EFFECT OF THERMAL STRESSES AND REDUCED DIFFERENTIAL TRANSFORM ON BENDING OF THE RECTANGULAR PLATE

Chandrashekhar S. Sutar[1], and Sandip N. Patil[2]

[1]**Department of Mathematics and Statistics, PSGVP Mandals Arts, Science and Commerce College, Shahada, India, sutarchandu@gmail.com**
[2]**N.Y.N.C. Arts, Science and Commerce College, Chalisgaon, India, snpatil2007@gmail.com**

Abstract. The bending of the rectangular plate exists for a large length comparing with its width as we have considered the simply supported plate. Also, it gets the accuracy, if the plate is attached between enough distance from the ends. This paper concern with the study of bending moment in the rectangular plate by using the thermal stresses. In this paper, we have considered the mathematical model of the unsteady state three-dimensional heat conduction equation for a thick rectangular plate with non-zero temperature initially. We have determined the thermal stresses and the bending moment in the rectangular plate by using the reduced differential transform method. The results are discussed by considering the special cases for copper material. Also, results are shown graphically. The graphs are drawn by using mathematical software Scilab.

Key words : Bending moment, Heat conduction, Reduced differential transform, Thermal stresses.

1. INTRODUCTION

Material's important properties are dependent on temperature. Thus temperature field acts as an important role in thermoelasticity. Nowacki [14] has studied the thermoelastic model in a circular plate subjected to an axisymmetric temperature distribution on the upper surface with zero temperature on the lower surface and with the circular edge thermally insulated. Noda et al. [13] have studied the fundamental theory of thermal stresses. Also, they have discussed the applications. Ozicik [15] in his book discussed the heat conduction in various solids. He has considered various examples of solids and determined the solutions. Recently Chaudhari and Sutar [4] analyzed the thermal stresses of two-dimensional rectangular plate with hyperbolic heat conduction. In their mathematical model they have used differential transform method to find the solution. Mansfield [9] discussed the elastic plate behavior and the bending and stretching of plates. Also, he has studied the deflection theory and thermal effects of various-shaped plates. Timoshenko and Woinowsky-Krieger [19] have studied bending of plates and described small deflections and pure bending of plates. Also they have analyzed the bending of plates depends on the thickness and the dimensions of the plate. Srivastava

et al.[17] have discussed the reduced differential transform method. Deshmukh et al.[6] have considered the simply supported rectangular plate and determined the deflection. Yuemei and Rui [21] studied the bending analysis of a rectangular plate, where he considered the rectangular plate with two adjacent edges free and other edges are clamped. Boley and Weiner [2] explain the theory of thermal stresses for various solids. Jane and Hong [7] have studied the thermal bending analysis of orthotropic plates by the generalized differential quadrature method. Yos and Kraiwood [20] have given the bending behaviors of simply supported rectangular plates with an internal line sagged and unsagged supports. Cattaneo and Mehta [3],[10] have studied the heat conduction analysis. Barletta [1], Chen [5], Mallick [8], Morimoto [12] and Sherief [16] have discussed the thermoelastic behavior. Al-Amr [11] and Taha [18] gives the details of the reduced differential transform method.

In this paper, we have considered the thick rectangular plate which has simply supported edges with thickness $'h'$ and dimensions as $0 \leq x \leq a$, $0 \leq y \leq b$, $0 \leq z \leq h$. The bending moment along direction $'z'$ has been determined by the reduced differential transform method.

2. REDUCED DIFFERENTIAL TRANSFORM METHOD

If $u(x, y, z, t)$ is analytic and continuosly differentiable with repect to x, y, z, t then the reduced differential transform function of $u(x, y, z, t)$ is defined as Srivastava et al.[17],

$$U_k(x, y, z, t) = \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}u(x, y, z, t)\right]_{t=t_0} \tag{1}$$

The inverse differential reduced transformed function of $U_k(x, y, z, t)$ is given by Srivastava et al.[17],

$$u(x, y, z, t) = \sum_{k=0}^{\infty} U_k(x, y, z, t)(t - t_0)^k \tag{2}$$

3. STATEMENT OF THE PROBLEM

Consider the three dimensional rectangular plate with simply supported edges [Fig.1] with dimensions $0 \leq x \leq a$, $0 \leq y \leq b$, $0 \leq z \leq h$. The bending moment along direction $'z'$ and thickness $'h'$.
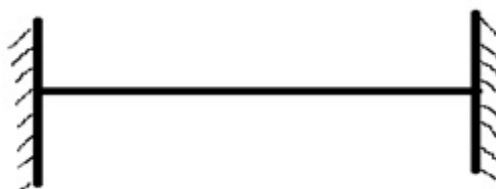

FIGURE 1. Rectangular plate with simply supported edges

The heat conduction equation in three dimensions, Ozicik [15] is given by

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{3}$$

where α = Thermal diffusivity of material,
with initial condition

$$[T(x, y, z, t)]_{t=0} = a_0(x, y, z) \tag{4}$$

The stress function χ is given by the equation, Noda et al. [13],

$$\chi = \chi_c + \chi_p \tag{5}$$

Where the χ_c is complementary function and χ_p is the particular solution of χ.
The χ_c satisfies the following equation, Noda et al. [13],

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\left(\frac{\partial^2 \chi_c}{\partial x^2} + \frac{\partial^2 \chi_c}{\partial y^2} + \frac{\partial^2 \chi_c}{\partial z^2}\right) = 0 \tag{6}$$

and χ_p is obtained from the equation, Noda et al. [13],

$$\frac{\partial^2 \chi_p}{\partial x^2} + \frac{\partial^2 \chi_p}{\partial y^2} + \frac{\partial^2 \chi_p}{\partial z^2} = -\alpha E \Gamma \tag{7}$$

where $\Gamma = T - [T]_{t=0} = b$ (let).
E = Young's Modulus of the material.
The thermal stresses are given by the equations, Noda et al. [13],

$$\sigma_{xx} = \frac{\partial^2 \chi}{\partial y^2}, \sigma_{yy} = \frac{\partial^2 \chi}{\partial x^2}, \sigma_{xy} = -\frac{\partial^2 \chi}{\partial x \partial y} \tag{8}$$

The bending moment regarding thermal stresses in z direction are given by the equation

$$M_x = \int_{\frac{-h}{2}}^{\frac{h}{2}} \sigma_{xx} z \, dz, M_y = \int_{\frac{-h}{2}}^{\frac{h}{2}} \sigma_{yy} z \, dz, M_{xy} = \int_{\frac{-h}{2}}^{\frac{h}{2}} \sigma_{xy} z \, dz. \tag{9}$$
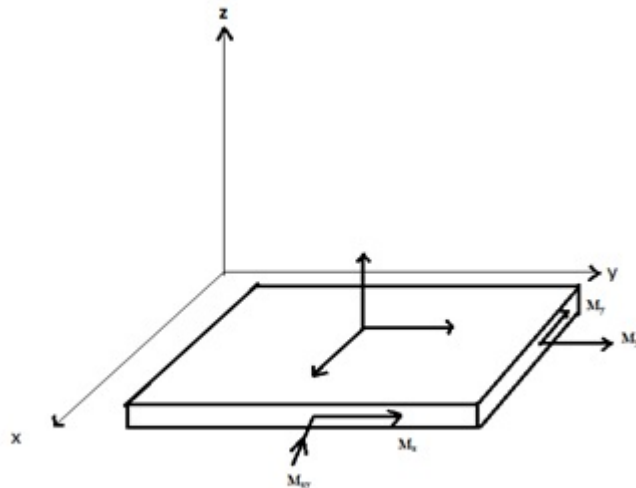

FIGURE 2. Direction of bending moments

4. SOLUTION OF THE PROBLEM

Applying Reduced differential transform to equation (3) and (4) we get

$$(k+1) T_{k+1}(x,y,z) = \alpha \left[\frac{\partial^2}{\partial x^2} T_k(x,y,z) + \frac{\partial^2}{\partial y^2} T_k(x,y,z) + \frac{\partial^2}{\partial z^2} T_k(x,y,z) \right] \tag{10}$$

$$T_0(x,y,z) = a_0(x,y,z) \tag{11}$$

Using (11) in (10) we get,

$$T_k(x,y,z) = \frac{\alpha^k}{k!} a_k, \tag{12}$$

where $a_k = (a_{k-1})_{xx} + (a_{k-1})_{yy} + (a_{k-1})_{zz}$, $k = 1,2,3....$
Applying inverse reduced differential transform [(Equation 2)] we get

$$T(x,y,z,t) = a_0(x,y,z) + \sum_{k=1}^{\infty} \frac{\alpha^k}{k!} a_k t^k. \tag{13}$$

5. DETERMINATION OF STRESS FUNCTION

Assume that the χ_c , which satisfies the equation (6) as

$$\chi_c = f_0 + f_1 x + f_2 x^2 + f_3 x^3 - \frac{1}{24}[(p_0)_{yy} + (p_0)_{zz} + 2(f_2)_{yy} + 2(f_2)_{zz}]x^4$$

$$- \frac{1}{120}[(p_1)_{yy} + (p_1)_{zz} + 6(f_3)_{yy} + 6(f_3)_{zz}]x^5 + \sum_{k=6}^{\infty} \frac{f_k}{k(k-1)(k-2)(k-3)} x^k, \tag{14}$$

where, at $x = 0$, $\chi_c = f_0$,

$$\frac{\partial \chi_c}{\partial x} \quad = \quad f_1(y,z), \frac{\partial^2 \chi_c}{\partial x^2} \quad = \quad 2f_2(y,z), \frac{\partial^3 \chi_c}{\partial x^3} \quad = \quad 6f_3(y,z), \quad (15)$$

and $f_k = -(p_{k-4})_{yy} - (p_{k-4})_{zz} - (k-4)(k-5)\left[(f_{k-2})_{yy} + (f_{k-2})_{zz}\right]$,
$\quad f_4 = (p_0)_{yy} + (p_0)_{zz} + 2(f_2)_{yy} + 2(f_2)_{zz}$,
$\quad f_5 = (p_1)_{yy} + (p_1)_{zz} + 6(f_3)_{yy} + 6(f_3)_{zz}$,
$\quad p_k = (k+1)(k+2)(\chi_c)_{k+2} + \frac{\partial^2}{\partial y^2}(\chi_c)_k + \frac{\partial^2}{\partial z^2}(\chi_c)_k$.
Also assume that the χ_p which satisfies the equation (7)as

$$\chi_p = -\frac{\alpha E}{2} b_0 x^2 - \frac{\alpha E}{6} b_1 x^3 + \frac{1}{12}\{-\alpha E b_2 + \frac{\alpha E}{2}[(b_0)_{yy} + (b_0)_{zz}]\}x^4$$

$$+ \frac{1}{20}\{-\alpha E b_3 + \frac{\alpha E}{6}[(b_1)_{yy} + (b_1)_{zz}]\}x^5 + \sum_{k=6}^{\infty} \frac{q_k}{k(k-1)} x^k, \quad (16)$$

where $\chi_p = 0$ at $x = 0$,
$\quad \frac{\partial \chi_p}{\partial x} = 0$ at $x = 0$,
$\quad q_k = -\alpha E b_{k-2} - \frac{1}{(k-2)(k-3)}[(q_{k-2})_{yy} + (q_{k-2})_{zz}]$,
$\quad q_4 = -\alpha E b_2 + \frac{\alpha E}{2}[(b_0)_{yy} + (b_0)_{zz}]$,

$$q_5 = -\alpha E b_3 + \frac{\alpha E}{6}[(b_1)_{yy} + (b_1)_{zz}].$$

From equations (5),(14) and (16) we get,

$$\chi = f_0 + f_1 x + f_2 x^2 + f_3 x^3 - \frac{1}{24}[(p_0)_{yy} + (p_0)_{zz} + 2(f_2)_{yy} + 2(f_2)_{zz}]x^4$$

$$- \frac{1}{120}[(p_1)_{yy} + (p_1)_{zz} + 6(f_3)_{yy} + 6(f_3)_{zz}]x^5 + \sum_{k=6}^{\infty} \frac{f_k}{k(k-1)(k-2)(k-3)}x^k$$

$$- \frac{\alpha E}{2}b_0 x^2 - \frac{\alpha E}{6}b_1 x^3 + \frac{1}{12}\{-\alpha E b_2 + \frac{\alpha E}{2}[(b_0)_{yy} + (b_0)_{zz}]\}x^4$$

$$+ \frac{1}{20}\{-\alpha E b_3 + \frac{\alpha E}{6}[(b_1)_{yy} + (b_1)_{zz}]\}x^5 + \sum_{k=6}^{\infty} \frac{q_k}{k(k-1)}x^k. \quad (17)$$

6. DETERMINATION OF THERMAL STRESS

The thermal stresses are given by the equation (8) and (17)

$$\sigma_{xx} = (f_0)_{yy} + x(f_1)_{yy} + x^2(f_2)_{yy} + x^3(f_3)_{yy} - \left\{\frac{1}{24}[(p_0)_{yy} + (p_0)_{zz} + 2(f_2)_{yy} + 2(f_2)_{zz}]\right\}_{yy}x^4$$

$$- \{\frac{1}{120}[(p_1)_{yy} + (p_1)_{zz} + 6(f_3)_{yy} + 6(f_3)_{zz}]\}_{yy}x^5 + \sum_{k=6}^{\infty} \frac{(f_k)_{yy}}{k(k-1)(k-2)(k-3)}x^k$$

$$- \frac{\alpha E}{2}(b_0)_{yy}x^2 - \frac{\alpha E}{6}(b_1)_{yy}x^3 + \frac{1}{12}\{-\alpha E(b_2)_{yy} + \frac{\alpha E}{2}[(b_0)_{yy} + (b_0)_{zz}]_{yy}\}x^4$$

$$+ \frac{1}{20}\{-\alpha E(b_3)_{yy} + \frac{\alpha E}{6}[(b_1)_{yy} + (b_1)_{zz}]_{yy}\}x^5 + \sum_{k=6}^{\infty} \frac{(q_k)_{yy}}{k(k-1)}x^k. \quad (18)$$

$$\sigma_{yy} = (f_0)_{xx} + (f_1)_{xx}x + 2f_2 + (f_2)_{xx}x^2 + 6f_3 x + (f_3)_{xx}x^3$$

$$- \frac{1}{24}\{[(p_0)_{yy} + (p_0)_{zz} + 2(f_2)_{yy} + 2(f_2)_{zz}]\}_{xx}x^4 - \frac{1}{2}[(p_0)_{yy} + (p_0)_{zz} + 2(f_2)_{yy} + 2(f_2)_{zz}]x^2$$

$$- \frac{1}{120}\{[(p_1)_{yy} + (p_1)_{zz} + 6(f_3)_{yy} + 6(f_3)_{zz}]\}_{xx}x^5 - \frac{1}{6}[(p_1)_{yy} + (p_1)_{zz} + 6(f_3)_{yy} + 6(f_3)_{zz}]x^3$$

$$+ \sum_{k=6}^{\infty}\{\frac{(f_k)_{xx}}{k(k-1)(k-2)(k-3)}x^k + \frac{f_k k(k-1)}{k(k-1)(k-2)(k-3)}x^{k-2}\}$$

$$- \frac{\alpha E}{2}((b_0)_{xx}x^2 + 2b_0) - \frac{\alpha E}{6}((b_1)_{xx}x^3 + 6xb_1) + \{-\alpha E b_2 + \frac{\alpha E}{2}[(b_0)_{yy} + (b_0)_{zz}]\}x^2$$

$$+ \frac{1}{12}\{-\alpha E(b_2)_{xx} + \frac{\alpha E}{2}[(b_0)_{yy} + (b_0)_{zz}]_{xx}\}x^4 + \{-\alpha E b_3 + \frac{\alpha E}{6}[(b_1)_{yy} + (b_1)_{zz}]\}x^3$$

$$+ \frac{1}{20}\{-\alpha E(b_3)_{xx} + \frac{\alpha E}{6}[(b_1)_{yy} + (b_1)_{zz}]_{xx}\}x^5 + \sum_{k=6}^{\infty}\{\frac{k(k-1)q_k}{k(k-1)}x^{k-2} + \frac{(q_k)_{xx}}{k(k-1)}x^k\}.$$

$$(19)$$

$$\sigma_{xy} = -(f_0)_{xy} - x(f_1)_{xy} - 2x(f_2)_y - x^2(f_2)_{xy} - 3x^2(f_3)_y - (f_3)_{xy}x^3$$

$$+\frac{1}{24}\{[(p_0)_{yy}+(p_0)_{zz}+2(f_2)_{yy}+2(f_2)_{zz}]\}_{xy}x^4+\frac{1}{6}[(p_0)_{yy}+(p_0)_{zz}+2(f_2)_{yy}+2(f_2)_{zz}]_yx^3$$

$$-\frac{1}{120}\{[(p_1)_{yy}+(p_1)_{zz}+6(f_3)_{yy}+6(f_3)_{zz}]\}_{xy}x^5-\frac{1}{24}[(p_1)_{yy}+(p_1)_{zz}+6(f_3)_{yy}+6(f_3)_{zz}]_yx^4$$

$$-\sum_{k=6}^{\infty}\{\frac{(f_k)_{xy}}{k\,(k-1)\,(k-2)\,(k-3)}x^k + \frac{(f_k)_y}{(k-1)\,(k-2)\,(k-3)}x^k\}$$

$$+\frac{\alpha E}{2}((b_0)_{xy}x^2+2(b_0)_yx)+\frac{\alpha E}{6}((b_1)_{xy}x^3+3x^2(b_1)_y)-\frac{1}{12}\{-\alpha Eb_2+\frac{\alpha E}{2}[(b_0)_{yy}+(b_0)_{zz}]\}_{xy}x^4$$

$$-\frac{1}{3}\{-\alpha Eb_2+\frac{\alpha E}{2}[(b_0)_{yy}+(b_0)_{zz}]\}_yx^3-\frac{1}{20}\{-\alpha Eb_3+\frac{\alpha E}{6}[(b_1)_{yy}+(b_1)_{zz}]\}_{xy}x^5$$

$$-\frac{1}{4}\{-\alpha Eb_3+\frac{\alpha E}{6}[(b_1)_{yy}+(b_1)_{zz}]\}_yx^4-\sum_{k=6}^{\infty}\{\frac{(q_k)_{xy}}{k(k-1)}x^k+\frac{(q_k)_y}{(k-1)}x^{k-1}\}. \quad (20)$$

7. DETERMINATION OF BENDING MOMENT

The bending moments of rectangular plate in $'z'$ direction by using equation (9) and equations (18) to (20) are given by

$$M_x = \int_{-\frac{h}{2}}^{\frac{h}{2}}\{(f_0)_{yy} + x(f_1)_{yy} + x^2(f_2)_{yy} + x^3(f_3)_{yy}$$

$$-\left\{\frac{1}{24}[(p_0)_{yy} + (p_0)_{zz} + 2(f_2)_{yy} + 2(f_2)_{zz}]\right\}_{yy}x^4$$

$$-\{\frac{1}{120}[(p_1)_{yy} + (p_1)_{zz} + 6(f_3)_{yy} + 6(f_3)_{zz}]\}_{yy}x^5$$

$$+\sum_{k=6}^{\infty}\frac{(f_k)_{yy}}{k\,(k-1)\,(k-2)\,(k-3)}x^k$$

$$-\frac{\alpha E}{2}(b_0)_{yy}x^2 - \frac{\alpha E}{6}(b_1)_{yy}x^3 + \frac{1}{12}\{-\alpha E(b_2)_{yy}+\frac{\alpha E}{2}[(b_0)_{yy}+(b_0)_{zz}]_{yy}\}x^4$$

$$+\frac{1}{20}\{-\alpha E(b_3)_{yy}+\frac{\alpha E}{6}[(b_1)_{yy}+(b_1)_{zz}]_{yy}\}x^5+\sum_{k=6}^{\infty}\frac{(q_k)_{yy}}{k(k-1)}x^k\}zdz. \quad (21)$$

$$M_y = \int_{\frac{-h}{2}}^{\frac{h}{2}} \{(f_0)_{xx} + (f_1)_{xx}x + 2f_2 + (f_2)_{xx}x^2 + 6f_3x + (f_3)_{xx}x^3$$

$$-\frac{1}{24}\{[(p_0)_{yy}+(p_0)_{zz}+2(f_2)_{yy}+2(f_2)_{zz}]\}_{xx}x^4 - \frac{1}{2}[(p_0)_{yy}+(p_0)_{zz}+2(f_2)_{yy}+2(f_2)_{zz}]x^2$$

$$-\frac{1}{120}\{[(p_1)_{yy}+(p_1)_{zz}+6(f_3)_{yy}+6(f_3)_{zz}]\}_{xx}x^5 - \frac{1}{6}[(p_1)_{yy}+(p_1)_{zz}+6(f_3)_{yy}+6(f_3)_{zz}]x^3$$

$$+\sum_{k=6}^{\infty}\{\frac{(f_k)_{xx}}{k\,(k-1)\,(k-2)\,(k-3)}x^k + \frac{f_k k(k-1)}{k\,(k-1)\,(k-2)\,(k-3)}x^{k-2}\}$$

$$-\frac{\alpha E}{2}((b_0)_{xx}x^2 + 2b_0) - \frac{\alpha E}{6}((b_1)_{xx}x^3 + 6xb_1) + \{-\alpha E b_2 + \frac{\alpha E}{2}[(b_0)_{yy}+(b_0)_{zz}]\}x^2$$

$$+\frac{1}{12}\{-\alpha E(b_2)_{xx} + \frac{\alpha E}{2}[(b_0)_{yy}+(b_0)_{zz}]_{xx}\}x^4 + \{-\alpha E b_3 + \frac{\alpha E}{6}[(b_1)_{yy}+(b_1)_{zz}]\}x^3$$

$$+\frac{1}{20}\{-\alpha E(b_3)_{xx} + \frac{\alpha E}{6}[(b_1)_{yy}+(b_1)_{zz}]_{xx}\}x^5 + \sum_{k=6}^{\infty}\{\frac{k(k-1)q_k}{k(k-1)}x^{k-2} + \frac{(q_k)_{xx}}{k(k-1)}x^k\}\}zdz.$$

$$(22)$$

$$M_{xy} = \int_{\frac{-h}{2}}^{\frac{h}{2}} \{-(f_0)_{xy} - x(f_1)_{xy} - 2x(f_2)_y - x^2(f_2)_{xy} - 3x^2(f_3)_y - (f_3)_{xy}x^3$$

$$+\frac{1}{24}\{[(p_0)_{yy}+(p_0)_{zz}+2(f_2)_{yy}+2(f_2)_{zz}]\}_{xy}x^4 + \frac{1}{6}[(p_0)_{yy}+(p_0)_{zz}+2(f_2)_{yy}+2(f_2)_{zz}]_y x^3$$

$$-\frac{1}{120}\{[(p_1)_{yy}+(p_1)_{zz}+6(f_3)_{yy}+6(f_3)_{zz}]\}_{xy}x^5 - \frac{1}{24}[(p_1)_{yy}+(p_1)_{zz}+6(f_3)_{yy}+6(f_3)_{zz}]_y x^4$$

$$-\sum_{k=6}^{\infty}\{\frac{(f_k)_{xy}}{k\,(k-1)\,(k-2)\,(k-3)}x^k + \frac{(f_k)_y}{(k-1)\,(k-2)\,(k-3)}x^k\}$$

$$+\frac{\alpha E}{2}((b_0)_{xy}x^2 + 2(b_0)_y x) + \frac{\alpha E}{6}((b_1)_{xy}x^3 + 3x^2(b_1)_y) - \frac{1}{12}\{-\alpha E b_2 + \frac{\alpha E}{2}[(b_0)_{yy}+(b_0)_{zz}]\}_{xy}x^4$$

$$-\frac{1}{3}\{-\alpha E b_2 + \frac{\alpha E}{2}[(b_0)_{yy}+(b_0)_{zz}]\}_y x^3 - \frac{1}{20}\{-\alpha E b_3 + \frac{\alpha E}{6}[(b_1)_{yy}+(b_1)_{zz}]\}_{xy}x^5$$

$$-\frac{1}{4}\{-\alpha E b_3 + \frac{\alpha E}{6}[(b_1)_{yy}+(b_1)_{zz}]\}_y x^4 - \sum_{k=6}^{\infty}\{\frac{(q_k)_{xy}}{k(k-1)}x^k + \frac{(q_k)_y}{(k-1)}x^{k-1}\}\}zdz.$$

$$(23)$$

8. SPECIAL CASES

Here we consider the copper plate with thickness $h = 4m, t = 1sec$ for the special case. All numerical values have been considered regarding the copper plate.

Consider some special functions which satisfies all the conditions in equation (4) and (15) as

$a_0(x,y,z) = x^2 y^2 z$
$f_0 = z^6 - y^6 + 2y^3 z^3$
$f_1 = 0 = f_2$
$f_3 = 2y^3 + 2z^3$.

From equation (14) and (16) we get,

$\chi_c = z^6 - y^6 + 2y^3 z^3 + 2x^3 y^3 + 2x^3 z^3 + 15x^4 y^2 - 6x^4 yz - 15z^2 x^4 - \frac{6}{5}x^5 y - \frac{6}{5}x^5 z$

$\chi_p = -\alpha E x^2 \{\alpha y^2 z t + 2\alpha^2 z t^2\}.$

Hence from equation (5) we get,

$\chi = z^6 - y^6 + 2y^3 z^3 + 2x^3 y^3 + 2x^3 z^3 + 15x^4 y^2 - 6x^4 yz - 15z^2 x^4$

$-\dfrac{6}{5}x^5 y - \dfrac{6}{5}x^5 z - \alpha E x^2 \{\alpha y^2 z t + 2\alpha^2 z t^2\}.$

From equation (8) we get thermal stresses are

$\sigma_{xx} = -30y^4 + 12yz^3 + 12x^3 y + 30x^4 - 2\alpha^2 z E x^2 t$

$\sigma_{yy} = 12xy^3 + 12xz^3 + 180x^2 y^2 - 72x^2 yz - 180z^2 x^2 - 24x^3 y - 24x^3 z$

$-\alpha E\{2\alpha y^2 z t + 4\alpha^2 z t^2\}$

$\sigma_{xy} = -18x^2 y^2 - 120x^3 y + 6x^4 + 24x^3 yz + 4\alpha^2 E xyzt.$

From equation (9) we get the bending moments as

$M_x = \dfrac{768y}{5} - \dfrac{32\alpha^2 E x^2 t}{3}$

$M_y = \dfrac{768x}{5} - 384x^2 y - 128x^3 - \dfrac{16\alpha E}{3}\{2\alpha y^2 t + 4\alpha^2 t^2\}$ $M_{xy} = 128x^3 y + \dfrac{64\alpha^2 E xyt}{3}.$
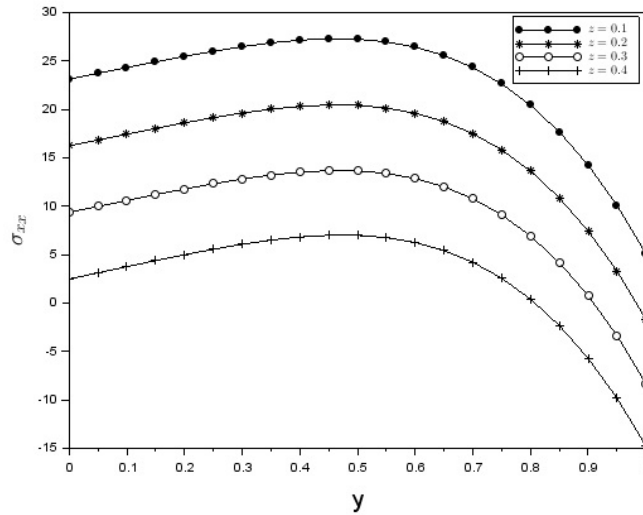

FIGURE 3. σ_{xx} Vs $'y'$ with variation in $'z'$

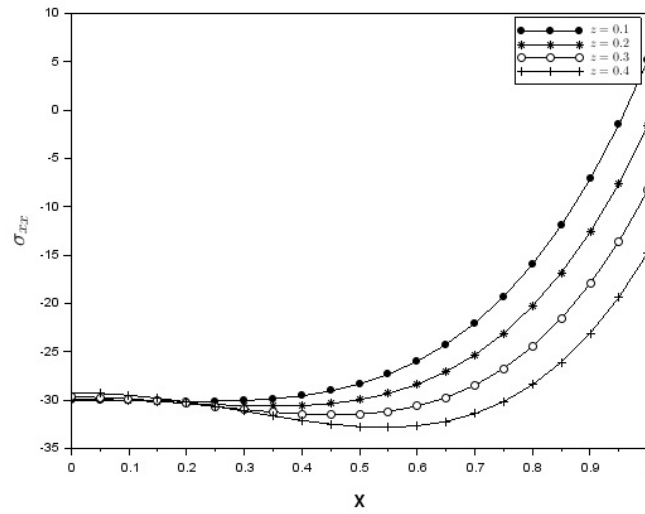FIGURE 4. σ_{xx} Vs $'x'$ with variation in $'z'$

Fig-3 and Fig-4 gives the relations of thermal stresses along with breadth(y) and length(x) respectively for given changes in heights. Whenever changes in breadth the stress decreases suddenly and an increase in length implies the sudden increase in stress.
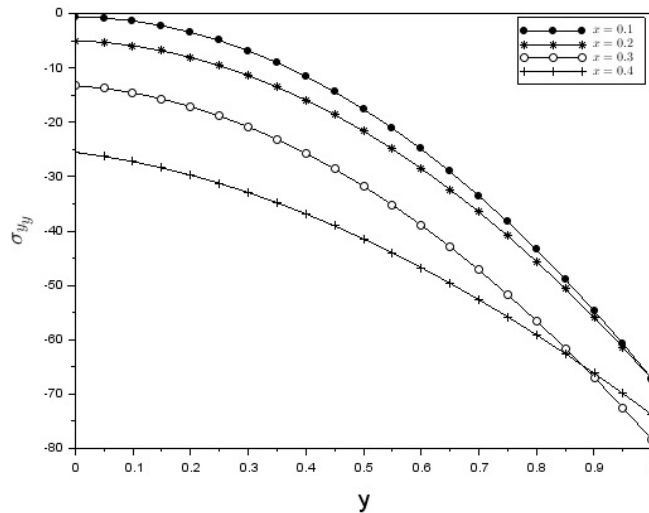

FIGURE 5. σ_{yy} Vs $'y'$ with variation in $'x'$

Fig-5 explains the nature of thermal stress along with the breadth of the plate for the variations in length of the plate at constant heights. For the change in breadth of the plate there is a sudden change in stress and the intersection point sometime indicates that whenever there is not much changes in length, the thermal
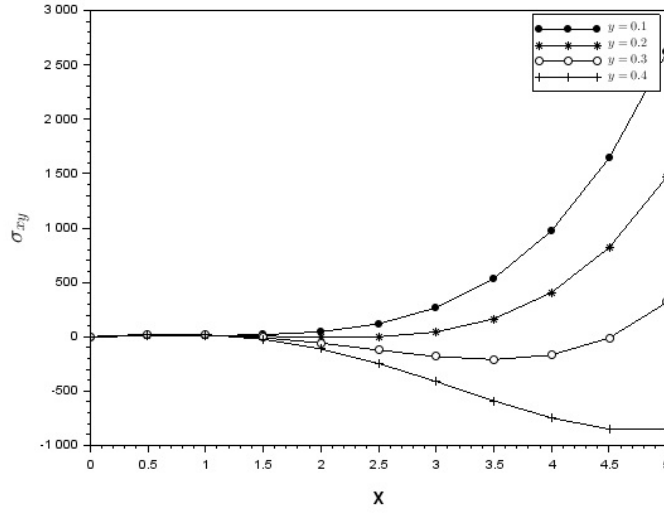
stresses remains constant.

Figure 6. σ_{xy} Vs $'y'$ with variation in $'x'$

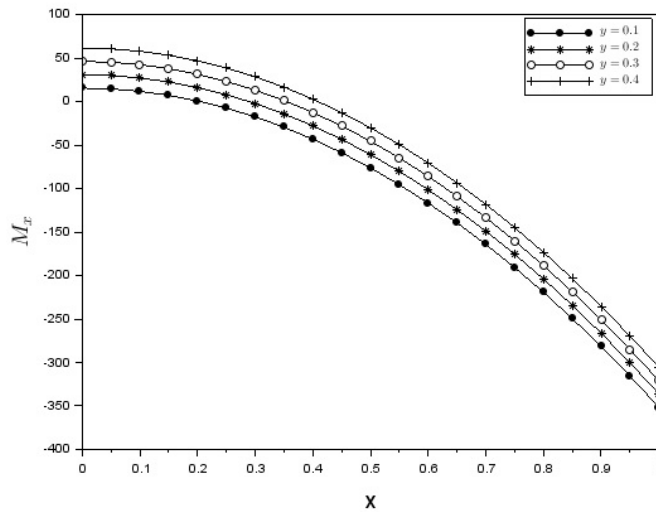Fig-6 gives the behavior of thermal stress with respective length at constant heights for changes in breadth.

Figure 7. M_x Vs $'x'$ with variation in $'y'$

Fig-7 explains the nature of bending moments with the length of the plate and the variations in breadth. The above figures explain the result that the bending
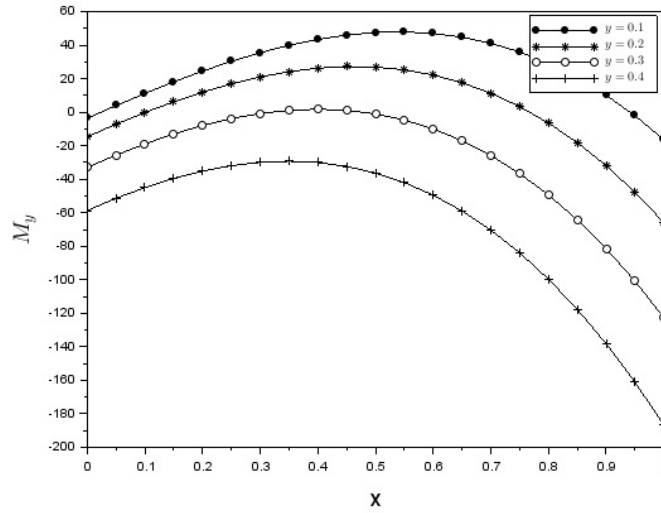
moment decreases along the length.

FIGURE 8. M_y Vs $'x'$ with variation in $'y'$

Fig-8 gives the variation of bending moments of the plate along the z-direction. The above figures explain the result that the bending moment increases along the length and the breadth of the rectangular plate, but sudden change in the bending moment decreases along the length.
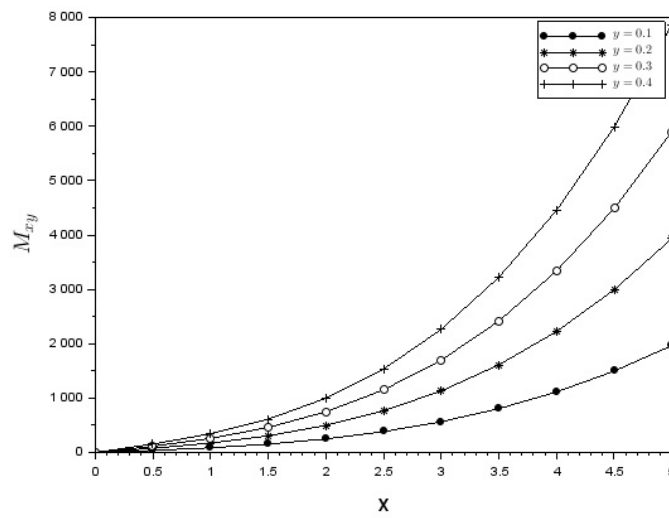

FIGURE 9. M_{xy} Vs $'x'$ with variation in $'y'$

Fig-9 gives the variation of bending moments of the plate along the z-direction. The above figures explain the result that the bending moment increases along the length and the breadth of the rectangular plate. That means the bending moment depends on the dimensions of the rectangular plate.

9. CONCLUSIONS

In this paper, we consider the thick rectangular plate with simply-supported edges. The results are obtained by using the reduced differential transform method. We have determined the thermal stresses with non-homogeneous initial boundary conditions. The bending moments and thermal stresses are obtained in the form of Taylor's series. By considering the special case for the copper plate the results are discussed graphically. It is observed that the bending of the plate depends on the dimensions of the plate. The thermal stresses are also related to the dimension of the plate.

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