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ON φ-2-ABSORBING AND φ-2-ABSORBING PRIMARY HYPERIDEALS IN MULTIPLICATIVE HYPERRINGS

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Abstract. In this paper, we extend the notion of 2-absorbing hyperideals and 2-absorbing primary hyperideals to the concept φ-2-absorbing hyperideals and φ-2-absorbing primary hyperideals. Suppose that $E(R)$ is the set of hyperideals of R and $\varphi : E(R) \longrightarrow E(R) \cup \{\phi\}$ is a function. A nonzero proper hyperideal I of R is said to be a φ- 2-absorbing hyperideal if for $x, y, z \in R, xoyoz \subseteq I - \varphi(I)$ implies $xoy \subseteq I$ or $xoz \subseteq I$ or $yoz \subseteq I$. Also, a nonzero proper hyperideal I of R is called a φ- 2-absorbing primary hyperideal if for all $x, y, z \in R,\ xoyoz \subseteq I - \varphi(I)$ implies $xoy \subseteq I$ or $xoz \subseteq r(I)$ or $yoz \subseteq r(I)$. A number of results concerning them are given.

Key words and Phrases: φ-2-absorbing hyperideal; φ-2-absorbing primary hyperideal; Hyperring.

Abstrak. Dalam paper ini, diperluas konsep 2-*absorbing hyperideal* dan 2-*absorbing primary hyperideal* menjadi φ-2-*absorbing hyperideal* dan φ-2-*absorbing primary hyperideal*. Misalkan $E(R)$ merupakan himpunan *hyperideal* dari R, dimana $\varphi : E(R) \longrightarrow E(R) \cup \{\phi\}$ merupakan suatu fungsi. Suatu proper *hyperideal* tak-nol I dari R disebut φ- 2-*absorbing hyperideal* jika untuk $x, y, z \in R, xoyoz \subseteq I - \varphi(I)$ mengakibatkan $xoy \subseteq I$ or $xoz \subseteq I$ or $yoz \subseteq I$. Lebih jauh, suatuproper *hyperideal* tak-nol I dari R disebut φ- 2-*absorbing primary hyperideal* jika untuk semua $x, y, z \in R,\ xoyoz \subseteq I - \varphi(I)$ mengakibatkan $xoy \subseteq I$ or $xoz \subseteq r(I)$ or $yoz \subseteq r(I)$. Sejumlah hasil terkait perluasan konsep ini diberikan.

Key words and Phrases: φ-2-*absorbing hyperideal*; φ-2-*absorbing primary hyperideal*; *Hyperring*.

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1. Introduction

Hyperstructures (or hypersystems), as a natural generalization of ordinary algebraic structures were introduced by Marty, the French mathematician, in 1934 [10]. Later on, it was found that the theory also have many applications in both applied and pure sciences. They can be studied in [5],[8],[11], [6] and [13]. The concept of multiplicative hyperring is introduced and studied by R. Rota [12] in 1982. For example, their applications in physics and chemistry can be found in Chapter 8 in [8].

An algebra system $(R, +, o)$ is a multiplicative hyperring if

(a) $(R, +)$ is an abelian group;
(b) (R, o) is semihypergroup;
(c) We have $ao(b+c) \subseteq aob + aoc$ and $(b+c)oa \subseteq boa + coa$, for all $a, b, c \in R$;
(d) We have $ao(-b) = (-a)ob = -(aob)$, for all $a, b \in R$.

Throughout this paper R denotes a multiplicative hyperring. Let A and B be two nonempty subsets of R and $x \in R$. We define

$$Aox = Ao\{x\}, \qquad AoB = \bigcup_{a \in A,\ b \in B} aob$$

Let I be a non empty subset of a multiplicative hyperring R. I is a hyperideal of R if

(1) $a - b \in I$, for all $a, b \in I$;
(2) $rox \subseteq I$, for all $r \in R$ and $x \in I$.

The notion of ϕ-ideal was studied by Anderson in [2]. The purpose of the paper is to generalize the idea of ϕ-prime ideal as in [2], [3] and [4] to the context of hyperideals of hyperrings.

In a conference, a researcher introduced the context of 2-absorbing hyperideal and got many results[9]. Indeed, it is a generalization of prime hyperideal. A proper hyperideal I of R is 2-absorbing if $xoyoz \subseteq I$ with $x, y, z \in R$, then $xoy \subseteq I$ or $xoz \subseteq I$ or $yoz \subseteq I$. Moreover, recall from [1] that a proper hyperideal I of R is said to be 2-absorbing primary hyperideal if $x, y, z \subseteq I$ with $x, y, z \in R$, then $xoy \subseteq I$ or $yoz \subseteq r(I)$ or $xoz \subseteq r(I)$.
A proper hyperideal of R is a prime hyperideal if $xoy \subseteq P$ with $x, y \in R$ implies that $x \in P$ or $y \in P$. The prime radical of I is the intersection of all prime hyperideals of R containing I which being denoted by $r(I)$. If R has no prime hyperideal containing I, we say $r(I) = R$. Let $\mathbf{C} = \{r_1 o r_2 o ... o r_n \ : \ r_i \in R, n \in \mathbb{N}\} \subseteq P^*(R)$. A hyperideal I of R is called a $\mathbf{C}$-hyperideal if $A \cap I \neq \varnothing$ for any $A \in \mathbf{C}$ then $A \subseteq I$. Let $D = \{r \in R : r^n \subseteq I \ for \ some \ n \in \mathbb{N}\}$ then $D \subseteq r(I)$. The equality holds where I is a $\mathbf{C}$-hyperideal of R(see proposition 3.2 in [7]). Throughout this paper, we suppose that all hyperideals are $\mathbf{C}$-hyperideal.
A hyperideal I of R is a strong $\mathbf{C}$-hyperideal of R if , $E \cap I \neq \varnothing$ for any $E \in \mathfrak{U}$ then $E \subseteq I$, where $\mathbf{C} = \{r_1 o r_2 o ... o r_n \ : \ r_i \in R, n \in \mathbb{N}\}$ and

$\mathfrak{U} = \{\sum_{i=1}^{n} A_i \ : \ A_i \in \mathbf{C}, n \in \mathbb{N}\}$. Let I be a hyperideal of R and $E(R)$ be the set of hyperideals of R. Throughout this paper, we assume that $\varphi(I) \subseteq I$ where $\varphi : E(R) \longrightarrow E(R) \cup \{\phi\}$ is a function.

A nonzero proper hyperideal I of R is said a φ-prime hyperideal if $xoy \subseteq I - \varphi(I)$ for some $x, y \in R$, implies $x \in I$ or $y \in I$.

In this paper, among many results , it is shown that for some function φ if I is a φ-2-absorbing hyperideal of R that is not a 2-absorbing hyperideal of R, then $I^3 \subseteq \varphi(I)$ (Theorem 2.6). It is shown (Theorem 2.10) that if R_1 and R_2 be two multiplicative hyperrings with scalar identity such that
$\varphi_1 : E(R_1) \longrightarrow E(R_1) \cup \{\phi\}, \varphi_2 : E(R_2) \longrightarrow E(R_2) \cup \{\phi\}$ be functions and $\varphi = \varphi_1 \times \varphi_2$. Then:

(1) $I_1 \times I_2$ is a φ- 2-absorbing hyperideal of $R_1 \times R_2$, such that I_1 and I_2 are two proper hyperideals of R_1 and R_2, respectively, with $\varphi_1(I_1) = I_1$ and $\varphi_2(I_2) = I_2$.

(2) $I_1 \times R_2$ is a φ- 2-absorbing hyperideal of $R_1 \times R_2$, such that I_1 is a φ_1- 2-absorbing hyperideal of R_1 which is 2-absorbing hyperideal if $\varphi_2(R_2) \neq R_2$.

(3) $R_1 \times I_2$ is a φ- 2-absorbing hyperideal of $R_1 \times R_2$, such that I_2 is a φ_2- 2-absorbing hyperideal of R_2 which is 2-absorbing hyperideal if $\varphi_1(R_1) \neq R_1$.

(4) If $I_1 \times I_2$ is a φ-2-absorbing hyperideal of $R_1 \times R_2$, then I_1 is a φ_1- 2-absorbing hyperideal of R_1 and I_2 is a φ_2- 2-absorbing hyperideal of R_2.

It is shown (Theorem 3.11) that if I and J are proper hyperideals of R with $J \subseteq \varphi(I)$ where $\varphi : E(R) \longrightarrow E(R) \cup \{\phi\}$ is a function. Then followings are equivalent.

i) Hyperideal I of R is a φ-2-absorbing primary hyperideal.
ii) Hyperideal I/J of R/J is a φ_J -2-absorbing primary hyperideal.
iii) For every $n \geq 1$, hyperideal I/J^n of R/J^n is a $\varphi_{(n)}$ -2-absorbing primary hyperideal.

2. φ-2-ABSORBING HYPERIDEALS

Definition 2.1. *Suppose that $E(R)$ be the set of hyperideals in R and $\varphi : E(R) \longrightarrow E(R) \cup \{\phi\}$ be a function. A nonzero proper hyperideal I of R is a φ- 2-absorbing hyperideal if $xoyoz \subseteq I - \varphi(I)$ with $x, y, z \in R$ implies $xoy \subseteq I$ or $yoz \subseteq I$ or $xoz \subseteq I$.*

Lemma 2.2. *Every 2-absorbing hyperideal and every φ-prime hyperideal are φ-2-absorbing hyperideal.*

Example 2.3. *Let $\mathbb{Z}_A$ be a multiplicative hyperring of integers with $A = \{5, 7\}$. In this hyperring, $< 2 > \cap < 3 >$ is φ- 2-absorbing hyperideal.*

Definition 2.4. *Let $\varphi_1, \varphi_2 : E(R) \longrightarrow E(R) \cup \{\phi\}$ be functions. If $\varphi_1(I) \subseteq \varphi_2(I)$ for every $I \in E(R)$, then we define $\varphi_1 \leq \varphi_2$.*

Theorem 2.5. *Let I be a hyperideal in R and $\varphi_1, \varphi_2 : E(R) \longrightarrow E(R) \cup \{\phi\}$ be functions with $\varphi_1 \leq \varphi_2$. Let I be a φ_1-2-absorbing hyperideal, then hyperideal I is a φ_2-2-absorbing hyperideal.*

Proof. Let $xoyoz \subseteq I - \varphi_2(I)$, with $x, y, z \in R$. Since $\varphi_1 \leq \varphi_2$, we obtain $xoyoz \subseteq I - \varphi_1(I)$. Since hyperideal I is a φ_1-2-absorbing hyperideal, then we conclude $xoy \subseteq I$ or $yoz \subseteq I$ or $xoz \subseteq I$. Therefore I is φ_2-2-absorbing. □

Theorem 2.6. *Let $\varphi : E(R) \longrightarrow E(R) \cup \{\phi\}$ be a function and hyperideal $I \in E^*(R)$ is not 2-absorbing. If I is φ- 2-absorbing strong* **C***-hyperideal then $I^3 \subseteq \varphi(I)$.*

Proof. We assume that $I^3 \nsubseteq \varphi(I)$ and look for a contradiction. Let $xoyoz \subseteq I$ where $x, y, z \in R$. Since hyperideal I is a φ-2-absorbing hyperideal, If $xoyoz \nsubseteq \varphi(I)$, then $xoy \subseteq I$ or $xoz \subseteq I$ or $yoz \subseteq I$. Let us suppose $xoyoz \subseteq \varphi(I)$. We assume $xoyoI \nsubseteq \varphi(I)$. So $xoyow \nsubseteq \varphi(I)$ for some $w \in I$. Thus $xoyoz + xoyow \subseteq I - \varphi(I)$ and so $xoyo(z+w) \subseteq I$. Hence $xoy \subseteq I$ or $xo(z+w) \subseteq I$ or $yo(z+w) \subseteq I$. Since I is a strong **C**-hyperideal and $xow, yow \subseteq I$, we obtain $xoy \subseteq I$ or $xoz \subseteq I$ or $yoz \subseteq I$. Thus, let us suppose $xoyoI \subseteq \varphi(I)$ and also $xozoI \subseteq \varphi(I)$ and $yozoI \subseteq \varphi(I)$. Since we supposed that $I^3 \nsubseteq \varphi(I)$, then $w_1ow_2ow_3 \nsubseteq \varphi(I)$ for some $w_1, w_2, w_3 \in I$. So $(x + w_1)o(y + w_2)o(z + w_3) \subseteq I - \varphi(I)$. Since hyperideal I is φ- 2-absorbing, we conclude that $(x+w_1)o(y+w_2) \subseteq I$ or$(y+w_2)o(z+w_3) \subseteq I$ or$(x+w_1)o((z+w_3) \subseteq I$. Since I is a strong **C**-hyperideal we have $xoy \subseteq I$ or $xoz \subseteq I$ or $yoz \subseteq I$. But this is a contradiction. □

Corollary 2.7. *Let I be a φ- 2-absorbing strong* **C***-hyperideal and $I^3 \nsubseteq \varphi(I)$. Then hyperideal I is 2-absorbing.*

Theorem 2.8. *Let $\varphi : E(R) \longrightarrow E(R) \cup \{\phi\}$ be a function and I be a proper strong* **C***-hyperidea of R. The followings are equivalent:*

1) *hyperideal I is φ- 2-absorbing.*
2) *$(I : ab) = (\varphi(I) : aob) \cup (I : a) \cup (I : b)$, for $a, b \in R$ such that $aob \subseteq R - I$.*
3) *$(I : ab) = (I : a)$ or $(I : ab) = (I : b)$ or $(I : ab) = (\varphi(I) : aob)$, for $a, b \in R$ such that $aob \subseteq R - I$.*
4) *If $I_1oI_2oI_3 \subseteq I$ for some hyperideals I_1, I_2, I_3 of R with $I_1oI_2oI_3 \nsubseteq \varphi(I)$, then $I_1oI_2 \subseteq I$ or $I_2oI_3 \subseteq I$ or $I_1oI_3 \subseteq I$.*

Proof. (1)⇒(2) Let $aob \subseteq R - I$ and $c \in (I : aob)$. Thus $aoboc \subseteq I$. Suppose that $aoboc \nsubseteq \varphi(I)$. It means $aoc \subseteq I$ or $boc \subseteq I$, and then $c \in (I : a)$ or $c \in (I : b)$. Now, assume that $aoboc \subseteq \varphi(I)$. So $c \in (\varphi(I) : aob)$ and then $(I : aob) \subseteq (\varphi(I) : aob) \cup (I : a) \cup (I : b)$. Since $\varphi(I) \subseteq I$, then $(\varphi(I) : aob) \cup (I : a) \cup (I : b) \subseteq (I : aob)$. Thus $(I : aob) = (\varphi(I) : aob) \cup (I : a) \cup (I : b)$.
(2)⇒(3) Since $(I : ab) = (\varphi(I) : aob) \cup (I : a) \cup (I : b)$, then it is equel to one of them.
(3)⇒(4) Let $I_1oI_2oI_3 \subseteq I$ for some hyperideals I_1, I_2, I_3 of R. Assume that $I_1oI_2 \nsubseteq I$ and $I_2oI_3 \nsubseteq I$ and $I_1oI_3 \nsubseteq I$, we look for a contradiction. Let $aob \subseteq I_1oI_2$ and

$aob \nsubseteq I$. Since $aoboI_3 \subseteq I$, then $I_3 \subseteq (I : aob)$. Since $I_1oI_3 \nsubseteq I$ and $I_2I_3 \nsubseteq I$, we have $(I : aob) = (\varphi(I) : aob)$. Thus $aoboI_3 \subseteq \varphi(I)$. Now, suppose that $aob \subseteq I \cap (I_1oI_2)$ and $xoy \subseteq (I_1oI_2) - I$ for some $x, y \in R$. Thus $(aob + xoy)oI_3 \subseteq \varphi(I)$. Thus we have $(aob+xoy)oc \subseteq \varphi(I)$ for some $c \in I_3$. Since I is a strong **C**-hyperideal, then $aoboc + xoyoc \subseteq \varphi(I)$ and so $aoboc \subseteq \varphi(I)$. Consequently, $I_1oI_2oI_3 \subseteq \varphi(I)$ and this is a contradiction.
(4)⇒(1) Assume that $aoboc \subseteq I - \varphi(I)$. Hence $\prec a \succ o \prec b \succ o \prec c \succ \subseteq I - \varphi(I)$. Thus $\prec a \succ o \prec b \succ \subseteq I$ or $\prec a \succ o \prec c \succ \subseteq I$ or $\prec b \succ o \prec c \succ \subseteq I$. Hence $aob \subseteq I$ or $aoc \subseteq I$ or $boc \subseteq I$. □

Theorem 2.9. *Assume that R_1 and R_2 are two multiplicative hyperrings with scalar identity. If I_1 and I_2 are hyperideasls of R_1 and R_2, respectively. Then:*

(i) *Hyperideal I_1 is 2-absorbing if and only if hyperideal $I_1 \times R_2$ of $R_1 \times R_2$ is 2-absorbing .*

(ii) *Hyperideal I_2 is 2-absorbing if and only if hyperideal $R_1 \times I_2$ of $R_1 \times R_2$ is 2-absorbing.*

Proof. (i) Suppose that I_1 is a 2-absorbing hyperideal of R_1 and $(x, y)o(z, u)o(v, w) \subseteq I_1 \times R_2$. Since

$$(x, y)o(z, u)o(v, w) = \{(a, b) \mid a \in xozov, b \in youow\},$$

we have $xozov \subseteq I_1$, and hence $xoz \subseteq I_1$ or $xov \subseteq I_1$ or $xov \subseteq I_1$. Thus $(x, y)o(z, u) \subseteq I_1 \times R_2$ or $(z, u)o(v, w) \subseteq I_1 \times R_2$ or $(x, y)o(v, w) \subseteq I_1 \times R_2$. Conversely, Assume that $xoyoz \subseteq I_1$. Thus $(x, 1)o(y, 1)o(z, 1) \subseteq I_1 \times R_2$. Hyperideal $I_1 \times R_2$ is 2-absorbing. It means $(x, 1)o(y, 1) \subseteq I_1 \times R_2$ or $(x, 1)o(z, 1) \subseteq I_1 \times R_2$ or $(y, 1)o(z, 1) \subseteq I_1 \times R_2$. Thus $xoy \subseteq I_1$ or $xoz \subseteq I_1$ or $yoz \subseteq I_1$, and hence I_1 is a 2-absorbing hyperideal of R_1. The proof of (ii) is similar to (i). □

Theorem 2.10. *Let R_1 and R_2 be multiplicative hyperrings with scalar identity and let $\varphi_1 : E(R_1) \longrightarrow E(R_1) \cup \{\phi\}, \varphi_2 : E(R_2) \longrightarrow E(R_2) \cup \{\phi\}$ be functions. Let $\varphi = \varphi_1 \times \varphi_2$. Then:*

(1) *$I_1 \times I_2$ is a φ- 2-absorbing hyperideal of $R_1 \times R_2$, such that I_1 and I_2 are proper hyperideals of R_1 and R_2, respectively, with $\varphi_1(I_1) = I_1$ and $\varphi_2(I_2) = I_2$.*

(2) *$I_1 \times R_2$ is a φ- 2-absorbing hyperideal of $R_1 \times R_2$, such that I_1 is a φ_1- 2-absorbing hyperideal of R_1 which is 2-absorbing hyperideal when $\varphi_2(R_2) \neq R_2$.*

(3) *$R_1 \times I_2$ is a φ- 2-absorbing hyperideal of $R_1 \times R_2$, such that I_2 is a φ_2- 2-absorbing hyperideal of R_2 which is 2-absorbing hyperideal when $\varphi_1(R_1) \neq R_1$.*

(4) *If hyperideal $I_1 \times I_2$ of $R_1 \times R_2$ is φ-2-absorbing, then hyperideal I_1 of R_1 is φ_1- 2-absorbing and hyperideal I_2 of R_2 is φ_2- 2-absorbing.*

Proof. (1) It follows by the fact that $I_1 \times I_2 - \varphi(I_1 \times I_2) = I_1 \times I_2 - \varphi_1(I_1) \times \varphi_2(I_2) = I_1 \times I_2 - I_1 \times I_2 = \phi$.
(2) Let hyperideal I_1 of R_1 be 2-absorbing. Then hyperideal $I_1 \times R_2$ is 2-absorbing and so φ-2-asorbing hyperideal. Now, assume that I_1 is a φ_1-2-asorbing hyperideal and $\varphi_2(R_2) = R_2$. Also, assume that $(x_1, y_1)o(x_2, y_2)o(x_3, y_3) \subseteq I_1 \times R_2 - \varphi_1(I_1) \times \varphi_2(R_2)$. Since $I_1 \times R_2 - \varphi_1(I_1) \times R_2 = (I_1 - \varphi_1(I_1)) \times R_2$.Clearly, $x_1 o x_2 o x_3 \subseteq I_1 - \varphi_1(I_1)$. Thus $x_1 o x_2 \subseteq I_1$ or $x_2 o x_3 \subseteq I_1$ or $x_1 o x_3 \subseteq I_1$. Hence $(x_1, y_1)o(x_2, y_2) \subseteq I_1 \times R_2$ or $(x_2, y_2)o(x_3, y_3) \subseteq I_1 \times R_2$ or $(x_1, y_1)o(x_3, y_3) \subseteq I_1 \times R_2$ and So $I_1 \times R_2$ is a φ- 2-absorbing hyperideal.
(3)The proof of case (3) is similar to (2).
(4) Suppose that hyperideal $I_1 \times I_2$ of $R_1 \times R_2$ is φ- 2-absorbing. Let $xoyoz \subseteq I_1 - \varphi_1(I_1)$. Therefore $(x, 0)o(y, 0)o(z, 0)$ is a subset of $I_1 \times I_2 - \varphi(I_1 \times I_2)$ and so $(x, 0)o(y, 0) \subseteq I_1 \times I_2$ or $(y, 0)o(z, 0) \subseteq I_1 \times I_2$ or $(x, 0)o(z, 0) \subseteq I_1 \times I_2$.Thus $xoy \subseteq I$ or $xoz \subseteq I$ or $yoz \subseteq I$and so I_1 is a φ_1- 2-absorbing hyperideal of R_1. Similarly, I_2 is a φ_2- 2-absorbing hyperideal of R_2. □

3. φ-2-ABSORBING PRIMARY HYPERIDEALS

Definition 3.1. *Let $\varphi : E(R) \longrightarrow E(R) \cup \{\phi\}$ be a function such that $E(R)$ be the set of hyperideals of R. A nonzero proper hyperideal I in R is called a φ- 2-absorbing primary hyperideal if $xoyoz \subseteq I - \varphi(I)$, for all $x, y, z \in R$ implies $xoy \subseteq I$ or $xoz \subseteq r(I)$ or $yoz \subseteq r(I)$.*

If $\varphi(I) = I^n$ for every $I \in E(R)$ and $n \geq 2$, then we define $\varphi = \varphi_{(n)}$ and say that I is a $\varphi_{(n)}$-2-absorbing primary hyperideal.

Theorem 3.2. *Let $r(I) = I$. Hyperideal I of R is $\varphi_{(n)}$ - 2-absorbing primary if and only if hyperideal I is $\varphi_{(n)}$ -2-absorbing.*

Proof. By Proposition 3.3 in [7], we have $r(r(I)) = r(I)$. Thus this is clear. □

Theorem 3.3. *Let $\varphi_1, \varphi_2 : E(R) \longrightarrow E(R) \cup \{\phi\}$ be functions with $\varphi_1 \leq \varphi_2$. If hyperideal I is φ_1-2-absorbing primary, then I is a φ_2-2-absorbing primary hyperideal.*

Proof. Let $xoyoz \subseteq I - \varphi_2(I)$ with $x, y, z \in R$. Therefore $xoyoz \subseteq I - \varphi_1(I)$. Consequently, we are done. □

Theorem 3.4. *Let $\varphi : E(R) \longrightarrow E(R) \cup \{\phi\}$ be a function and hyperideal $I \in E^*(R)$ is not 2-absorbing primary. If I is φ- 2-absorbing primary strong* **C***-hyperideal, then $I^3 \subseteq \varphi(I)$.*

Proof. The proof is similar to Theorem 2.6 □

Corollary 3.5. *Let I is a φ-2-absorbing primary strong* **C***-hyperideal of R. If I is not 2-absorbing primary, then $r(I) = r(\varphi(I))$.*

Proof. Assume that hyperideal I is not 2-absorbing primary. By Theorem 3.4, $I^3 \subseteq \varphi(I)$. Therefore $r(I) \subseteq r(\varphi(I))$. Moreover, we have $\varphi(I) \subseteq I$. Thus $r(\varphi(I)) \subseteq r(I)$. This completes the proof. □

Corollary 3.6. *Let I be a proper strong* $\mathbf{C}$*-hyperideal of R where $r(\varphi(I))$ is a prime hyperideal of R and let φ be a function. Then I is a 2-absorbing primary hyperideal of R if and only if hyperideal I is φ-2-absorbing primary hyperideal.*

Proof. ($\Longrightarrow$) It is clear.
($\Longleftarrow$) Assume that hyperideal I is φ-2-absorbing primary but is not 2-absorbing primary. By Corollary 3.5, $r(\varphi(I)) = r(I)$. Hence $r(I)$ is a prime hyperideal. Let $xoyoz \subseteq I$ and $xoy \nsubseteq I$. Since $(xoz)o(yoz) \subseteq xoyoz^2 \subseteq I \subseteq r(I)$, we conclude that $yoz \subseteq r(I)$ or $xoz \subseteq r(I)$. Thus the proof is completed.

□

Corollary 3.7. *Let I be a proper φ-2-absorbing primary strong* $\mathbf{C}$*-hyperideal of R and let $\varphi \leq \varphi_{(4)}$. Then for every $n \geq 3$, hyperideal I is $\varphi_{(n)}$-2-absorbing primary.*

Proof. Let I be a 2-absorbing primary hyperideal, then we are done. Let us suppose that hyperideal I is not 2-absorbing primary. Hence, by Theorem 3.4, we have $I^3 \subseteq \varphi(I)$. Since $\varphi \leq \varphi_{(4)}$, we conclude that $I^3 \subseteq \varphi(I) \subseteq I^4$. Thus for every $n \geq 3$, we obtain $I^3 = I^n = \varphi(I)$. Thus the claim is obvious. □

Theorem 3.8. *Let I, J be proper hyperideals of R with $J \subseteq I$. If I is a $\varphi_{(n)}$ -2-absorbing primary hyperideal, for every $n \geq 2$, then I/J is a $\varphi_{(n)}$ -2-absorbing primary hyperideal of R/J.*

Proof. Let I be a $\varphi_{(n)}$ -2-absorbing primary hyperideal of R. Suppose that $(x + J)o(y + J)o(z + J) \subseteq I/J - (I/J)^n$ with $x, y, z \in R$. Since $J \subseteq I$, then $xoyoz \subseteq I - I^n$. Since hyperideal I is $\varphi_{(n)}$ -2-absorbing primary, then we get $xoy \subseteq I$ or $yoz \subseteq r(I)$ or $xoz \subseteq r(I)$. Also, Since $J \subseteq I$, we have $r(I/J) = r(I)/J$. Consecuently $(x + J)o(y + J) \subseteq I/J$ or $(y + J)o(z + J) \subseteq r(I)/J$ or $(x + J)o(z + J) \subseteq r(I)/J$. □

Definition 3.9. *Suppose that $\varphi : E(R) \longrightarrow E(R) \cup \{\phi\}$ is a function. Let I and J be proper hyperideals of R with $J \subseteq I$. A hyperideal I/J of R/J is φ_J -2-absorbing primary hyperideal if $xoyoz \subseteq I/J - (\varphi(I) + J)/J$ with $x, y, z \in R/J$ implies $xoy \subseteq I/J$ or $yoz \subseteq r(I/J)$ or $xoz \subseteq r(I/J)$.*

Theorem 3.10. *Let I, J be two proper hyperideals of R with $J \subseteq I$ and let $\varphi : E(R) \longrightarrow E(R) \cup \{\phi\}$ be a function. If hyperideal I is φ-2-absorbing primary, then I/J is a φ_J -2-absorbing primary hyperideal in R/J.*

Proof. Assume that $(x + J)o(y + J)o(z + J) \subseteq xoyoz + J \subseteq I/J - (\varphi(I) + J)/J$, with $x, y, z \in R$. Thus $xoyoz \subseteq I - \varphi(I)$. Since hyperideal I is φ-2-absorbing primary, then $xoy \subseteq I$ or $yoz \subseteq r(I)$ or $xoz \subseteq r(I)$. Thus $(x + J)o(y + J) \subseteq I/J$ or $(y + J)o(z + J) \subseteq r(I)/J$ or $(x + J)o(z + J) \subseteq r(I)/J$. □

Theorem 3.11. *Let I, J be proper two hyperideals of R and let $\varphi : E(R) \longrightarrow E(R) \cup \{\phi\}$ be a function with $J \subseteq \varphi(I)$. The followings are equivalent.*

1) *Hyperideal I is φ-2-absorbing primary.*
2) *Hyperideal I/J of R/J is φ_J -2-absorbing primary.*
3) *For every $n \geq 1$, hyperideal I/J^n of R/J^n is $\varphi_{(n)}$ -2-absorbing primary.*

Proof. (1)⇒(2) It follows by Theorem 3.10.
(2)⇒(3) Since $J \subseteq \varphi(I)$, then for $n \geq 1$, $J^n \subseteq J \subseteq \varphi(I)$. Assume that $(x+J^n)o(y+J^n)o(z+J^n) \subseteq I/J^n - \varphi(I)/J^n$ with $x, y, z \in R$. Thus $xoyoz \nsubseteq \varphi(I)$. Since $J \subseteq \varphi(I)$ and $xoyoz \nsubseteq \varphi(I)$, then $xoyoz \nsubseteq J$. Hence $(x + J)o(y + J)o(z + J) \subseteq I/J - \varphi(I)/J$. Since hyperideal I/J is φ_J -2-absorbing primary and $r(I/J) = r(I/J^n) = r(I)/J^n$, we obtain $xoy \subseteq I$ or $yoz \subseteq r(I)$ or $xoz \subseteq r(I)$. Hence $xoy + J^n \subseteq I/J^n$ or $yoz \subseteq r(I)/J^n$ or $xoz \subseteq r(I)/J^n$.
(3)⇒(1) Let $xoyoz \subseteq I - \varphi(I)$ with $x, y, z \in R$ and $n = 1$. Since $J \subseteq \varphi(I) \subset I$, we have $xoyoz \nsubseteq J$ and $(x + J)o(y + J)o(z + J) \subseteq xoyoz + J \subseteq I/J - \varphi(I)/J$. Since hyperideal I/J is φ_J -2-absorbing primary and $r(I/J) = r(I)/J$, then $xoy \subseteq I$ or $yoz \subseteq r(I)$ or $xoz \subseteq r(I)$.

□

Corollary 3.12. *Let $I \in E^*(R)$ be a strong* **C***-hyperideal such that is not a 2-absorbing primary hyperideal and let $\varphi : E(R) \longrightarrow E(R) \cup \{\phi\}$ be a function. The followings are equivalent.*

1) *Hyeperideal I is φ-2-absorbing primary.*
2) *Hyperideal I/I^3 of R/I^3 is φ_{I^3} -2-absorbing primary.*
3) *For every $n \geq 3$, hyperideal I/I^n of R/I^n is φ_{I^n} -2-absorbing primary.*

Proof. It follows by Theorem 3.4 and Corollary 3.11. □

Definition 3.13. *Let hyperideal I of R be a φ-2-absorbing primary and let $xoyoz \subseteq \varphi(I)$ with $x, y, z \in R$ where $xoy \nsubseteq I$, $yox \nsubseteq r(I)$ and $xoz \nsubseteq r(I)$, then (x, y, z) is called a φ-triple-zero of I .*

A proper hyperideal I of R is φ-2-absorbing primary such that is not 2-absorbing primary if and only if there exists a φ-triple-zero of I.

Theorem 3.14. *Let R_1 and R_2 be multiplicative hyperrings with scalar identity and let $\varphi_1 : E(R_1) \longrightarrow E(R_1) \cup \{\phi\}, \varphi_2 : E(R_2) \longrightarrow E(R_2) \cup \{\phi\}$ be functions such that $\varphi_2(R_2) \neq R_2$. Let $\varphi = \varphi_1 \times \varphi_2$. Then the followings are equivalent.*

1) *Hyperideal $I_1 \times R_2$ of $R_1 \times R_2$ is φ-2-absorbing primary.*
2) *Hyperideal $I_1 \times R_2$ of $R_1 \times R_2$ is 2-absorbing primary.*
3) *Hyperideal I_1 of R_1 is 2-absorbing primary.*

Proof. (1)⇒(2). Since hyperideal I_1 of R_1 is φ_1 -2-absorbing primary, then If hyperideal I_1 is 2-absorbing primary, then the claim is proved. Thus, we suppose that I_1 is not a 2-absorbing hyperideal of R_1. Hence there exist a φ_1 -triple-zero (x, y, z) with $x, y, z \in R_1$ for I_1 . Since $\varphi_2(R_2) \neq R_2$, we get

$(x,1)o(y,1)o(z,1) \subseteq I_1 \times R_2 - \varphi_1(I_1) \times \varphi_2(R_2)$. This implies that $xoy \subseteq I_1$ or $xoz \subseteq r(I_1)$ or $yoz \subseteq r(I_1)$ which is a contradiction. Hence hyperideal I_1 of R_1 is 2-absorbing primary. Thus $I_1 \times R_2$ is a 2-absorbing primary hyperideal of $R_1 \times R_2$. (2)⇒(3) and(3)⇒(1) are obvious. □

REFERENCES

[1] Anbarloei, M., "On 2-absorbing and 2-absorbing primary hyperideals of a multiplicative hyperrings",*Cogent Mathematics* (2017), No. 4, 1-8.
[2] Anderson, DD., Bataineh, M., "Generalizations of prime ideals", *Comm Algebra,* **36**(2008), 686-696.
[3] Badawi, A., Tekir, U., Yetkin, E., "On weakly 2-absorbing primary ideals of commutative rings", *J. Korean Math Soc.,* **52** (2015), No. 1, 97-111.
[4] Badawi, A., Tekir, U., Ugurlu, E., Ulucak, G., Yetkin, E. "Generalizations of 2-absorbing primary ideals of commutative rings", *Turkish Journal of Mathematics*, **40** (2016), No. 3, doi:10.3906/ma1-1505-43.
[5] Corsini, S.,"Prolegomena of hypergroup theory", *Second edition, Aviani editor, Italy* , (1993).
[6] Corsini, S., Leoreanu, V., "Applications of hyperstructure theory", *Advances in Mathematics* , vol. 5, Kluwer Academic Publishers, (2003).
[7] Dasgupta, U., "On prime and primary hyperideals of a multiplicative hyperrings", *Bull. Austral. Math. Soc.*, **75**(2007), 417-429.
[8] Davvaz, B., Leoreanu-Fotea, V., "Hyperring Theory and Applications", *International Academic Press, Palm Harbor, USA*, (2007).
[9] Ghiasvand, P., "On 2-absorbing hyperideals of multiplicative hyperrings", *Second Seminar on Algebra and its Applications*, (2014) 58-59.
[10] Marty, F., "Sur une generalization de la notion de groupe", 8^{th} *Congress Math. Scandenaves, Stockholm,* (1934) 45-49.
[11] Omidi, S., Davvaz, B., "Contribution to study special kinds of hyperideals in ordered semihyperrings", *J. Taibah Univ. Sci.*, (2016), http://dx.doi.org/10.1016/j.jtusci.2016.09.001.
[12] Rota, R., "Sugli iperanelli moltiplicativi", *Rend. Di Math.,* Series VII, **4**(1982) 711-724.
[13] Vougiouklis, T., "Hyperstructures and their representations", *Hadronic Press Inc., Florida,* (1994).