be the soft open covering of $\tilde{U}_E$. Since $(\tilde{U},\tilde{\tau},E)$ is soft S-paracompact, there is a locally finite soft semi-open refinement $\mathcal{C}$ that covers $\tilde{U}_E$. Let H be a subcollection of $\mathcal{C}$ consisting of all elements of $\mathcal{C}$ that intersect $(F,\tilde{D})$. Let $\mathcal{V}$ be defined as union of H i.e., $\mathcal{V}=\bigcup\{H\in\mathcal{C};H\cap(F,\tilde{D})\neq\phi\}$. Then $\mathcal{V}$ is a soft semi-open set containing $(F,\tilde{D})$ and $Cl(\mathcal{V})=\{Cl(H);H\in\mathcal{C}$ and $H\cap(F,\tilde{D})\neq\phi\}$, where the soft closure of $\mathcal{V}$ is disjoint from e_x. Consequently, $(F,\tilde{P})=\tilde{U}_E-Cl(\mathcal{V})$ is a soft open set containing e_x, such that $(F,\tilde{P})\cap(F,\tilde{B_{e_y}})=\phi$.

□

Theorem 3.11. *Every soft S-paracompact soft regular space is soft normal.*

Proof. Consider the soft S-paracompact soft regular space $(\tilde{U},\tilde{\tau},E)$. Let $e_x,e_y\in\tilde{U}_E$ and let $(F,\tilde{C})$ and $(F,\tilde{D})$ be the soft closed set of $\tilde{U}_E$, which are disjoint from each other, where $e_x\in(F,\tilde{C})$ and $e_y\in(F,\tilde{D})$. By the soft regular condition, for every soft point e_y of $(F,\tilde{D})$, there exists a soft open set $(F,\tilde{B_{e_y}})$ about e_y and disjoint Soft open set $(F,\tilde{P_{e_x}})$, such that $e_y\in(F,\tilde{D})\subseteq(F,\tilde{B_{e_y}})$ and $e_x\in(F,\tilde{C})\subseteq(F,\tilde{P_{e_x}})$ which is implies that $(F,\tilde{C})\cap(F,\tilde{B_{e_y}})=\phi$ as every point $(F,\tilde{C})$ has a neighborhood $(F,\tilde{P_{e_x}})$ that is disjoint from the $(F,\tilde{B_{e_y}})$. Since $(\tilde{U},\tilde{\tau},E)$ is soft S-paracompact, there is a locally finite soft semi-open refinement $\mathcal{C}$ that covers $\tilde{U}_E$. Let H be a subcollection of $\mathcal{C}$ consisting of all elements of $\mathcal{C}$ that intersect $(F,\tilde{D})$. Let $\mathcal{V}$ be defined as union of H i.e., $\mathcal{V}=\bigcup\{H\in\mathcal{C};H\cap(F,\tilde{D})\neq\phi\}$. Then a soft open neighborhood of $(F,\tilde{D})$, whose soft closure is disjoint from $(F,\tilde{C})$. Then $Cl(\mathcal{V})=\{Cl(H);H\in\mathcal{C}$ and $H\cap(F,\tilde{D})\neq\phi\}$, where the soft closure of $\mathcal{V}$ is disjoint from e_x. We completed this proof as mentioned in Theorem3.10, in the same way to get normality. Instead of $e_x\notin Cl(F,\tilde{B_{e_y}})$, we used $(F,\tilde{C})\cap Cl(F,\tilde{B_{e_y}})=\phi$, whenever relevant.

□

Theorem 3.12. *Each soft S-paracompact T_2 space is soft semiregular i.e., $\tilde{\tau} = \tilde{\tau_S}$.*

Proof. Consider the soft S-paracompact T_2 space $(\tilde{U}, \tilde{\tau}, E)$. Let a soft point $e_x \in \tilde{U}_E$ and $(F, \tilde{D})$ be the soft closed set of $\tilde{U}_E$, which is disjoint from e_x. For every soft point e_y of $(F, \tilde{D})$, there exists a soft semi-open set $(F, \tilde{V_{e_y}})_s$ for which $e_y \in (F, \tilde{V_{e_y}})_s$ and its closure is disjoint from e_x i.e., $e_x \notin Cl(F, \tilde{V_{e_y}})_s$. Let $\mathcal{A} = \{(F, \tilde{V_{e_y}})_s; e_y \in (F, \tilde{D})\} \bigcup (F, \tilde{D})^c$. $\mathcal{A}$ be the soft open covering of $\tilde{U}_E$. Since $(\tilde{U}, \tilde{\tau}, E)$ is soft S-paracompact, there is a locally finite soft semi-open refinement $\mathcal{C}$ that covers $\tilde{U}_E$. Let H be a subcollection of $\mathcal{C}$ consisting of all elements of $\mathcal{C}$ that intersect $(F, \tilde{D})$, i.e., $\mathcal{C} = \bigcup\{H \in \mathcal{C}; H \cap (F, \tilde{D}) \neq \phi\}$. Then $\mathcal{C}$ is a soft semi-open set containing $(F, \tilde{D})$. From Lemma 3.7 $Cl_s(\mathcal{C}) = Int(Cl(\mathcal{C}))$. Then $Cl_s(\mathcal{C}) = \{Cl_s(H); H \in \mathcal{C}$ and $H \cap (F, \tilde{D}) \neq \phi\}$, where the soft semi-closure of $\mathcal{C}$ is disjoint from e_x. Consequently, $(F, \tilde{P}) = \tilde{U}_E - Cl_s(\mathcal{C})$ is a soft semi-open set containing e_x and $Int(Cl((F, \tilde{P})))$ is soft regular open for every soft open subset $(F, \tilde{P})$ of $\tilde{U}_E$, such that $(F, \tilde{P}) \cap (F, \tilde{V_{e_y}})_s = \phi$. □

Theorem 3.13. *Each soft extremely disconnected soft S-paracompact T_2 space is soft regular.*

Proof. Consider the soft S-paracompact T_2 space $(\tilde{U}, \tilde{\tau}, E)$. Let $e_x \in \tilde{U}_E$ and $(F, \tilde{D})$ be the soft closed set of $\tilde{U}_E$, which is disjoint from e_x. By the soft T_2 condition, for every soft point e_y of $(F, \tilde{D})$, there exists a soft open set $(F, \tilde{B_{e_y}})$ about e_y, such that its closure is disjoint from e_x. Let $\mathcal{A} = \{(F, \tilde{B_{e_y}}); e_y \in (F, \tilde{D})\} \bigcup (F, \tilde{D})^c$. $\mathcal{A}$ be the soft open covering of $\tilde{U}_E$. Since $(\tilde{U}, \tilde{\tau}, E)$ is soft extremely disconnected soft S-paracompact space, there is a locally finite soft semi-open refinement $\mathcal{C}$ that covers $\tilde{U}_E$. Let H be a subcollection of $\mathcal{C}$ consisting of all elements of $\mathcal{C}$ that intersect $(F, \tilde{D})$. Let $\mathcal{V}$ be defined as union of H i.e., $\mathcal{V} = \bigcup\{H \in \mathcal{C}; H \cap (F, \tilde{D}) \neq \phi\}$. Then $\mathcal{V}$ is a soft semi-open set containing $(F, \tilde{D})$ and $Cl_s(\mathcal{V}) = Cl(\mathcal{V})$ (by Lemma 2.30). $Cl_s(\mathcal{V}) = \{Cl_s(H); H \in \mathcal{C}$ and $H \cap (F, \tilde{D}) \neq \phi\}$, where the soft semi-closure of $\mathcal{V}$ is soft semi-open and disjoint from e_x. Consequently, $(F, \tilde{P_{e_x}}) = \tilde{U}_E - Cl_s(\mathcal{V})$ is a soft semi-open set containing e_x, such that $(F, \tilde{P_{e_x}}) \cap (F, \tilde{B_{e_y}}) = \phi$. □

Theorem 3.14. *Every soft closed subspace of a soft S-paracompact space is a soft S-paracompact.*

Proof. Consider a soft S-paracompact space $(\tilde{U}, \tilde{\tau}, E)$ and any soft closed set $(F, \tilde{D})$ within $\tilde{U}_E$. A soft open covering $(F, \tilde{C})$ of $(F, \tilde{D})$ exists, where $(F, \tilde{C}) = \bigcup_{i=1}^{n}(F, C_i)$ for each $(F, C_i) \in (F, \tilde{C})$, encompassing $(F, \tilde{D})$. For every $(F, C_i) \in (F, \tilde{C})$, there is a corresponding $(F, C_i)^C$ such that $(F, C_i) = (F, C_i)^C \bigcap (F, \tilde{D})$. The soft semi-open set $(F, C_i)^C$, in conjunction with the soft semi-open set $(F, \tilde{D})^C$, forms a cover for $\tilde{U}_E$. A soft locally finite soft semi-open refinement $\mathcal{B}$ of this soft covering exists, which covers $\tilde{U}_E$. The set $\mathcal{G} = \{(F, \tilde{B}) \bigcap (F, \tilde{D}); (F, \tilde{B}) \in \mathcal{B}\}$ constitutes the required locally finite soft semi-open refinement of $(F, \tilde{C})$. □

Theorem 3.15. *Let $(\tilde{U}, \tilde{\tau}, E)$ is the soft extremely disconnected soft regular space. If each soft open cover of $\tilde{U}_E$ has a soft S-locally finite soft semi-open refinement. Every soft open cover of $\tilde{U}_E$ has a soft locally finite soft open refinement.*

Proof. Let $(F, \tilde{P})$ denote the soft open cover of $\tilde{U}_E$, where each $e_x \in \tilde{U}_E$ corresponds to a selected member $P_{e_x} \in (F, \tilde{P})$ and by soft regularity and the existence of a soft open subset $V_{e_x} \in \tilde{\tau}$ such that $e_x \in V_{e_x} \subseteq Cl(V_{e_x}) \subseteq P_{e_x}$. Consequently, the collection $\mathcal{V} = \{V_{e_x}; e_x \in \tilde{U}_E\}$ forms a soft open cover of $\tilde{U}_E$ and, as assumed, possesses a soft S-locally finite soft semi-open refinement denoted by $\mathcal{W} = \{W_{e_y}; e_y \in (F, \tilde{B})\}$. For each $e_y \in (F, \tilde{B})$, select a soft open set $\mathcal{H}_{e_y}$ such that $\mathcal{H}_{e_y} \subseteq W_{e_y} \subseteq Cl(\mathcal{H}_{e_y})$. However, if $e_y \in (F, \tilde{B})$, then $Cl(\mathcal{H}_{e_y}) = Cl(\mathcal{W}_{e_y}) \subseteq Cl(V_{e_x})$ for some $V_{e_x} \in \mathcal{V}$, implying that $Cl(\mathcal{H}_{e_y}) \subseteq P$ for some $P \in (F, \tilde{P})$. Conversely, as $(\tilde{U}, \tilde{\tau}, E)$ is soft extremely disconnected, $Cl(\mathcal{H}_{e_y}) \in \tilde{\tau}$ for every $e_y \in (F, \tilde{B})$. We now demonstrate that the set $H = Cl(\mathcal{H}_{e_y}); e_y \in (F, \tilde{B})$ is soft locally finite. Given $e_x \in \tilde{U}_E$, according to Lemma 2.26, the set $\{Cl_s(\mathcal{W}_{e_y}); e_y \in (F, \tilde{B})\}$ is locally finite. Therefore, the set H is soft S-locally finite ($Cl(\mathcal{H}_{e_y}) = Cl(\mathcal{W}_{e_y}) = Cl_s(\mathcal{W}_{e_y})$ for all $e_y \in (F, \tilde{B})$ according to Lemma 2.30). Choose $\mathcal{O}_{e_x} \in SSO(\tilde{U}, \tilde{\tau}, E)$ such that $e_x \in \mathcal{O}_{e_x}$ and $\mathcal{O}_{e_x}$ intersects at most finitely many members of H. Select $\mathcal{A}_{e_x} \in \tilde{\tau}$ such that $\mathcal{A}_{e_x} \subseteq \mathcal{O}_{e_x} \subseteq Cl(\mathcal{A}_{e_x})$. Given that $(\tilde{U}, \tilde{\tau}, E)$ is soft extremely disconnected, it follows that $Cl(\mathcal{A}_{e_x})$ is a soft open set that includes e_x and satisfies $Cl(\mathcal{A}_{e_x}) \cap Cl(\mathcal{H}_{e_x}) \neq \phi$ if and only if $\mathcal{O}_{e_x} \cap \mathcal{H}_{e_x} \neq \phi$. Consequently, H serves as a locally finite soft open refinement of $(F, \tilde{P})$.

□

Corollary 3.16. *Each soft, extremely disconnected soft S-paracompact T_2 space is a soft paracompact.*

Definition 3.17. *Let $(\tilde{U}, \tilde{\tau}, E)$ is said to be the soft S-compact space if every soft cover of $\tilde{U}_E$ by semi-open sets has a finite subcover.*

Definition 3.18. *Let $(\tilde{U}, \tilde{\tau}, E)$ is said to be the soft S-Lindelof space if every cover of $\tilde{U}_E$ by soft semi-open subsets of $\tilde{U}_E$ has a countable subcover.*

Proposition 3.19. *Every soft S-compact space is a soft S-Lindelof space, and every soft S-Lindelof space is a soft S-paracompact space.*

Proposition 3.20. *Let $(\tilde{U}, \tilde{\tau}, E)$ be the soft S-paracompact space and if $\tilde{A} = \{e\}$, then $(\tilde{U}, \tilde{\tau}, E)$ is soft S-paracompact if and only if the collection $(F, \tilde{C}) = \{(F, \tilde{A}_i); i \in \mathbb{I}; (F, \tilde{A}_i) \in \tilde{\tau}\}$ is a soft paracompact on $\tilde{U}_E$.*

Based on Proposition 3.20, it can be inferred that not every soft S-compact space is necessarily a soft S-paracompact space. For instance, a universal set $\tilde{U}$, a soft topological space $(\tilde{U}, \tilde{\tau}_{dis}, E)$ can be a soft S-paracompact space without being a soft S-compact space.

Definition 3.21. *A soft topological space $(\tilde{U}, \tilde{\tau}, E)$ is defined as soft S-closed if every soft semi-open cover of $\tilde{U}_E$ has finite subcover whose soft closures covers $\tilde{U}_E$.*

Theorem 3.22. *In countably soft S-closed spaces, every soft S-locally finite collection of soft semi-open sets is finite.*

Proof. Let $(\tilde{U}, \tilde{\tau}, E)$ be the countably soft S-closed space. Assume $\mathcal{A} = \{A_i; i \in \mathbb{I}\}$ is an infinite soft s-locally finite collection of soft semi-open sets in space $(\tilde{U}, \tilde{\tau}, E)$. For each $n \in \mathbb{I}$, we define $F_n = \bigcup_{j=n} Cl_s(A_j) = Cl_s\left(\bigcup_{j=n} A_j\right)$ (from Lemma 2.27). Since $\bigcup_{j=n} A_j$ is an element of $SSO(\tilde{U}, \tilde{\tau}, E)$ (as noted in Theorem 3.2 [14]), we can conclude that $Cl(F_n)$ belongs to $SRC(\tilde{U}, \tilde{\tau}, E)$. Thus, we have the following chain of inclusions: $Cl(F_1) \supseteq Cl(F_2) \supseteq Cl(F_3) \supseteq \ldots$. Our objective is to prove that $\bigcap\{Int(Cl(F_n)); n \in \mathbb{I}\} = \emptyset$. We will prove this by contradiction. Suppose there is a point t in the intersection $\bigcap\{Int(Cl(F_n)); n \in \mathbb{I}\}$. For each natural number n, we can find $V_n(t)$ in $\tilde{\tau}$ such that t is in $V_n(t)$ and $V_n(t)$ is contained in the closure of F_n. Because $\mathcal{A}$ is soft s-locally finite, there exists $\mathcal{O}_t \in (\tilde{U}, \tilde{\tau}, E)$ containing t that intersects only finitely many members of $\mathcal{A}$. We can choose $\mathcal{U}_t \in \tilde{\tau}$ satisfying $\mathcal{U}_t \subseteq \mathcal{O}_t \subseteq Cl(\mathcal{U}_t)$. Then, for every $n \in \mathbb{I}$, $V_n(t) \cap \mathcal{U}_t$ is non-empty, allowing us to select $\mathsf{K}_n \in V_n(t) \cap \mathcal{U}_t \subseteq Cl(F_n) \bigcap \mathcal{U}_t$. As a result, $(F_n) \cap \mathcal{U}_t \neq \phi$, which implies that $\mathcal{O}_t$ intersects numerous elements of $\mathcal{A}$. Therefore, we can conclude that $\bigcap\{Int(Cl(F_n)); n \in \mathbb{I}\} = \phi$. This contradicts the assertion that for a space $(\tilde{U}, \tilde{\tau}, E)$ to be countably soft S-closed, any countable cover consisting of soft semi-open sets must have a finite subset whose members closures collectively cover $\tilde{U_E}$. □

Corollary 3.23. *Each soft S-paracompact countably soft S-closed space is soft compact.*

Theorem 3.24. *Every soft, extremely disconnected, soft compact space is soft S-closed.*

Proof. Let us consider $(\tilde{U}, \tilde{\tau}, E)$ is the soft extremely disconnected space, and we know that by the definition of soft extremely disconnected space that is soft closure of any soft open set is soft open. Then, the interior of the soft semi-open set is dense in it. Then we will assume $\{Int(Cl(V_t)); t \in \tilde{\tau}\}$ instead of given soft semi-open set. In other words, we consider soft pre-open sets rather than soft semi-open sets. □

Corollary 3.25. *Let $(\tilde{U}, \tilde{\tau}, E)$ be the soft extremely disconnected space the following are equivalent:*

(a) $(\tilde{U}, \tilde{\tau}, E)$ *is soft S-paracompact and soft S-closed.*
(b) $(\tilde{U}, \tilde{\tau}, E)$ *is soft compact.*

Note 3.1. *From the Definition 2.24, The family of all soft α-open set of a space $(\tilde{U}, \tilde{\tau}, E)$ is denoted as $\tilde{\tau}_\alpha$, forms a topology on $\tilde{U_E}$ and it is finer than the $\tilde{\tau}$ such that $\tilde{\tau} \subseteq \tilde{\tau}_\alpha \subseteq SSO(\tilde{U}, \tilde{\tau}, E)$ and $SSO(\tilde{U}, \tilde{\tau}, E) = SSO(\tilde{U}, \tilde{\tau}_\alpha, E)$.*

Theorem 3.26. *If the soft α-open space $(\tilde{U}, \tilde{\tau}_\alpha, E)$ is soft S-paracompact space then $(\tilde{U}, \tilde{\tau}, E)$ is soft S-paracompact space.*

Proof. Consider a soft S-paracompact space $(\tilde{U}, \tilde{\tau}, E)$ and a soft open cover $(F, \tilde{C})$ of the soft S-paracompact space $(\tilde{U}, \tilde{\tau}_\alpha, E)$. As noted in Note 3.1, $\tilde{\tau} \subseteq \tilde{\tau}_\alpha$, so $(F, \tilde{C})$ is also a soft open cover of $(\tilde{U}, \tilde{\tau}_\alpha, E)$. Consequently, $(F, \tilde{C})$ possesses a locally finite soft semi-open refinement $(F, \tilde{D})$ in $(\tilde{U}, \tilde{\tau}_\alpha, E)$. Note 3.1 also states that $SSO(\tilde{U}, \tilde{\tau}, E) = SSO(\tilde{U}, \tilde{\tau}_\alpha, E)$. Our objective is to demonstrate that $(F, \tilde{D})$ is locally finite in $(\tilde{U}, \tilde{\tau}, E)$. For any $e_x \in \tilde{U_E}$, there exists a soft open subset $(F, \tilde{M}) \in \tilde{\tau}_\alpha$ that intersects only a finite number of members of $(F, \tilde{D})$, denoted as $\{D_1, D_2, D_3..., D_n\}$. For each $D \in (F, \tilde{C})$, we can find $W_D \in \tilde{\tau}$ such that $W_D \subseteq D \subseteq Cl(W_D)$. We claim that $Int(Cl(int(F, \tilde{G})))$ is a soft open set in $(\tilde{U}, \tilde{\tau}, E)$ containing e_x and satisfying $Int(Cl(int(F, \tilde{G}))) \cap D = \phi$ for all $D \in (F, \tilde{D}) - \{D_1, D_2, D_3..., D_n\}$. If we assume $Int(Cl(int(F, \tilde{G}))) \cap D \neq \phi$, then $Int(Cl(int(F, \tilde{G}))) \cap Cl(W_D) \neq \phi$ and $(F, \tilde{G}) \cap W_D \neq \phi$, implying $(F, \tilde{G}) \cap D \neq \phi$, and thus $D \in \{D_1, D_2, D_3..., D_n\}$. Therefore, we can conclude that $(F, \tilde{D})$ is a locally finite soft semi-open refinement of $(F, \tilde{C})$ in $(\tilde{U}, \tilde{\tau}, E)$, establishing that $(\tilde{U}, \tilde{\tau}, E)$ is indeed a soft S-paracompact space. □

The converse of Theorem 3.26 is given in the following example.

Example 3.27. *Consider $\tilde{U} = \{a, b, c, d\}$ is the universal set and $E = \{e_1, e_2, e_3\}$ is the collection of parameters and $\tilde{A} = E$. Then the soft set $(F, \tilde{A})$ is defined as $(F, \tilde{A}) = \{(e_1, \{a, b, c, d\}), (e_2, \{a, b, c, d\}), (e_3, \{a, b, c, d\})\}$ and subsets of $(F, \tilde{A})$ are:*
$(F, \tilde{A}_1) = \{(e_1, \{a\}), (e_2, \{b, c\}), (e_3, \{a, d\})\}$,
$(F, \tilde{A}_2) = \{(e_1, \{b, d\}), (e_2, \{a, c, d\}), (e_3, \{a, b, d\})\}$,
$(F, \tilde{A}_3) = \{(e_2, \{c\}), (e_3, \{a\})\}$,
$(F, \tilde{A}_4) = \{(e_1, \{a, b, d\}), (e_2, \{a, b, c, d\}), (e_3, \{a, b, c, d\})\}$,
$(F, \tilde{A}_5) = \{(e_1, \{a, c\}), (e_2, \{b, d\}), (e_3, \{b\})\}$,
$(F, \tilde{A}_6) = \{(e_1, \{a\}), (e_2, \{b\})\}$,
$(F, \tilde{A}_7) = \{(e_1, \{a, c\}), (e_2, \{b, c, d\}), (e_3, \{a, b, d\})\}$,
$(F, \tilde{A}_8) = \{(e_2, \{d\}), (e_3, \{b\})\}$,
$(F, \tilde{A}_9) = \{(e_1, \{a, b, c, d\}), (e_2, \{a, b, c, d\}), (e_3, \{a, b, c\})\}$,
$(F, \tilde{A}_{10}) = \{(e_1, \{a, c\}), (e_2, \{b, c, d\}), (e_3, \{a, b\})\}$,
$(F, \tilde{A}_{11}) = \{(e_1, \{b, c, d\}), (e_2, \{a, b, c, d\}), (e_3, \{a, b, c\})\}$,
$(F, \tilde{A}_{12}) = \{(e_1, \{a\}), (e_2, \{b, c, d\}), (e_3, \{a, b, d\})\}$,
$(F, \tilde{A}_{13}) = \{(e_1, \{a\}), (e_2, \{b, d\}), (e_3, \{b\})\}$,
$(F, \tilde{A}_{14}) = \{(e_1, \{c, d\}), (e_2, \{a, b\})\}$,
$(F, \tilde{A}_{15}) = \{(e_1, \{a\}), (e_2, \{b, c\}), (e_3, \{a\})\}$,
$(F, \tilde{A}_{16}) = \{(e_1, \{b\}), (e_2, \{c\}), (e_3, \{a, c\})\}$.
Let us consider a soft topology $\tilde{\tau} = \{(F, \tilde{A}_\phi), (F, \tilde{A}), (F, \tilde{A}_1), (F, \tilde{A}_2), (F, \tilde{A}_3), (F, \tilde{A}_4), (F, \tilde{A}_5), (F, \tilde{A}_6), (F, \tilde{A}_7), (F, \tilde{A}_8), (F, \tilde{A}_9), (F, \tilde{A}_{10}), (F, \tilde{A}_{11}), (F, \tilde{A}_{12}), (F, \tilde{A}_{13}), (F, \tilde{A}_{14}), (F, \tilde{A}_{15})\}$.
Let $(F, \tilde{C}) = \{(F, \tilde{A}_4), (F, \tilde{A}_9)\}$ *represent the soft open cover of* $(F, \tilde{A})$. *Suppose*

$(F, \tilde{D}) = \{(F, \tilde{A}_1), (F, \tilde{A}_2), (F, \tilde{A}_{10}), \{(F, \tilde{A}_{11})\}$ *denotes the soft semi-open refinement set of* $(F, \tilde{C})$. *This set fulfills the S-paracompactness condition. Thus* $(\tilde{U}, \tilde{\tau}, E)$ *is classified as a soft S-paracompact space.*

On the other hand, if $\tilde{\tau}_\alpha$ *denotes the soft topology on* $(F, \tilde{A})$, *then* $(\tilde{U}, \tilde{\tau}_\alpha, E)$ *does not qualify as a soft S-paracompact space. Assume* $(F, \tilde{M}) = \{(F, \tilde{A}_4), (F, \tilde{A}_9)\}$ *represents the soft open cover of* $(F, \tilde{A})$.
Let $(F, \tilde{N}) = \{(F, \tilde{A}_1), (F, \tilde{A}_2), (F, \tilde{A}_3), (F, \tilde{A}_{16})\}$ *be the soft semi-open refinement of* $(F, \tilde{M})$. *For* $(F, \tilde{A}_{16})$, *we have* $Int(F, \tilde{A}_{16}) = (F, \tilde{A}_3)$ *and* $Cl(Int(F, \tilde{A}_{16})) = (F, \tilde{A}_5)^c$. *However,* $(F, \tilde{A}_{16}) \nsubseteq Int(Cl(Int(F, \tilde{A}_{16})))$, *indicating that it is not a soft* α*-open set.*

Given that $\tilde{\tau} \subseteq \tilde{\tau}_\alpha$ *and* $(\tilde{U}, \tilde{\tau}, E)$ *is soft S-paracompact,* $(\tilde{U}, \tilde{\tau}_\alpha, E)$ *fails to be a soft S-paracompact space. This is because the collection of soft open covers of* $(\tilde{U}, \tilde{\tau}_\alpha, E)$ *does not allow for locally finite semi-open refinements in* $(\tilde{U}, \tilde{\tau}_\alpha, E)$, *as* $(F, \tilde{A}_{16})$ *belongs to* $\tilde{\tau}$ *but is not a soft* α*-open set.*

Let us consider $(\tilde{U}, \tilde{\tau}, E)$ is the soft topological space and $\tilde{\tau}_{sso}$ be the soft topology on $\tilde{U}_E$. And it has soft subbase $SSO(\tilde{U}, \tilde{\tau}, E)$. The collection $SSO(\tilde{U}, \tilde{\tau}, E)$ is soft topology on $\tilde{U}_E$ if and only if $SSO(\tilde{U}, \tilde{\tau}, E)$ is soft extremely disconnected. In such cases $\tilde{\tau}_{sso} = SSO(\tilde{U}, \tilde{\tau}, E)$.

Corollary 3.28. *Let* $(\tilde{U}, \tilde{\tau}, E)$ *be a soft extremely disconnected space. Then* $(\tilde{U}, \tilde{\tau}, E)$ *is soft S-paracompact if* $(\tilde{U}, \tilde{\tau}_{sso}, E)$ *is soft S-paracompact.*

Proof. Let $(\tilde{U}, \tilde{\tau}, E)$ be the soft extremely disconnected space. Then $SSO(\tilde{U}, \tilde{\tau}, E) \subseteq SPO(\tilde{U}, \tilde{\tau}, E)$(based on Proposition 2.31), hence $\tilde{\tau}_\alpha = SPO(\tilde{U}, \tilde{\tau}, E) \cap SSO(\tilde{U}, \tilde{\tau}, E)$ (from [24])$= SSO(\tilde{U}, \tilde{\tau}, E) = \tilde{\tau}_{sso}$. Then from the Theorem 3.26 we conclude that $(\tilde{U}, \tilde{\tau}, E)$ is soft S-paracompact if $(\tilde{U}, \tilde{\tau}_{sso}, E)$ is soft S-paracompact. □

The converse of Corollary 3.28 can be demonstrated using Example 3.1 from [14]. It is established that every soft open set in soft topological space qualifies as a soft semi-open set; however, not every soft semi-open set is classified as a soft open set in soft topological space.

Theorem 3.29. *If* $(\tilde{U}, \tilde{\tau}, E)$ *is the soft* T_2 *space, then* $(\tilde{U}, \tilde{\tau}, E)$ *is soft S-paracompact space if and only if each soft open cover* $(F, \tilde{B})$ *of* $\tilde{U}_E$ *has a locally finite soft semi-closed refinement* $(F, \tilde{C})$ *(that is* $C \in SSC(\tilde{U}, \tilde{\tau}, E)$ *for every* $C \in (F, \tilde{C})$*).*

Proof. To establish the necessary condition, consider that $(F, \tilde{B})$ constitutes a soft open cover of $\tilde{U}_E$, and for each $e_x \in \tilde{U}_E$, we choose a member $B_{e_x} \in (F, \tilde{B})$. According to Theorem 3.10, there exists a soft open subset $C_{e_x} \in \tilde{\tau}$ such that $e_x \in C_{e_x} \subseteq Cl_s(C_{e_x}) \subseteq B_{e_x}$. Thus, $(F, \tilde{C}) = \{C_{e_x}; e_x \in \tilde{U}_E\}$ functions as a soft open cover of $\tilde{U}_E$ and, as assumed, it has a locally finite soft semi-open refinement, labeled $D = \{D_{e_y}; e_y \in \mathbb{I}\}$. Examine the set $Cl_s(D) = \{Cl_s(D_{e_y}); e_y \in \mathbb{I}\}$ (Lemma 2.26). Clearly, $Cl_s(D)$ creates a locally finite group of soft semi-closed subsets of $(\tilde{U}, \tilde{\tau}, E)$. For each $e_y \in \mathbb{I}$, $Cl_s(D_{e_y}) \subseteq Cl_s(C_{e_x}) \subseteq B_{e_x})$ for a particular $B_{e_x} \in (F, \tilde{B})$, showing that $Cl_s(D)$ acts as a soft refinement of $(F, \tilde{B})$.

To demonstrate the sufficient condition, let $(F, \tilde{B})$ form a soft open cover of $\tilde{U}_E$ and $(F, \tilde{C})$ be the locally finite soft semi-closed refinement of $(F, \tilde{B})$. For each $e_x \in \tilde{U}_E$, let D_{e_x} be the soft open set containing e_x that intersects with at most a finite number of elements from $(F, \tilde{C})$. Let $(F, \tilde{G})$ be the soft semi-closed locally refinement of $D = \{D_{e_x}; e_x \in \tilde{U}_E\}$. For every $C \in (F, \tilde{C})$, define $C' = \tilde{U}_E - \{G \in (F, \tilde{G}); G \cap C = \phi\}$. The set $\{C' : C \in (F, \tilde{C})\}$ then forms a soft semi-open cover of $\tilde{U}_E$. Next, for each $C \in (F, \tilde{C})$, pick $B_C \in (F, \tilde{B})$ such that $C \subseteq B_C$. Consequently, the collection $\{B_C \cap C' : V \in (F, \tilde{C})\}$ serves as a locally finite soft semi-open (Lemma 3.6) refinement of $F, \tilde{B})$, thereby proving that $(\tilde{U}, \tilde{\tau}, E)$ is a soft S-paracompact space.

□

Theorem 3.30. *If $(\tilde{U}, \tilde{\tau}, E)$ be the soft regular space, then $(\tilde{U}, \tilde{\tau}, E)$ is soft S-paracompact space if and only if each soft open cover $(F, \tilde{B})$ of $\tilde{U}_E$ has a locally finite soft semi-closed refinement $(F, \tilde{C})$ (that is $C \in SRC(\tilde{U}, \tilde{\tau}, E)$ for every $C \in (F, \tilde{C})$).*

Proof. Let us establish the necessary conditions. Suppose $(F, \tilde{B})$ forms a soft open cover of $\tilde{U}_E$, and for each $e_x \in \tilde{U}_E$, we select a member $B_{e_x} \in (F, \tilde{B})$. According to Theorem 3.10, there exists a soft open subset $C_{e_x} \in \tilde{\tau}$ such that $e_x \in C_{e_x} \subseteq Cl_s(C_{e_x}) \subseteq B_{e_x}$. Consequently, $(F, \tilde{C}) = \{C_{e_x}; e_x \in \tilde{U}_E\}$ serves as a soft open cover of $\tilde{U}_E$ and, as assumed, it possesses a locally finite soft semi-open refinement, denoted as $D = \{D_{e_y}; e_y \in \mathbb{I}\}$. Consider the set $Cl(D) = \{Cl(D_{e_y}); e_y \in \mathbb{I}\}$ (Lemma 2.26). It is evident that $Cl(D)$ forms a locally finite group of soft semi-closed subsets of $(\tilde{U}, \tilde{\tau}, E)$. For each $e_y \in \mathbb{I}$, $Cl(D_{e_y}) \subseteq Cl_s(C_{e_x}) \subseteq B_{e_x})$ for a certain $B_{e_x} \in (F, \tilde{B})$, $Cl(D) \in SRC(\tilde{U}, \tilde{\tau}, E)$ for each $D \in SSO(\tilde{U}, \tilde{\tau}, E)$, indicating that $Cl(D)$ serves as a soft refinement of $(F, \tilde{B})$.
Now we will establish the sufficient condition. $(F, \tilde{B})$ forms a soft open cover of $\tilde{U}_E$ and $(F, \tilde{C})$ be the a locally finite soft semi-closed refinement of $(F, \tilde{B})$ or each $e_x \in \tilde{U}_E$, Let D_{e_x} be the soft open set containing e_x intersecting at most finitely many element of $(F, \tilde{C})$. Let $(F, \tilde{G})$ be the soft semi-closed locally refinement of $D = \{D_{e_x}; e_x \in \tilde{U}_E\}$. For every $C \in (F, \tilde{C})$, define $C' = \tilde{U}_E - \{G \in (F, \tilde{G}); G \cap C = \phi\}$. Then the set $\{C' : C \in (F, \tilde{C})\}$ forms a soft semi-open cover of $\tilde{U}_E$. Subsequently, for each $C \in (F, \tilde{C})$, select $B_C \in (F, \tilde{B})$ such that $C \subseteq B_C$. Hence, the collection $\{B_C \cap C' : V \in (F, \tilde{C})\}$ serves as a locally finite soft semi-open(Lemma 3.6) refinement of $F, \tilde{B})$ and therefore $(\tilde{U}, \tilde{\tau}, E)$ is a soft S-paracompact space.

□

Theorem 3.31. *Let $(\tilde{U}, \tilde{\tau}, E)$ be a soft, extremely disconnected, and semi-regular space. Then the following are equivalent:*

a) *$(\tilde{U}, \tilde{\tau}, E)$ is soft nearly paracompact*
b) *$(\tilde{U}, \tilde{\tau}, E)$ is soft paracompact*
c) *$(\tilde{U}, \tilde{\tau}, E)$ is soft S-paracompact*

Proof. $a) \longrightarrow b)$: Let $(\tilde{U}, \tilde{\tau}, E)$ represent a soft nearly paracompact space, and let $(F, \tilde{C}) = \{C_\alpha; \alpha \in I\}$ denote a soft open cover for its soft semi-regularization space $(\tilde{U}, \tilde{\tau}_S, E)$. For each set C_α, there exists an index set J_α such that $C_\alpha = \bigcup_{\beta \in J_\alpha} W_\beta$, where W_β is soft regularly open in $\tilde{U_E}$ for every $\beta \in J_\alpha$. Let $\mathcal{U} = \{U_r; r \in I^*\}$ be a soft regularly open and soft locally finite refinement of the collection $\{W_\beta; \beta \in J_\alpha; \alpha \in I\}$. For any $r \in J^*$, there exists an $\alpha_r \in I$ such that $U_r \subseteq W_\beta \subseteq \bigcup_{\beta \in J_\alpha} W_\beta \subseteq C_\alpha$. Since the soft regularly open envelope of any soft neighborhood disjoint with almost all U_r also has this property, the family $\mathcal{U}$ can be accepted as the soft locally finite refinement of $(F, \tilde{C})$ is the space$(\tilde{U}, \tilde{\tau}_S, E)$ that is $(\tilde{U}, \tilde{\tau}_S, E)$ is soft paracompact. Conversely $(\tilde{U}, \tilde{\tau}_S, E)$ be the soft paracompact and consequently a soft nearly paracompact and let $(F, \tilde{C})^*$ be a soft regular open cover of $\tilde{U_E}$. Since $(F, \tilde{C})^*$ is also a soft regular open cover of $(\tilde{U}, \tilde{\tau}_S, E)$ is being soft regular open in $\tilde{U_E}$. It has a soft, regularly open, soft, locally finite refinement $\mathcal{U}^*$ by Theorem 3.1 of [18]. Consequently, $\mathcal{U}^*$ is soft regularly open, soft locally finite refinement of $(F, \tilde{C})^*$ in $\tilde{U_E}$. So $(\tilde{U}, \tilde{\tau}, E)$ verify the necessary and sufficient conditions for being soft nearly paracompact.
$b) \longrightarrow c)$: The proof is obvious as every soft open refinement is a soft semi-open refinement. □

4. SOFT αS-PARACOMPACT SPACE

This section introduces the notion of a soft αS-paracompact space and examines the fundamental characteristics of soft S-paracompact spaces. We define soft semi-generalized closed set(sg-closed), soft θ-open set, soft θ-closed set, soft θ_s-open set,soft θ_s-closed set in soft topological space.

Definition 4.1. *Consider a soft topological space denoted by $(\tilde{U}, \tilde{\tau}, E)$. Within this space, let $(F, \tilde{A})$ represent a soft subset. This subset $(F, \tilde{A})$ is defined as a soft αS-paracompact space within $(\tilde{U}, \tilde{\tau}, E)$ if any cover of $(F, \tilde{A})$ composed of soft open subsets of $(\tilde{U}, \tilde{\tau}, E)$ possesses a locally finite soft semi-open refinement in $(\tilde{U}, \tilde{\tau}, E)$.*

From [25], we recall soft g-closed set.

Definition 4.2. [25] *A soft set $(F, \tilde{A})$ is said to be soft generalized closed set (soft g-closed) in a soft topological space $(\tilde{U}, \tilde{\tau}, E)$ if $Cl(F, \tilde{A}) \subset (F, \tilde{B})$, whenever $(F, \tilde{A}) \subset (F, \tilde{B})$ and $(F, \tilde{B}) \subset (\tilde{U}, \tilde{\tau}, E)$.*

This allows us to define the following terms.

Definition 4.3. *A soft set $(F, \tilde{A})$ is said to be soft semi-generalized closed set (soft sg-closed) in a soft topological space $(\tilde{U}, \tilde{\tau}, E)$ if $Cl_s(F, \tilde{A}) \subset (F, \tilde{B})$, whenever $(F, \tilde{A}) \subset (F, \tilde{B})$ and $(F, \tilde{B}) \subset SSO(\tilde{U}, \tilde{\tau}, E)$.*

Theorem 4.4. *Every soft g-closed subset of a soft S-paracompact space is soft αS-paracompact space.*

Proof. Consider a soft S-paracompact space $(\tilde{U}, \tilde{\tau}, E)$ and an element $e_x \in \tilde{U_E}$. Suppose $(F, \tilde{A})$ is a soft g-closed subset of this space. Consider a collection of soft open subsets $(F, \tilde{C}) = \{C_\alpha; \alpha \in I\}$ in $\tilde{U_E}$ that covers $(F, \tilde{A})$, such that $(F, \tilde{A}) \subseteq \bigcup\{C_\alpha; \alpha \in I\}$. As $(F, \tilde{A})$ is soft g-closed, we have $Cl(F, \tilde{A}) \subseteq \bigcup\{C_\alpha; \alpha \in I\}$. For any e_x in $\tilde{U_E}$ not in $Cl(F, \tilde{A})$, there exists a soft open set $(F, \tilde{W}_{e_x})$ in $\tilde{U_E}$ where $(F, \tilde{A}) \cap (F, \tilde{W}_{e_x}) = \phi$. We can now define $(F, \tilde{O}) = \{C_\alpha; \alpha \in I\} \bigcup \{\tilde{W}_{e_x}; e_x \notin Cl(F, \tilde{A})\}$, which forms a soft open cover of $(\tilde{U}, \tilde{\tau}, E)$. Let $(F, \tilde{H}) = \{H_\beta; \beta \in (F, \tilde{B})\}$ be a soft locally finite soft semi-open refinement of $(F, \tilde{O})$. For each $\beta \in (F, \tilde{B})$, either $H_\beta \subseteq C_{\alpha(\beta)}$ for some $\alpha(\beta) \in I$ or $H_\beta \subseteq W_{e_x(\beta)}$ for some $e_x(\beta) \in I$. We can then define $(F, \tilde{B})' = \{\beta \in (F, \tilde{B}); H_\beta \subseteq C_{\alpha(\beta)}\}$. Thus, $(F, \tilde{H})' = \{H_\beta; \beta \in (F, \tilde{B})'\}$ is a soft locally finite soft semi-open refinement of $(F, \tilde{C})$ and $(F, \tilde{A}) \subseteq \bigcup\{H_\beta; \beta \in (F, \tilde{B})\}$. As a result, $(F, \tilde{A})$ is classified as a soft αS-paracompact space. □

Theorem 4.5. *Every soft open subset of a soft αS-paracompact space of $(\tilde{U}, \tilde{\tau}, E)$ is soft S-paracompact.*

Proof. Consider a soft αS-paracompact space $(\tilde{U}, \tilde{\tau}, E)$ and a soft open subset $(F, \tilde{A})$ within it. Suppose $(F, \tilde{C}) = \{C_\alpha; \alpha \in I\}$ is a cover of $(F, \tilde{A})$ consisting of soft open subsets of the subspace $(\tilde{U_s}, \tilde{\tau_y}, \tilde{A})$. Given that $(F, \tilde{A})$ is soft open and $(F, \tilde{C})$ covers it with soft open subsets of $(\tilde{U}, \tilde{\tau}, E)$, there exists a soft locally finite soft semi-open refinement $(F, \tilde{W})$ in $(\tilde{U}, \tilde{\tau}, E)$. Consequently, $(F, \tilde{W}_{\tilde{A}}) = \{(F, W) \cap (F, \tilde{A}); W \in (F, \tilde{W})\}$ forms a soft locally finite soft semi-open refinement of $(F, \tilde{C})$ within $(\tilde{U_s}, \tilde{\tau_y}, \tilde{A})$. □

Theorem 4.6. *Let $(\tilde{U}, \tilde{\tau}, E)$ be the soft topological space and $(F, \tilde{A})$ is the soft clopen subspace of a soft space $(\tilde{U}, \tilde{\tau}, E)$. Then $(F, \tilde{A})$ is soft αS-paracompact space if and only if it is soft S-paracompact.*

Proof. To establish the necessary condition, consider a soft αS-paracompact space $(\tilde{U}, \tilde{\tau}, E)$ and a soft open subset $(F, \tilde{A})$ within it. Let $(F, \tilde{C}) = \{C_\alpha; \alpha \in I\}$ be a cover of $(F, \tilde{A})$ consisting of soft open subsets of the subspace $(\tilde{U_s}, \tilde{\tau_y}, \tilde{A})$. Given that $(F, \tilde{A})$ is soft open and $(F, \tilde{C})$ covers it with soft open subsets of $(\tilde{U}, \tilde{\tau}, E)$, there exists a soft locally finite soft semi-open refinement $(F, \tilde{W})$ in $(\tilde{U}, \tilde{\tau}, E)$. Consequently, $(F, \tilde{W}_{\tilde{A}}) = \{(F, W) \cap (F, \tilde{A}); W \in (F, \tilde{W})\}$ forms a soft locally finite soft semi-open refinement of $(F, \tilde{C})$ in $(\tilde{U_s}, \tilde{\tau_y}, \tilde{A})$. This demonstrates that $(F, \tilde{A})$ is soft S-paracompact. We will now establish a sufficient condition. Consider $(F, \tilde{C}) = \{C_\alpha; \alpha \in I\}$ as the cover of $(F, \tilde{A})$ by the soft open subset of $(\tilde{U}, \tilde{\tau}, E)$. Consequently, $(F, \tilde{C})' = \{(F, \tilde{A}) \cap C_\alpha; \alpha \in I\}$ forms a soft open cover of the soft S-paracompact subspace $(\tilde{U_s}, \tilde{\tau_y}, \tilde{A})$, which possesses a soft locally finite soft semi-open refinement $(F, \tilde{W})$ in $(\tilde{U_s}, \tilde{\tau_y}, \tilde{A})$. According to Lemma 3.6(c), for every $W \in (F, \tilde{W})$, $W \in SSO(\tilde{U}, \tilde{\tau}, E)$. Our objective is to demonstrate that $(F, \tilde{W})$ is soft locally finite in $(\tilde{U}, \tilde{\tau}, E)$. Let $e_x \in \tilde{U_E}$. If $e_x \in (F, \tilde{A})$, there exists $(F, \tilde{O}_{e_x}) \in \tilde{\tau}_A \subseteq \tilde{\tau}$ containing e_x such that $(F, \tilde{O}_{e_x})$ intersects with at most a finite number of members of $(F, \tilde{W})$. Alternatively, $(F, \tilde{A})^c$ is a soft open set in $(\tilde{U}, \tilde{\tau}, E)$ containing e_x that does not intersect with any member of $(F, \tilde{W})$. Thus, $(F, \tilde{W})$ is locally finite in $(\tilde{U}, \tilde{\tau}, E)$ such that $(F, \tilde{A}) \subset \bigcup\{W; W \in (F, \tilde{W})\}$. Therefore $(F, \tilde{A})$ is soft αS-paracompact. □

Corollary 4.7. *Each soft clopen subspace of soft S-paracompact space is soft S-paracompact.*

Definition 4.8. *Let $(F, \tilde{A})$ be the soft subset of of a soft space $(\tilde{U}, \tilde{\tau}, E)$ is called soft θ-open if for each $e_x \in (F, \tilde{A})$, there exist on soft open subset $(F, \tilde{O})$ of $(\tilde{U}, \tilde{\tau}, E)$ such that $e_x \in (F, \tilde{O}) \subseteq Cl(F, \tilde{O}) \subseteq (F, \tilde{A})$. The complement of soft θ-open set is called soft θ-closed set.*

Definition 4.9. *Let $(F, \tilde{A})$ be the soft subset of of a soft space $(\tilde{U}, \tilde{\tau}, E)$ is called soft θ_s-open if for each $e_x \in (F, \tilde{A})$, there exist on soft open subset $(F, \tilde{O})$ of $(\tilde{U}, \tilde{\tau}, E)$ such that $e_x \in (F, \tilde{O}) \subseteq Cl_s(F, \tilde{O}) \subseteq (F, \tilde{A})$. The complement of soft θ_s-open set is called soft θ_s-closed set.*

Note 4.1. *Soft θ_s-closed $\Rightarrow$ soft closed $\Rightarrow$ soft g-closed.*

Theorem 4.10. *Consider $(\tilde{U}, \tilde{\tau}, E)$ is the soft T_2 space and $(F, \tilde{A})$ is the soft αS-paracompact space, then $(F, \tilde{A})$ is soft θ_s- closed.*

Proof. Consider $(\tilde{U}, \tilde{\tau}, E)$ as a soft T_2 space and $(F, \tilde{A})$ as a soft αS-paracompact space. Assume $e_x \notin (F, \tilde{A})$ and $e_y \in (F, \tilde{A})$. There exists a soft open set (F, B_{e_y}) where $e_y \in (F, B_{e_y})$ and $e_x \notin (F, \tilde{A})$. The collection of such open sets $(F, \tilde{B}) = \{(F, B_{e_y}); e_y \in (F, \tilde{A})\}$ forms a soft open cover of the soft αS-paracompact subset $(F, \tilde{A})$ of $\tilde{U_E}$. Let $(F, \tilde{C})$ be the soft locally finite soft semi open refinement in $(F, \tilde{B})$ of $(\tilde{U}, \tilde{\tau}, E)$. Define $(F, D) = \cup\{C; C \in (F, \tilde{C})\}$ and $(F, \tilde{D})^c = \tilde{U_E} - Cl(F_E, \tilde{D})$. Then $(F, \tilde{D}) \in SSO(\tilde{U}, \tilde{\tau}, E)$ and $(F, \tilde{D})^c \in \tilde{\tau}$. Additionally, $e_x \in (F, \tilde{D})^c \subseteq Cl_s(F, \tilde{D})^c \subseteq \tilde{U_E} - (F, \tilde{A})$, demonstrating that $\tilde{U_E} - (F, \tilde{A})$ is soft θ_s-open. Hence $(F, \tilde{A})$ is soft θ_s-open. □

Corollary 4.11. *Let $(\tilde{U}, \tilde{\tau}, E)$ be the soft S-paracompact T_2 space and $(F, \tilde{A})$ be the soft subset of $\tilde{U_E}$. Then, the following are equivalent:*

(1) $(F, \tilde{A})$ is soft αS-paracompact space,
(2) $(F, \tilde{A})$ is the soft θ_s-closed space,
(3) $(F, \tilde{A})$ is soft closed,
(4) $(F, \tilde{A})$ is soft g-closed.

Proposition 4.12. *Consider $(F, \tilde{A})$ as any soft sg-closed subspace within $(\tilde{U}, \tilde{\tau}, E)$, and $(F, \tilde{B})$ as any soft subset of $\tilde{U_E}$. Suppose $(F, \tilde{A})$ is a soft αS-paracompact space and $(F, \tilde{A}) \subseteq (F, \tilde{B}) \subseteq Cl_s(F, \tilde{A})$. Under these conditions, $(F, \tilde{B})$ will also be a soft αS-paracompact space in $(\tilde{U}, \tilde{\tau}, E)$.*

Proposition 4.13. *Consider two soft subsets $(F,\tilde{A})$ and $(F,\tilde{B})$ of $(\tilde{U},\tilde{\tau},E)$, where $(F,\tilde{A})$ is contained within $(F,\tilde{B})$, and $(F,\tilde{B})$ is a soft open set. Under these conditions, $(F,\tilde{A})$ is a soft αS-paracompact space in $(\tilde{U}_S,\tilde{\tau}_B,\tilde{B})$ if and only if it is also a soft αS-paracompact space in $(\tilde{U},\tilde{\tau},E)$.*

5. SUM AND PRODUCT OF SOFT S-PARACOMPACT SPACE

From [26, 27, 28], we recall the definition of sum, product, and mappings of soft topological space.

Definition 5.1. [26] *Let $\{\ (\tilde{U_\alpha},\tilde{\tau}_\alpha,E):\alpha\in I\ \}$ be a family of pairwise disjoint soft topological space and $\tilde{U}=\cup_{n\in I}\tilde{U_\alpha}$. Then the collection $\tilde{\tau}=\{(F,E) over \cup_{n\in I}\tilde{U_\alpha}:\tilde{U_\alpha}\cap(F,E)=(e,F(e))\cap\tilde{U_\alpha}:e\in E\}$ is a soft open set in $(\tilde{U_\alpha},\tilde{\tau}_\alpha,E)$ for every $\alpha\in I\}$ defines a soft topology on $\tilde{U}$ with a fixed set of parameters E. Then the soft topological space $(\tilde{U},\tilde{\tau},E)$is said to be sum of soft topological space and it is denoted by $(\oplus_{\alpha\in I}\tilde{U_\alpha},\tilde{\tau},E)$.*

Theorem 5.2. *Let $(\tilde{U},\tilde{\tau},E)$ be the soft topological space. Then sum of soft topological space $(\oplus_{\alpha\in I}\tilde{U_\alpha},\tilde{\tau},E)$ is soft S-paracompact space if and only if $(\tilde{U_\alpha},\tilde{\tau}_\alpha,E)$ is soft S-paracompact for each $\alpha\in I$.*

Proof. To prove necessary condition, let $(\oplus_{\alpha\in I}\tilde{U_\alpha},\tilde{\tau},E)$ is soft S-paracompact space. From [26], we have, all soft sets $\tilde{U_E}$ are soft clopen in $(\oplus_{\alpha\in I}\tilde{U_\alpha},\tilde{\tau},E)$. Since $(\tilde{U_\alpha},\tilde{\tau}_\alpha,E)$ is a soft clopen subspace of $(\oplus_{\alpha I}\tilde{U_\alpha},\tilde{\tau},E)$, it follows from Corollary 4.7 that $(\tilde{U_\alpha},\tilde{\tau}_\alpha,E)$ is also a soft S-paracompact space.

To prove sufficient condition, let $(F,\tilde{C})=\{C_\alpha;\alpha\in I\}$ be a soft open cover of $\oplus_{\alpha\in I}\tilde{U_\alpha}$. For every $\alpha\in I$, the collection $C_\alpha=\{V\cap\tilde{U_\alpha}:V\in(F,\tilde{C})\}$ forms a soft open cover of the soft S-paracompact space $(\tilde{U_\alpha},\tilde{\tau}_\alpha,E)$. Consequently, C_α possesses a locally finite soft semi-open refinement W_α within $(\tilde{U_\alpha},\tilde{\tau}_\alpha,E)$. Define $\tilde{W}=\cup_{\alpha\in I}W_\alpha$. Evidently, $\tilde{W}$ constitutes a locally finite soft semi-open refinement of $(F,\tilde{C})$ such that $W\in SSO(\tilde{U},\tilde{\tau},E)$ for every $W\in\tilde{W}$. Hence, $\oplus_{\alpha\in I}\tilde{U_\alpha}$ exhibits soft S-paracompactness.

□

We recall from [29], the function $f:(\tilde{U},\tilde{\tau},E)\longrightarrow(\tilde{V},\tilde{\sigma},E)$ is said to be soft irresoluteness if inverse image of every soft semi-open set is soft semi-open. Every soft continuous open surjective function is soft irresolute.

Theorem 5.3. *Let $(\tilde{U},\tilde{\tau},E)$ be compact and $(\tilde{V},\tilde{\tau},E)$ be the soft S-paracompact space then product of $(\tilde{U},\tilde{\tau},E)\times(\tilde{V},\tilde{\sigma},E)$ is soft S-paracompact space.*

Proof. Consider $(F,\tilde{C})=\{C_\alpha:\alpha\in I\}$ as a soft open cover for the product soft topological space $(\tilde{U},\tilde{\tau},E)\times(\tilde{V},\tilde{\sigma},E)$. This implies that $(F,\tilde{C})$ also serves as a soft open cover for the soft compact subspace $\pi_{\tilde{V}}^{-1}(v)=\tilde{U}\times\{v\}$ for each $v\in\tilde{V}$, where $\pi_{\tilde{V}}$ denotes the natural projection from $(\tilde{U}, \tilde{\tau}, E) \times (\tilde{V}, \tilde{\sigma}, E)$ onto $(\tilde{V}, \tilde{\sigma}, E)$. There exists a finite subset $I(v)$ of I such that $\pi_{\tilde{V}}^{-1}(v) \subseteq \bigcup_{\alpha \in I(v)} C_\alpha = C_v$, and C_v is soft open. Given that $\pi_{\tilde{V}}$ is a soft closed function, for each $v \in \tilde{V}$, we can identify a soft open subset D_v of $\tilde{V}$ such that v is an element of D_v and $\pi_{\tilde{V}}^{-1}(D_v) \subseteq C_v$. Consequently, the collection $\tilde{D} = \{D_v; v \in \tilde{V}\}$ forms a soft open cover of the soft S-paracompact space $(\tilde{V}, \tilde{\sigma}, E)$, which therefore possesses a locally finite soft semi-open refinement, denoted as $\tilde{W} = \{W_\beta; \beta \in I\}$. Since $\pi_{\tilde{V}}$ is both soft continuous and soft irresolute, the family $\pi_{\tilde{V}}^{-1}(\tilde{W}) = \{\pi_{\tilde{V}}^{-1}(W_\beta); \beta \in I\}$ constitutes a soft semi-open locally finite cover of $(\tilde{U}, \tilde{\tau}, E) \times (\tilde{V}, \tilde{\sigma}, E)$, such that for each $\beta \in I$, $\pi_{\tilde{V}}^{-1}(W_\beta) = C_v$ for some $v \in \tilde{V}$. Ultimately, the collection $\{\pi_{\tilde{V}}^{-1}(W_\beta) \cap C_v; v \in \tilde{V}, \beta \in I\}$ provides a locally finite soft semi-open refinement of $(F, \tilde{C})$, where $\pi_{\tilde{V}}^{-1}(W_\beta) \cap C_v = \{\pi_{\tilde{V}}^{-1}(W_\beta) \cap C_{\alpha(u)} : \beta \in I, \alpha(u) \in I(v)\}$. Thus, $(\tilde{U}, \tilde{\tau}, E) \times (\tilde{V}, \tilde{\sigma}, E)$ is confirmed to be a soft S-paracompact space. □

6. ADVANTAGE & LIMITATIONS OF SOFT S-PARACOMPACTNESS

In this section, we examine the characteristics of soft S-paracompactness. In soft topology, soft paracompactness is essential for broadening the applicability of compact-like attributes to a wider range of soft topological spaces. Soft Sparacompactness, which pertains to soft open covers with locally finite semi-open refinements rather than entirely open ones, presents a more flexible alternative to soft paracompactness. This comparative analysis will highlight the connections between S-paracompactness and recognized soft topological properties and their advantages and limitations.

(1) **Advantage of soft S-paracompactness**
   - **A condition less strict than soft paracompactness:** Soft S- paracompactness represents a less stringent criterion than full soft paracompactness, allowing for its application to a broader spectrum of soft topological spaces.
   - **Useful in generalizing theorems:** The enhanced adaptability of soft semi-open sets over soft open sets enables the extension of results applicable to soft paracompact spaces to a wider variety of soft topological spaces under the framework of soft S-paracompactness.
   - **Applications in the field of analysis and functional spaces:** The notion of soft semi-open covers proves beneficial in examining functional spaces, particularly in scenarios where full soft paracompactness may be excessively stringent.
   - **Bridges the gap between soft compactness and soft paracompactness:** It offers a middle ground for spaces that do not fully meet the criteria for soft paracompactness.

(2) **Limitations of soft S-paracompactness**
The main drawback of soft S-paracompactness is that it does not inherently ensure normality, unlike paracompactness, which offers a stronger separation property. Moreover, since its refinements rely on soft semi-open sets rather than soft open sets, fundamental results like partitions of unity may not always apply. This weaker nature also reduces its effectiveness in establishing deeper soft topological results.

7. SOFT S-PARACOMPACT SPACE IN DECISION MAKING

This section presents an application of soft S-paracompact space in decision-making. Molodtsov [2] presented some applications of soft set theory in several directions: studies of smoothness function, game theory, operation research, Riemann integration, Perron integration, probability, theory of measurement, and other well-known theories. Maji [30] applied the soft set theory to solve a decision-making problem using rough mathematics. Mareay [31] gives a decision-making application of the theory of soft set. Atef [32] investigated covering soft and rough sets and their topological properties with application, and Sanjitha and Baiju [33] proposed ordered weighted aggregation operators on multiple sets with their application on decision-making problems.

Choosing from the available options can become tedious in many real-world scenarios. The selection process is complicated because it must consider all characteristics and factors. This situation often arises during the selection of cricket players. Soft set theory appears to be an essential tool for developing a framework that accommodates the vague evaluations involved in the selection process. Here, we apply soft S-paracompactness in a decision-making problem using a rough approach[34].

7.1. **Algorithm.**

Step 1: Define a soft set denoted as (F, E) within the universe $\tilde{U}$. The input parameters $\tilde{A}$ and $\tilde{B}$ serve as choice parameters for the selectors $\tilde{X}$ and $\tilde{Y}$, respectively, both of which are subsets of E.

Step 2: The evaluation of the player's skills differs based on the selection of parameters chosen by each selector represented as $(F, \tilde{A})$, $(F, \tilde{B}) \subseteq (F, E)$.

Step 3: Consider alternative selection of various type of selectors represented as $(F, E_i) \subseteq (F, E)$.

Step 4: Compute the soft topological space $\tilde{\tau} = \{(F, E_i) : (F, E_i) \subseteq (F, E), \forall i = 1, 2, 3, ...n\}$.

Step 5: Create a soft open cover that has all the player's skills and integrates the selector's choices.

Step 6: The choice value of an objective $h_i \in \tilde{U}$ is s_i, given by

$$s_i = \sum_j h_{ij}$$

where h_i represents players and h_{ij} are the entries in the table of the players with skills.

Step 7: Construct weighted-table (s_i) from the weightage of the attributes (w_i) given by the selectors in such a way that

$$s_i = \sum_j t_{ij}$$

where $t_{ij} = w_i \times h_{ij}$.

Step 8: Select the players with the maximum choice value from a pool of aggregate bases. We will get $s_k = max(s_i)$, then h_k's are the optimal choices of the respected selectors.

Step 9: Determine the result based on the decisions made by both selectors $\tilde{X}$ and $\tilde{Y}$.

- **Illustration:**
  Let $\tilde{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, ..., 20\}$ represent 20 cricket players who are waiting for their selection in a cricket team, and let E={ Better batting average, Good strike rate, All-rounder, Best fielder, Good bowling economy} denotes the set of parameters.

  Consider the soft set (F, E) that outlines the "skills" of these cricket players, defined as (F, E) = { Better batting average(e_1) = {1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 13, 15}, Good strike rate(e_2)={1, 2, 4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 20}, All rounder(e_3)={2, 5, 7, 8, 10, 12, 13, 14, 15, 16, 20}, Best fielder(e_4)={2, 4, 6, 8, 10, 12, 16} Good bowling economy (e_5)={6, 8, 12, 14, 16, 17, 18, 19, 20} }.

  Suppose selector $\tilde{X}$ wants to choose players based on his preferred parameters, which include 'better batting average', 'good strike rate', 'all rounder', and 'good bowling economy', forming the subset $\tilde{A}$ = {Better batting average, Good strike rate, All rounder, Good bowling economy} of the set E. This meant that from the available players in $\tilde{U}$, the selector selected the best ones who meet all the parameters in set $\tilde{A}$.

  Consider another selector, $\tilde{Y}$, aims to pick players based on his set of parameters, which includes 'better batting average', 'best fielders', 'all rounder', and 'good bowling economy'. These parameters form the subset $\tilde{B}$ = {Better batting average, Best fielders, All rounder, Good bowling economy} of the set E. The challenge is identifying the most appropriate players using $\tilde{X}$ and $\tilde{Y}$ selection parameters. Players deemed the best by $\tilde{X}$ may not necessarily be the top choice for $\tilde{Y}$, as each selector's decision is influenced by their specific set of parameters.
- Consider the reduct of a soft set (F, E), which represents alternative selections of various selectors. Those are as follows:
  (F, E_1) = { (e_1,{1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 13, 15})}.
  (F, E_2) = { (e_2,{1, 2, 4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 20})}.
  (F, E_3) = { (e_3,{2, 5, 7, 8, 10, 12, 13, 14, 15, 16, 20})}.
  (F, E_4) = { (e_4{2, 4, 6, 8, 10, 12, 16})}.

(F,E_5) = { (e_5,{6, 8, 12, 14, 16, 17, 18, 19, 20})}.
(F,E_6) = { (e_1,{1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 13, 15}), (e_2,{1, 2, 4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 20})}.
(F,E_7) = { (e_1,{1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 13, 15}),(e_3,{2, 5, 7, 8, 10, 12, 13, 14, 15, 16, 20})}.
(F,E_8) = {(e_2,{1, 2, 4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 20}), (e_3,{2, 5, 7, 8, 10, 12, 13, 14, 15, 16, 20})}.
(F,E_9) = { (e_1,{1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 13, 15}), (e_2,{1, 2, 4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 20}), (e_3,{2, 5, 7, 8, 10, 12, 13, 14, 15, 16, 20})}.
(F,E_{10}) = { (e_1,{1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 13, 15}), (e_4,{2, 4, 6, 8, 10, 12, 16})}.
(F,E_{11}) = {(e_2,{1, 2, 4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 20}), (e_4,{2, 4, 6, 8, 10, 12, 16})}.
(F,E_{12}) = { (e_1,{1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 13, 15}), (e_2,{1, 2, 4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 20}), (e_4,{2, 4, 6, 8, 10, 12, 16})}.
(F,E_{13}) = { (e_3,{2, 5, 7, 8, 10, 12, 13, 14, 15, 16, 20}), (e_4{2, 4, 6, 8, 10, 12, 16})}.
(F,E_{14}) = { (e_1,{1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 13, 15}), (e_3,{2, 5, 7, 8, 10, 12, 13, 14, 15, 16, 20}), (e_4,{2, 4, 6, 8, 10, 12, 16})}.
(F,E_{15}) = {(e_2,{1, 2, 4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 20}), (e_3,{2, 5, 7, 8, 10, 12, 13, 14, 15, 16, 20}), (e_4,{2, 4, 6, 8, 10, 12, 16})}.
(F,E_{16}) = { (e_1,{1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 13, 15}), (e_2,{1, 2, 4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 20}), (e_3,{2, 5, 7, 8, 10, 12, 13, 14, 15, 16, 20}), (e_4,{2, 4, 6, 8, 10, 12, 16})}.
(F,E_{17}) = { (e_1,{1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 13, 15}), (e_5,{6, 8, 12, 14, 16, 17, 18, 19, 20})}.
(F,E_{18}) = {(e_2,{1, 2, 4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 20}), (e_5,{6, 8, 12, 14, 16, 17, 18, 19, 20})}.
(F,E_{19}) = { (e_1,{1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 13, 15}), (e_2,{1, 2, 4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 20}), (e_5,{6, 8, 12, 14, 16, 17, 18, 19, 20})}.
(F,E_{20}) = { (e_3,{2, 5, 7, 8, 10, 12, 13, 14, 15, 16, 20}), (e_5,{6, 8, 12, 14, 16, 17, 18, 19, 20})}.
(F,E_{21}) = { (e_1,{1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 13, 15}),(e_3,{2, 5, 7, 8, 10, 12, 13, 14, 15, 16, 20}), (e_5,{6, 8, 12, 14, 16, 17, 18, 19, 20})}.
(F,E_{22}) = {(e_2,{1, 2, 4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 20}), (e_3,{2, 5, 7, 8, 10, 12, 13, 14, 15, 16, 20}), (e_5,{6, 8, 12, 14, 16, 17, 18, 19, 20})}.
(F,E_{23}) = { (e_1,{1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 13, 15}), (e_2,{1, 2, 4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 20}), (e_3,{2, 5, 7, 8, 10, 12, 13, 14, 15, 16, 20}), (e_5,{6, 8, 12, 14, 16, 17, 18, 19, 20})}.
(F,E_{24}) = { (e_4,{2, 4, 6, 8, 10, 12, 16}), (e_5,{6, 8, 12, 14, 16, 17, 18, 19, 20})}.
(F,E_{24}) = { (e_1,{1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 13, 15}), (e_4,{2, 4, 6, 8, 10, 12, 16}), (e_5,{6, 8, 12, 14, 16, 17, 18, 19, 20})}.
(F,E_{25}) = { (e_2,{1, 2, 4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 20}), (e_4,{2, 4, 6, 8, 10, 12, 16}), (e_5,{6, 8, 12, 14, 16, 17, 18, 19, 20})}.

(F, E_{26}) = { (e_1,{1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 13, 15}), (e_2,{1, 2, 4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 20}), (e_4,{2, 4, 6, 8, 10, 12, 16}), (e_5,{6, 8, 12, 14, 16, 17, 18, 19, 20})}.
(F, E_{27}) = { (e_3,{2, 5, 7, 8, 10, 12, 13, 14, 15, 16, 20}), (e_4,{2, 4, 6, 8, 10, 12, 16}), (e_5,{6, 8, 12, 14, 16, 17, 18, 19, 20})}.
(F, E_{28}) = { (e_1,{1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 13, 15}),(e_3,{2, 5, 7, 8, 10, 12, 13, 14, 15, 16, 20}), (e_4,{2, 4, 6, 8, 10, 12, 16}), (e_5,{6, 8, 12, 14, 16, 17, 18, 19, 20})}.
(F, E_{29}) = { (e_2,{1, 2, 4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 20}),(e_3,{2, 5, 7, 8, 10, 12, 13, 14, 15, 16, 20}), (e_4,{2, 4, 6, 8, 10, 12, 16}), (e_5,{6, 8, 12, 14, 16, 17, 18, 19, 20})}.

- Here, we define soft topology $\tilde{\tau}$,
  $\tilde{\tau} = \{(F, \phi), (F, E), (F, \tilde{A}), (F, \tilde{B}), (F, \tilde{E_1}), (F, \tilde{E_2}), (F, \tilde{E_3}), ..., (F, \tilde{E_{29}})\}$
  where, $(F, \tilde{A})$ and $(F, \tilde{B})$ are the selected parameters of the selectors $\tilde{X}$, and $\tilde{Y}$.
- Let us consider a soft open cover (F, P) defined as
  $(F, P) = \{(F, \tilde{A}), (F, \tilde{B})\}$.
- Now, we must find out the locally finite soft semi-open refinement of (F, P). We obtain the outstanding 11 players from the $\tilde{X}$ selection and $\tilde{Y}$.

- **Tabular representation of soft set**
  Lin[35] and Yao [36] previously introduced a tabular format for presenting soft sets. We offer a similar representation using a binary table. To do this, consider the soft set (F, E) based on the parameters E. This soft set can be depicted below. Such a representation is advantageous for storing a soft set in computer memory. If $h_i \in F(e)$, where h_i denotes the players numbers $(i = 1, 2, ..., 20)$, then the player's skill is indicated by $h_{ij} = 1$; otherwise, it is $h_{ij} = 0$.

Table 1: Professional Skill Evaluation of Players

$\tilde{U}$	e_1	e_2	e_3	e_4	e_5
h_1	1	1	0	0	0
h_2	1	1	1	1	0
h_3	1	0	0	0	0
h_4	1	1	0	0	0
h_5	1	1	1	1	0
h_6	1	1	0	1	1
h_7	0	1	1	0	0
h_8	1	1	1	1	1
h_9	0	1	0	0	0
h_{10}	1	1	1	1	0
h_{11}	1	0	0	0	0

h_{12}	1	1	1	1	1
h_{13}	1	1	1	0	0
h_{14}	0	1	1	1	1
h_{15}	1	1	1	1	0
h_{16}	0	1	1	1	1
h_{17}	0	0	0	0	1
h_{18}	0	0	0	0	1
h_{19}	0	0	0	0	1
h_{20}	0	1	1	0	1

- **Choice value of an objective** h_i
  The choice value of an objective $h_i \in \tilde{U}$ is s_i $\forall i = 1, 2, 3, ..., 20$, given by $s_i = \sum_{j=1}^{5} h_{ij}$ where h_{ij} are the entries in the table of the players with skills. From Table 1.

Table 2: Choice Value of Players

$\tilde{U}$	e_1	e_2	e_3	e_4	e_5	$s_i = \sum_{j=1}^{5} h_{ij}$
h_1	1	1	0	0	0	2
h_2	1	1	1	1	0	4
h_3	1	0	0	0	0	1
h_4	1	1	0	0	0	2
h_5	1	1	1	1	0	4
h_6	1	1	0	1	1	4
h_7	0	1	1	0	0	2
h_8	1	1	1	1	1	5
h_9	0	1	0	0	0	1
h_{10}	1	1	1	1	0	4
h_{11}	1	0	0	0	0	1
h_{12}	1	1	1	1	1	5
h_{13}	1	1	1	0	0	3
h_{14}	0	1	1	1	1	4
h_{15}	1	1	1	1	0	4
h_{16}	0	1	1	1	1	4
h_{17}	0	0	0	0	1	1
h_{18}	0	0	0	0	1	1
h_{19}	0	0	0	0	1	1
h_{20}	0	1	1	0	1	3

- **Weighted-table choice of an objective**
  The weightage choice value of an object $h_i \in \tilde{U}$ is s_i $\forall i = 1, 2, 3, ..., 20$, given by
  $s_i = \sum_{j=1}^{4} t_{ij}$ where $t_{ij} = w_i \times h_{ij}$.

- Imposing weights on the choice of selector $\tilde{X}$.

  Suppose selector $\tilde{X}$ sets the following weights for the parameters $\tilde{A}$: for the parameter 'better batting average' $w_1 = 0.8$, 'good strike rate' $w_2 = 0.7$, 'all rounder' $w_3 = 0.6$, and 'good bowling economy' $w_5 = 0.9$.

Table 3: Choice Value s_i and Weight-Table (t_{ij}) for The Selector $\tilde{X}$

$\tilde{U}$	e_1	e_2	e_3	e_5	$s_i = \sum_{j=1}^{4} h_{ij}$	$s_i = \sum_{j=1}^{4} t_{ij}$
h_1	1	1	0	0	2	1.5
h_2	1	1	1	0	3	2.1
h_3	1	0	0	0	1	0.8
h_4	1	1	0	0	2	1.5
h_5	1	1	1	0	3	2.1
h_6	1	1	0	1	3	2.4
h_7	0	1	1	0	2	1.3
h_8	1	1	1	1	4	3
h_9	0	1	0	0	1	0.7
h_{10}	1	1	1	0	3	2.1
h_{11}	1	0	0	0	1	0.8
h_{12}	1	1	1	1	4	3
h_{13}	1	1	1	0	3	2.1
h_{14}	0	1	1	1	3	2.2
h_{15}	1	1	1	0	3	2.1
h_{16}	0	1	1	1	3	2.2
h_{17}	0	0	0	1	1	0.9
h_{18}	0	0	0	1	1	0.9
h_{19}	0	0	0	1	1	0.9
h_{20}	0	1	1	1	3	2.2

From the Table 3, $max(s_i) = s_2, s_5, s_6, s_8, s_{10}, s_{12}, s_{13}, s_{14}, s_{15}, s_{16},$ and s_{20}.
Decision: The selector $\tilde{X}$ can choose 11 players, specifically h_2, h_5, h_6, h_8, h_{10}, h_{12}, h_{13}, h_{14}, h_{15}, h_{16}, and h_{20}.

- Now, imposing weights on the choice of selector $\tilde{Y}$.

  Suppose selector $\tilde{Y}$ sets the following weights for the parameters $\tilde{B}$: for the parameter 'better batting average' $w_1 = 0.8$, 'all rounder' $w_3 = 0.6$, 'best fielders' $w_4 = 0.5$ and 'good bowling economy' $w_5 = 0.9$.

Table 4: Choice Value (s_i) and Weight-Table(t_{ij}) for Selector $\tilde{Y}$

$\tilde{U}$	e_1	e_3	e_4	e_5	$s_i = \sum_{j=1}^{4} h_{ij}$	$s_i = \sum_{j=1}^{4} t_{ij}$
h_1	1	0	0	0	1	0.8
h_2	1	1	1	0	3	1.9

h_3	1	0	0	0	1	0.8
h_4	1	0	0	0	1	0.8
h_5	1	1	1	0	3	1.9
h_6	1	0	1	1	3	2.2
h_7	0	1	0	0	1	0.6
h_8	1	1	1	1	4	2.8
h_9	0	0	0	0	0	0
h_{10}	1	1	1	0	3	1.9
h_{11}	1	0	0	0	1	1.7
h_{12}	1	1	1	1	4	2.8
h_{13}	1	1	0	0	2	1.4
h_{14}	0	1	1	1	3	2
h_{15}	1	1	1	0	3	1.9
h_{16}	0	1	1	1	3	2
h_{17}	0	0	0	1	1	0.9
h_{18}	0	0	0	1	1	0.9
h_{19}	0	0	0	1	1	0.9
h_{20}	0	1	0	1	2	1.5

From the Table 4, $max(s_i) = s_2, s_5, s_6, s_8, s_{10}, s_{12}, s_{13}, s_{14}, s_{15}, s_{16}$ and s_{20}.

Decision: The selector $\tilde{Y}$ can choose 11 players, specifically h_2, h_5, h_6, h_8, h_{10}, h_{12}, h_{13}, h_{14}, h_{15}, h_{16} and h_{20}.

- The selectors $\tilde{X}$ and $\tilde{Y}$ chose 11 players according to the criteria $\tilde{A}$ and $\tilde{B}$. The players finally chosen are: h_2, h_5, h_6, h_8, h_{10}, h_{12}, h_{13}, h_{14}, h_{15}, h_{16}, and h_{20}.

8. CONCLUSION

This paper investigates some characteristics of soft semi-locally finite sets, soft semi-refinement sets, soft s-expandable spaces, and soft extremely disconnected spaces in soft topological spaces. We extend and generalize existing topological notions using the parameterized family of topological spaces induced by the soft topology. Additionally, we introduce soft S-paracompact spaces, a generalization of soft paracompact spaces, and explore their characteristics. Furthermore, we investigate concepts such as soft sg-closed sets, soft θ-open sets, soft θ-closed sets, and soft θ_s-open sets, and soft θ_s-closed sets in the context of soft topological spaces. These contributions expand the theoretical foundations of soft topology and provide a solid framework for future research.

We investigated a comparative analysis of soft S-paracompactness with concepts like soft compact and soft paracompactness in soft topological spaces. We apply soft S-paracompactness in decision-making, establishing a robust foundation for future investigations in soft topological spaces.

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