J. Indones. Math. Soc.
Vol. 24, No. 1 (2018), pp. 1–25.

CERTAIN BIPOLAR NEUTROSOPHIC COMPETITION GRAPHS

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Abstract. Bipolarity plays an important role in many research domains. A bipolar fuzzy model is a very important model in which positive information represents what is possible or preferred, while negative information represents what is forbidden or surely false. In this research paper, we first introduce the concept of p-competition bipolar neutrosophic graphs. We then define generalization of bipolar neutrosophic competition graphs called m-step bipolar neutrosophic competition graphs. Moreover, we present some related concepts of bipolar neutrosophic graphs. Finally, we describe an application of m-step bipolar neutrosophic competition graphs.

Key words and Phrases: p-competition bipolar neutrosophic graphs, m-step bipolar neutrosophic competition graphs, Algorithm.

Abstrak. Bipolariti memainkan peran penting dalam berbagai macam topik penelitian. Sebuah model Fuzzy bipolar adalah sebuah model yang sangat penting dalam hal informasi postif menyatakan apa yang mungkin dipilih, sedangkan informasi negatif menyatakan apa yang dilarang atau pasti salah. Pada paper ini, pertama kali kami memperkenalkan konsep graf neotrosopik bipolar p-kompetisi. Kemudian kami mendefinisikan perumuman dari graf kompetisi neutrosopik bipolar yang disebut dengan graf kompetisi neutrosopik bipolar m-langkah. Lebih jauh, kami menyajikan beberapa konsep yang terkait dengan graf neutrosopik bipolar. Akhirnya, kami menggambarkan sebuah aplikasi dari graf kompetisi neutrosopik bipolar m-langkah.

Kata kunci: Graf neotrosopik bipolar p-kompetisi, graf kompetisi neutrosopik bipolar m-langkah, algoritma.

2000 Mathematics Subject Classification: 03E72, 68R10, 68R05.
Received: 28-02-2017, revised: 11-04-2017, accepted: 09-05-2017.

1. Introduction

The notion of competition graphs was introduced by Cohen [10] in 1968, depending upon a problem in ecology. The competition graphs have many utilizations in solving daily life problems, including channel assignment, modeling of complex economic, phytogenetic tree reconstruction, coding and energy systems.

Fuzzy set theory [26] and intuitionistic fuzzy set theory [6] are useful models for dealing with uncertainty and incomplete information. But they may not be sufficient in modeling of indeterminate and inconsistent information encountered in real world. In order to cope with this issue, neutrosophic set theory was proposed by Smarandache [18] as a generalization of fuzzy sets and intuitionistic fuzzy sets. However, since neutrosophic sets are identified by three functions called truth-membership (t), indeterminacy-membership (i) and falsity-membership (f) whose values are real standard or non-standard subset of unit interval $]0^-, 1^+[$. There are some difficulties in modeling of some problems in engineering and sciences. To overcome these difficulties, in 2010, concept of single-valued neutrosophic sets and its operations defined by Wang *et al.* [22] as a generalization of intuitionistic fuzzy sets. Ye [24, 25] has presented several novel applications of neutrosophic sets.

Deli *et al.* [11] extended the ideas of bipolar fuzzy sets [28] and neutrosophic sets to bipolar neutrosophic sets and studied its operations and applications in decision making problems. Smarandache [20] proposed notion of neutrosophic graph and they separated them to four main categories. Wu [23] discussed fuzzy digraphs. The concept of fuzzy k-competition graphs and p-competition fuzzy graphs was first introduced by Samanta and Pal in [15], it was further studied in [5, 13, 16, 17]. Cho *et al.* [9] proposed the generalization of a digraphs known as m-step competition graphs. Samanta *et al.* [16] introduced the generalization of fuzzy competition graphs, called m-step fuzzy competition graphs. On the other hand, the concepts of bipolar fuzzy competition graphs and intuitionistic fuzzy competition graphs are discussed in [17, 13]. Samanta *et al.* [16] also introduced the concepts of fuzzy m-step neighbouthood graphs. The notion of bipolar fuzzy graphs was first introduced by Akram [1] in 2011 as a generalization of fuzzy graphs. On the other hand, Akram and Shahzadi [4] first introduced the notion of neutrosophic soft graphs and gave its applications. Akram [2] introduced the notion of single-valued neutrosophic planar graphs. Akram and Sarwar have shown that there are some flaws in Broumi et al. [8] 's definition, which cannot be applied in network models. All the predator-prey relations cannot only be represented by bipolar neutrosophic competition graphs. For example, in a food web, species may have a chain consisting of same number of preys by which they can reach to their common preys. This idea motivates the necessity of m-step bipolar neutrosophic competition graphs. In this research paper, we first introduce the concept of p-competition bipolar neutrosophic graphs. We then define generalization of bipolar neutrosophic competition graphs called m-step bipolar neutrosophic competition graphs. Moreover, we present some related concepts of bipolar neutrosophic graphs. Finally, we describe an application of m-step bipolar neutrosophic competition graphs.

2. Certain Bipolar Neutrosophic Competition Graphs

Definition 2.1. [26, 27]A *fuzzy set* μ in a universe X is a mapping $\mu : X \to [0,1]$. A fuzzy relation on X is a fuzzy set ν in $X \times X$.

Definition 2.2. [28]A *bipolar fuzzy set* on a non-empty set X has the form

$$A = \{(x, \mu_A^P(x), \mu_A^N(x)) : x \in X\}$$

where, $\mu_A^P : X \to [0,1]$ and $\mu_A^N : X \to [-1,0]$ are mappings. The positive membership value $\mu_A^P(x)$ represents the strength of truth or satisfaction of an element x to a certain property corresponding to bipolar fuzzy set A and $\mu_A^N(x)$ denotes the strength of satisfaction of an element x to some counter property of bipolar fuzzy set A. If $\mu_A^P(x) \neq 0$ and $\mu_A^N(x) = 0$ it is the situation when x has only truth satisfaction degree for property A. If $\mu_A^N(x) \neq 0$ and $\mu_A^P(x) = 0$, it is the case that x is not satisfying the property of A but satisfying the counter property to A. It is possible for x that $\mu_A^P(x) \neq 0$ and $\mu_A^N(x) \neq 0$ when x satisfies the property of A as well as its counter property in some part of X.

Definition 2.3. [1]Let X be a non-empty set. A mapping $B = (\mu_B^P, \mu_B^N) : X \times X \to [0,1] \times [-1,0]$ is a *bipolar fuzzy relation* on X such that $\mu_B^P(xy) \in [0,1]$ and $\mu_B^N(xy) \in [-1,0]$ for $x, y \in X$.

Definition 2.4. [1]A *bipolar fuzzy graph* on X is a pair $G = (A, B)$ where $A = (\mu_A^P, \mu_A^N)$ is a bipolar fuzzy set on X and $B = (\mu_B^P, \mu_B^N)$ is a bipolar fuzzy relation in X such that

$$\mu_B^P(xy) \leq \mu_A^P(x) \wedge \mu_A^P(y) \text{ and } \mu_B^N(xy) \geq \mu_A^N(x) \vee \mu_A^N(y) \text{ for all } x, y \in X.$$

Definition 2.5. [21]A *neutrosophic set* A on a non-empty set X is characterized by a truth-membership fuction $t_A : X \to [0,1]$, indeterminacy-membership function $i_A : X \to [0,1]$ and a falsity-membership function $f_A : X \to [0,1]$. There is no restriction on the sum of $t_A(x)$, $i_A(x)$ and $f_A(x)$ for all $x \in X$.

Definition 2.6. [11]A *bipolar neutrosophic set* A on a non-empty set X is an object of the form

$$A = \{(x, t_A^P(x), i_A^P(x), f_A^P(x), t_A^N(x), i_A^N(x), f_A^N(x)) : x \in X\},$$

where t_A^P, i_A^P, $f_A^P : X \to [0,1]$ and t_A^N, i_A^N, $f_A^N : X \to [-1,0]$. The positive values $t_A^P(x)$, $i_A^P(x)$, $f_A^P(x)$ denote respectively the truth, indeterminacy and false-memberships degrees of an element $x \in X$, whereas, $t_A^N(x)$, $i_A^N(x)$, $f_A^N(x)$ denote the implicit counter property of the truth,indeterminacy and false-memberships degrees of the element $x \in X$ corresponding to the bipolar neutrosophic set A.

Definition 2.7. The *height* of bipolar neutrosophic set $A = (t_A^P(x), i_A^P(x), f_A^P(x), t_A^N(x), i_A^N(x), f_A^N(x))$ in universe of discourse X is defined as,

$$\begin{aligned} h(A) &= (h_1(A), h_2(A), h_3(A), h_4(A), h_5(A), h_6(A)) \\ &= (\sup_{x\in X} t_A^P(x), \sup_{x\in X} i_A^P(x), \inf_{x\in X} f_A^P(x), \sup_{x\in X} t_A^P(x), \sup_{x\in X} i_A^P(x), \inf_{x\in X} f_A^P(x)), \end{aligned}$$

for all $x \in X$.

Definition 2.8. Let $\overrightarrow{G}$ be a bipolar neutrosophic digraph then *bipolar neutrosophic out-neighbourhoods* of a vertex x is a bipolar neutrosophic set

$$\mathcal{N}^+(x) = (X_x^+, t_x^{(P)^+}, i_x^{(P)^+}, f_x^{(P)^+}, t_x^{(N)^+}, i_x^{(N)^+}, f_x^{(N)^+}),$$

where,

$$X_x^+ = \{y | B_1^P\overrightarrow{(x,y)} > 0,\, B_2^P\overrightarrow{(x,y)} > 0,\, B_3^P\overrightarrow{(x,y)} > 0, B_1^N\overrightarrow{(x,y)} < 0,\, B_2^N\overrightarrow{(x,y)} < 0, \\ B_3^N\overrightarrow{(x,y)} < 0\},$$

such that $t_x^{(P)^+} : X_x^+ \to [0,1]$, defined by $t_x^{(P)^+}(y) = B_1^P\overrightarrow{(x,y)}$, $i_x^{(P)^+} : X_x^+ \to [0,1]$, defined by $i_x^{(P)^+}(y) = B_2^P\overrightarrow{(x,y)}$, $f_x^{(P)^+} : X_x^+ \to [0,1]$, defined by $f_x^{(P)^+}(y) = B_3^P\overrightarrow{(x,y)}$, $t_x^{(N)^+} : X_x^+ \to [-1,0]$, defined by $t_x^{(N)^+}(y) = B_1^N\overrightarrow{(x,y)}$, $i_x^{(N)^+} : X_x^+ \to [-1,0]$, defined by $i_x^{(N)^+}(y) = B_2^N\overrightarrow{(x,y)}$, $f_x^{(N)^+} : X_x^+ \to [-1,0]$, defined by $f_x^{(N)^+}(y) = B_3^N\overrightarrow{(x,y)}$.

Definition 2.9. Let $\overrightarrow{G}$ be a bipolar neutrosophic digraph then *bipolar neutrosophic in-neighbourhoods* of a vertex x is a bipolar neutrosophic set

$$\mathcal{N}^-(x) = (X_x^-, t_x^{(P)^-}, i_x^{(P)^-}, f_x^{(P)^-}, t_x^{(N)^-}, i_x^{(N)^-}, f_x^{(N)^-}),$$

where,

$$X_x^- = \{y | B_1^P\overrightarrow{(y,x)} > 0,\, B_2^P\overrightarrow{(y,x)} > 0,\, B_3^P\overrightarrow{(y,x)} > 0, B_1^N\overrightarrow{(y,x)} < 0,\, B_2^N\overrightarrow{(y,x)} < 0, \\ B_3^N\overrightarrow{(y,x)} < 0\},$$

such that $t_x^{(P)^-} : X_x^- \to [0,1]$, defined by $t_x^{(P)^-}(y) = B_1^P\overrightarrow{(y,x)}$, $i_x^{(P)^-} : X_x^- \to [0,1]$, defined by $i_x^{(P)^-}(y) = B_2^P\overrightarrow{(y,x)}$, $f_x^{(P)^-} : X_x^- \to [0,1]$, defined by $f_x^{(P)^-}(y) = B_3^P\overrightarrow{(y,x)}$, $t_x^{(N)^-} : X_x^- \to [-1,0]$, defined by $t_x^{(N)^-}(y) = B_1^N\overrightarrow{(y,x)}$, $i_x^{(N)^-} : X_x^- \to [-1,0]$, defined by $i_x^{(N)^-}(y) = B_2^N\overrightarrow{(y,x)}$, $f_x^{(N)^-} : X_x^- \to [-1,0]$, defined by $f_x^{(N)^-}(y) = B_3^N\overrightarrow{(y,x)}$.

Definition 2.10. A *bipolar neutrosophic competition graph* of a bipolar neutrosophic graph $\overrightarrow{G} = (A, \overrightarrow{B})$ is an undirected bipolar neutrosophic graph $\mathcal{C}\overrightarrow{(G)} = (A, R)$ which has the same vertex set as in $\overrightarrow{G}$ and there is an edge between two vertices x and y if and only if $\mathcal{N}^+(x) \cap \mathcal{N}^+(y)$ is non-empty. The positive truth-membership, indeterminacy-membership, falsity-membership and negative truth-membership, indeterminacy-membership, falsity-membership values of the edge (x, y) are defined as,

(1) $t_R^P(x,y) = (t_A^P(x) \wedge t_A^P(y))h_1(\mathcal{N}^+(x) \cap \mathcal{N}^+(y))$,
(2) $i_R^P(x,y) = (i_A^P(x) \wedge i_A^P(y))h_2(\mathcal{N}^+(x) \cap \mathcal{N}^+(y))$,
(3) $f_R^P(x,y) = (f_A^P(x) \vee f_A^P(y))h_3(\mathcal{N}^+(x) \cap \mathcal{N}^+(y))$,
(4) $t_R^N(x,y) = (t_A^N(x) \vee t_A^N(y))h_4(\mathcal{N}^+(x) \cap \mathcal{N}^+(y))$,
(5) $i_R^N(x,y) = (i_A^N(x) \vee i_A^N(y))h_5(\mathcal{N}^+(x) \cap \mathcal{N}^+(y))$,
(6) $f_R^N(x,y) = (f_A^N(x) \wedge f_A^N(y))h_6(\mathcal{N}^+(x) \cap \mathcal{N}^+(y))$, for all $x, y \in X$.

Example 2.11. Consider $\overrightarrow{G} = (A, B)$ is a bipolar single-valued neutrosophic digraph, such that, $X = \{a, b, c, d\}$, $A = \{(a,\ 0.3,\ 0.8,\ 0.2,\ -0.5,\ -0.2,\ -0.1)$, $(b,\ 0.8,\ 0.3,\ 0.1,\ -0.5,\ -0.4,\ -0.2)$, $(c, 0.4, 0.5, 0.6, -0.2, -0.3, -0.5)$, $(d,\ 0.7,\ 0.3,\ 0.4, -0.2, -0.3, -0.5)\}$, and $B = \{(\overrightarrow{(a,b)},\ 0.2,\ 0.1,\ 0.1,\ -0.4,\ -0.1,\ -0.2)$, $(\overrightarrow{(a,c)},\ 0.3,\ 0.5,\ 0.6,\ -0.2, -0.2, -0.1)$, $(\overrightarrow{(b,d)}, 0.6, 0.2, 0.2, -0.1, -0.2, -0.3)$, $(\overrightarrow{(d,c)},\ 0.2,\ 0.2,\ 0.2, -0.2, -0.3, -0.5)\}$ as shown in Fig. 1.

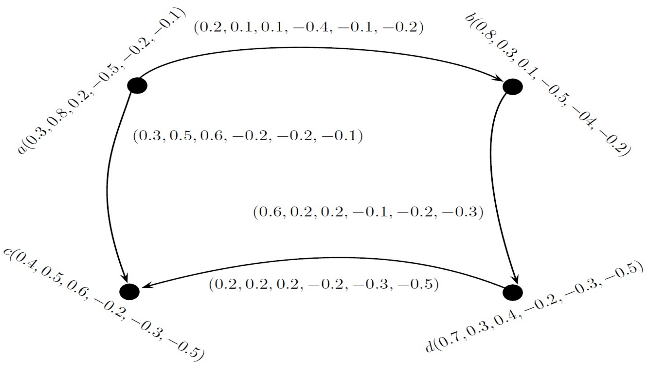

FIGURE 1. Bipolar single-valued neutrosophic digraph

By direct calculations we have Table 1 representing bipolar single-valued neutrosophic out-neighbourhoods.

TABLE 1. Bipolar single-valued neutrosophic out-neighbourhoods

x	$\mathcal{N}^+(x)$
a	{(b, 0.2, 0.1, 0.1,-0.4,-0.1,-0.2), (c, 0.3, 0.5, 0.6,-0.2,-0.2,-0.1)}
b	{(d, 0.6, 0.2, 0.2,-0.1,-0.2,-0.3)}
c	$\emptyset$
d	{(c, 0.2, 0.2, 0.2,-0.2,-0.3,-0.5)}

Then bipolar single-valued neutrosophic competition graph of Fig. 1 is shown in Fig. 2.

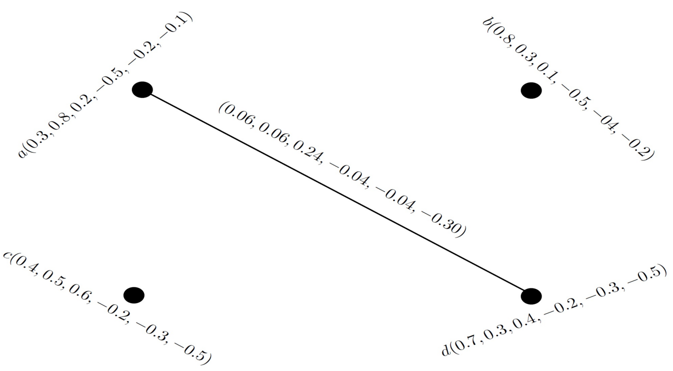

Figure 2. Bipolar single-valued neutrosophic competition graph

Definition 2.12. The *support* of a bipolar neutrosophic set $A = (x,\ t_A^P,\ i_A^P,\ f_A^P,\ t_A^N,\ i_A^N,\ f_A^N)$ in X is the subset of X defined by

$$supp(A) = \{x \in X : t_A^P(x) \neq 0,\ i_A^P(x) \neq 0,\ f_A^P(x) \neq 1,\ t_A^N(x) \neq -1,\ i_A^N(x) \neq -1,\ f_A^N(x) \neq 0\}$$

and $|supp(A)|$ is the number of elements in the set.

Example 2.13. The support of a bipolar neutrosophic set $A = \{(a,\ 0.5,\ 0.7,\ 0.2,\ -0.8, -0.9,\ -0.3),\ (b,\ 0.1,\ 0.2,\ 1,\ -0.5,\ -0.7,\ -0.6),\ (c,\ 0.3,\ 0.5,\ 0.3,\ -0.8,\ -0.6,\ -0.4),\ (d,\ 0,\ 0,\ 1,\ -1,\ -1,\ 0)\}$ in $X = \{a,\ b,\ c,\ d\}$ is $supp(A) = \{a, b, c\}$ and $|supp(A)| = 3$.

We now discuss p-competition bipolar neutrosophic graphs.

Definition 2.14. Let p be a positive integer. Then *p-competition bipolar neutrosophic graph* $\mathcal{C}^p(\overrightarrow{G})$ of the bipolar neutrosophic digraph $\overrightarrow{G} = (A,\ \overrightarrow{B})$ is an undirected bipolar neutrosophic graph $G = (A,\ B)$ which has same bipolar neutrosophic set of vertices as in $\overrightarrow{G}$ and has a bipolar neutrosophic edge between two vertices $x,\ y \in X$ in $\mathcal{C}^p(\overrightarrow{G})$ if and only if $|supp(\mathcal{N}^+(x) \cap \mathcal{N}^+(y))| \geq p$. The positive truth-membership value of edge $(x,\ y)$ in $\mathcal{C}^p(\overrightarrow{G})$ is $t_B^P(x,\ y) = \frac{(i-p)+1}{i}[t_A^P(x) \wedge t_A^P(y)]h_1(\mathcal{N}^+(x) \cap \mathcal{N}^+(y))$, the positive indeterminacy-membership value of edge $(x,\ y)$ in $\mathcal{C}^p(\overrightarrow{G})$ is $i_B^P(x,\ y) = \frac{(i-p)+1}{i}[i_A^P(x) \wedge i_A^P(y)]h_2(\mathcal{N}^+(x) \cap \mathcal{N}^+(y))$, positive falsity-membership value of edge $(x,\ y)$ in $\mathcal{C}^p(\overrightarrow{G})$ is $f_B^P(x,\ y) = \frac{(i-p)+1}{i}[f_A^P(x) \vee f_A^P(y)]h_3(\mathcal{N}^+(x) \cap \mathcal{N}^+(y))$, the negative truth-membership value of edge $(x,\ y)$

in $\mathcal{C}^p(\overrightarrow{G})$ is $t_B^N(x,\ y) = \frac{(i-p)+1}{i}[t_A^N(x) \vee t_A^N(y)]h_4(\mathcal{N}^+(x) \cap \mathcal{N}^+(y))$, the negative indeterminacy-membership value of edge (x, y) in $\mathcal{C}^p(\overrightarrow{G})$ is $i_B^N(x, y) = \frac{(i-p)+1}{i}[i_A^N(x) \vee i_A^N(y)]h_5(\mathcal{N}^+(x) \cap \mathcal{N}^+(y))$, negative falsity-membership value of edge $(x,\ y)$ in $\mathcal{C}^p(\overrightarrow{G})$ is $f_B^N(x,\ y) = \frac{(i-p)+1}{i}[f_A^N(x) \wedge f_A^N(y)]h_6(\mathcal{N}^+(x) \cap \mathcal{N}^+(y))$, where $i = |supp(\mathcal{N}^+(x) \cap \mathcal{N}^+(y))|$.

The $3-$competition bipolar neutrosophic graph is illustrated by the following example.

Example 2.15. Consider $\overrightarrow{G} = (A,\ \overrightarrow{B})$ is a bipolar neutrosophic digraph, such that, $X = \{x,\ y,\ z,\ a,\ b,\ c\}$, $A = \{(x,\ 0.7,\ 0.8,\ 0.5,\ -0.5,\ -0.6,\ -0.7),\ (y,\ 0.6,\ 0.7,\ 0.5,\ -0.3,\ -0.2,\ -0.7),\ (z,\ 0.6,\ 0.7,\ 0.3,\ -0.2,\ -0.3,\ -0.4),\ (a,\ 0.5,\ 0.6,\ 0.7,\ -0.5,\ -0.6,\ -0.8),\ (b,\ 0.5,\ 0.6,\ 0.7,\ -0.9,\ -0.8,\ -0.7),\ (c,\ 0.5,\ 0.6,\ 0.3,\ -0.1,\ -0.2,\ -0.4)\}$, and $B = \{(\overrightarrow{(x,a)}, 0.3,\ 0.4,\ 0.6,\ -0.4,\ -0.5,\ -0.7),\ (\overrightarrow{(x,b)}, 0.4,\ 0.5,\ 0.4,\ -0.4,\ -0.5,\ -0.5),\ (\overrightarrow{(x,c)}, 0.4,\ 0.5,\ 0.4,\ -0.1,\ -0.1,\ -0.6),\ (\overrightarrow{(y,a)}, 0.4,\ 0.5,\ 0.6,\ -0.2,\ -0.2,\ -0.6),\ (\overrightarrow{(y,b)}, 0.4,\ 0.4,\ 0.6,\ -0.2,\ -0.2,\ -0.6),\ (\overrightarrow{(y,c)}, 0.4,\ 0.5,\ 0.4,\ -0.1,\ -0.2,\ -0.3),\ (\overrightarrow{(z,b)}, 0.4,\ 0.5,\ 0.3,\ -0.1,\ -0.2,\ -0.6),\ (\overrightarrow{(z,c)}, 0.4,\ 0.5,\ 0.2,\ -0.1,\ -0.2,\ -0.3)\}$, as shown in Fig. 3. Then $\mathcal{N}^+(x) = \{(a,\ 0.3,\ 0.4,\ 0.6,\ -0.4,\ -0.5,\ -0.7),\ (b,\ 0.4,\ 0.5,\ 0.4,\ -0.4,\ -0.5,\ -0.5),\ (c,\ 0.4,\ 0.5,\ 0.4,\ -0.1,\ -0.1,\ -0.6)\}$, $\mathcal{N}^+(y) = \{(a,\ 0.4,\ 0.5,\ 0.6,\ -0.2,\ -0.2,\ -0.6),\ (b,\ 0.4,\ 0.4,\ 0.6,\ -0.2,\ -0.2,\ -0.6),\ (c,\ 0.4,\ 0.5,\ 0.4,\ -0.1,\ -0.2,\ -0.3)\}$, $\mathcal{N}^+(z) = \{(b,\ 0.4,\ 0.5,\ 0.3,\ -0.1,\ -0.2,\ -0.6),\ (c,\ 0.4,\ 0.5,\ 0.2,\ -0.1,\ -0.2,\ -0.3)\}$. So, $\mathcal{N}^+(x) \cap \mathcal{N}^+(y) = \{(a,\ 0.3,\ 0.4,\ 0.6,\ -0.2,\ -0.2,\ -0.7),\ (b,\ 0.4,\ 0.4,\ 0.6,\ -0.2,\ -0.2,\ -0.6),\ (c,\ 0.4,\ 0.5,\ 0.4,\ -0.1,\ -0.1,\ -0.6)\}$.

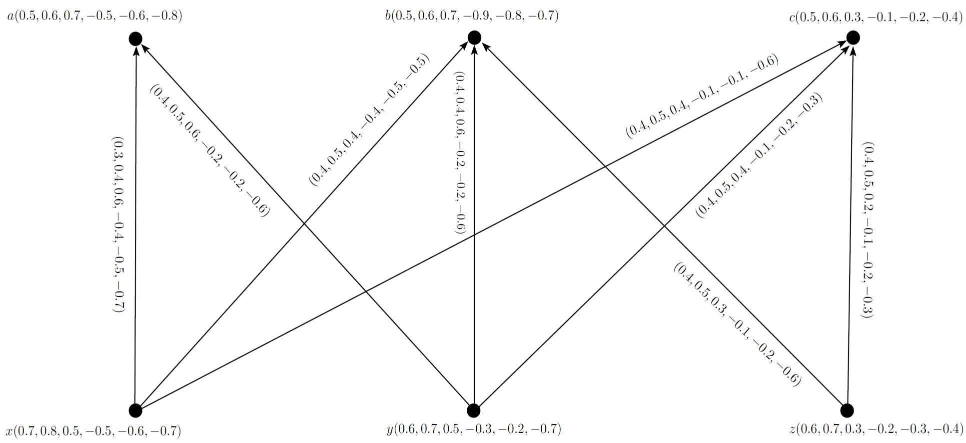

Figure 3. Bipolar neutrosophic digraph

Now $i = |supp(\mathcal{N}^+(x) \cap \mathcal{N}^+(y))| = 3$. For $p = 3$, $t_B^P(x,\ y) = 0.08$, $i_B^P(x,\ y) = 0.1166$, $f_B^P(x,\ y) = 0.066$, $t_B^N(x,\ y) = -0.04$, $i_B^N(x,\ y) = -0.033$, and $f_B^N(x,\ y) = -0.0933$. As shown in Fig. 4.

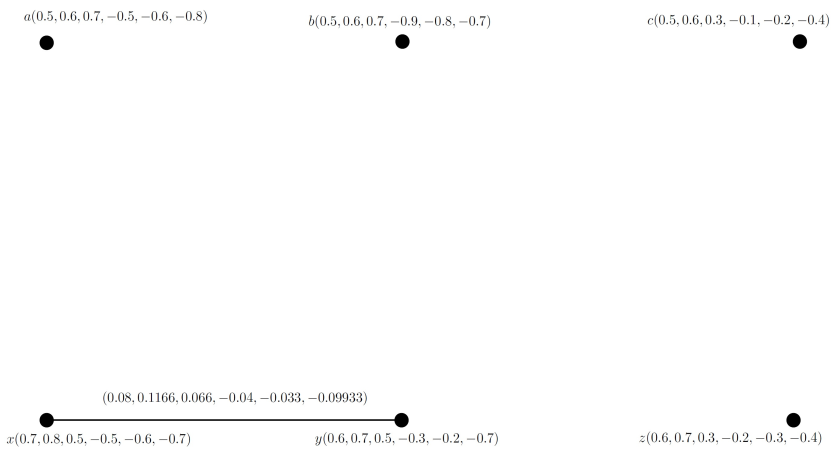

FIGURE 4. 3−competition bipolar neutrosophic graph

Theorem 2.16. Let $\overrightarrow{G} = (A,\ \overrightarrow{B})$ be a bipolar neutrosophic digraph. If

$$h_1(\mathcal{N}^+(x) \cap \mathcal{N}^+(y)) = 1, \quad h_2(\mathcal{N}^+(x) \cap \mathcal{N}^+(y)) = 1, \quad h_3(\mathcal{N}^+(x) \cap \mathcal{N}^+(y)) = 0,$$
$$h_4(\mathcal{N}^+(x) \cap \mathcal{N}^+(y)) = 1, \quad h_5(\mathcal{N}^+(x) \cap \mathcal{N}^+(y)) = 1, \quad h_6(\mathcal{N}^+(x) \cap \mathcal{N}^+(y)) = 0,$$

in $\mathcal{C}^{[\frac{i}{2}]}(\overrightarrow{G})$, then the edge (x, y) is strong, where $i = |supp(\mathcal{N}^+(x) \cap \mathcal{N}^+(y))|$. (Note that for any real number x, $[x]$=greatest integer not exceeding x.)

PROOF. Suppose $\overrightarrow{G} = (A,\ \overrightarrow{B})$ is a bipolar neutrosophic digraph. Let the corresponding $[\frac{i}{2}]$-bipolar neutrosophic competition graph be $\mathcal{C}^{[\frac{i}{2}]}(\overrightarrow{G})$, where $i = |supp(\mathcal{N}^+(x) \cap \mathcal{N}^+(y))|$. Also, assume that

$$h_1(\mathcal{N}^+(x) \cap \mathcal{N}^+(y)) = 1, \quad h_2(\mathcal{N}^+(x) \cap \mathcal{N}^+(y)) = 1, \quad h_3(\mathcal{N}^+(x) \cap \mathcal{N}^+(y)) = 0,$$
$$h_4(\mathcal{N}^+(x) \cap \mathcal{N}^+(y)) = 1, \quad h_5(\mathcal{N}^+(x) \cap \mathcal{N}^+(y)) = 1, \quad h_6(\mathcal{N}^+(x) \cap \mathcal{N}^+(y)) = 0,$$

for all $x,\ y \in X$. Now,

$$t_B^P(x,y) = \frac{(i-[\frac{i}{2}])+1}{i}[t_A^P(x) \wedge t_A^P(y)] \times 1, t_B^N(x,y) = \frac{(i-[\frac{i}{2}])+1}{i}[t_A^N(x) \vee t_A^N(y)] \times 1,$$
$$i_B^P(x,y) = \frac{(i-[\frac{i}{2}])+1}{i}[i_A^P(x) \wedge i_A^P(y)] \times 1, i_B^N(x,y) = \frac{(i-[\frac{i}{2}])+1}{i}[i_A^N(x) \vee i_A^N(y)] \times 1,$$
$$f_B^P(x,y) = \frac{(i-[\frac{i}{2}])+1}{i}[f_A^P(x) \vee f_A^P(y)] \times 0, f_B^N(x,y) = \frac{(i-[\frac{i}{2}])+1}{i}[f_A^N(x) \wedge f_A^N(y)] \times 0.$$

This gives the result,

$$\frac{t_B^P(x,y)}{[t_A^P(x)\wedge t_A^P(y)]}=\frac{(i-[\frac{i}{2}])+1}{i}>0.5,\quad \frac{t_B^N(x,y)}{[t_A^N(x)\vee t_A^N(y)]}=\frac{(i-[\frac{i}{2}])+1}{i}<0.5,$$
$$\frac{i_B^P(x,y)}{[i_A^P(x)\wedge i_A^P(y)]}=\frac{(i-[\frac{i}{2}])+1}{i}>0.5,\quad \frac{i_B^N(x,y)}{[i_A^N(x)\vee i_A^N(y)]}=\frac{(i-[\frac{i}{2}])+1}{i}<0.5,$$
$$\frac{f_B^P(x,y)}{[f_A^P(x)\vee f_A^P(y)]}=\frac{(i-[\frac{i}{2}])+1}{i}<0.5,\quad \frac{f_B^N(x,y)}{[f_A^N(x)\wedge f_A^N(y)]}=\frac{(i-[\frac{i}{2}])+1}{i}<0.5.$$

Hence, the edge (x, y) is strong. This proves the result.

We now define another extension of bipolar neutrosophic competition graph known as m-step bipolar neutrosophic competition graph.
In this paper, we will use the following notations:
$\mathcal{P}_{x,y}^m$: A bipolar neutrosophic path of length m from x to y.
$\overrightarrow{\mathcal{P}}_{x,y}^m$: A directed bipolar neutrosophic path of length m from x to y.
$\mathcal{N}_m^+(x)$: m-step bipolar neutrosophic out-neighbourhood of vertex x.
$\mathcal{N}_m^-(x)$: m-step bipolar neutrosophic in-neighbourhood of vertex x.
$\mathcal{N}_m(x)$: m-step bipolar neutrosophic neighbourhood of vertex x.
$\mathcal{N}_m(G)$: m-step bipolar neutrosophic neighbourhood graph of the bipolar neutrosophic graph G.
$\mathcal{C}_m(\overrightarrow{G})$: m-step bipolar neutrosophic competition graph of the bipolar neutrosophic digraph $\overrightarrow{G}$.

Definition 2.17. Suppose $\overrightarrow{G}=(A,\,\overrightarrow{B})$ is a bipolar neutrosophic digraph. The *m-step bipolar neutrosophic digraph* of $\overrightarrow{G}$ is denoted by $\overrightarrow{G}_m=(A,\,B)$, where bipolar neutrosophic set of vertices of $\overrightarrow{G}$ is same with bipolar neutrosophic set of vertices of $\overrightarrow{G}_m$ and has an edge between x and y in $\overrightarrow{G}_m$ if and only if there exists a bipolar neutrosophic directed path $\overrightarrow{\mathcal{P}}_{x,y}^m$ in $\overrightarrow{G}$.

Definition 2.18. The *bipolar neutrosophic m-step out-neighbourhood* of vertex x of a bipolar neutrosophic digraph $\overrightarrow{G}=(A,\,\overrightarrow{B})$ is bipolar neutrosophic set

$$\mathcal{N}_m^+(x)=(X_x^+,\,t_x^{(P)^+},\,i_x^{(P)^+},\,f_x^{(P)^+},\,t_x^{(N)^+},\,i_x^{(N)^+},\,f_x^{(N)^+}),\quad \text{where}$$

$X_x^+=\{y|$ there exists a directed bipolar neutrosophic path of length m from x to y, $\overrightarrow{P}_{x,y}^m\}$, $t_x^{(P)^+}:X_x^+\rightarrow[0,\,1]$, $i_x^{(P)^+}:X_z^+\rightarrow[0,\,1]$, $f_x^{(P)^+}:X_z^+\rightarrow[0,\,1]$ $t_x^{(N)^+}:X_x^+\rightarrow[-1,\,0]$, $i_x^{(N)^+}:X_z^+\rightarrow[-1,\,0]$, $f_x^{(N)^+}:X_z^+\rightarrow[-1,\,0]$ are defined by $t_x^{(P)^+}=\min\{t^P\overrightarrow{(x_1,\,x_2)}$, $(x_1,\,x_2)$ is an edge of $\overrightarrow{\mathcal{P}}_{x,y}^m\}$, $i_x^{(P)^+}=\min\{i^P\overrightarrow{(x_1,\,x_2)}$, $(x_1,$ $x_2)$ is an edge of $\overrightarrow{\mathcal{P}}_{x,y}^m\}$, $f_x^{(P)^+}=\max\{f^P\overrightarrow{(x_1,\,x_2)}$, $(x_1,\,x_2)$ is an edge of $\overrightarrow{\mathcal{P}}_{x,y}^m\}$, $t_x^{(N)^+}=\max\{t^P\overrightarrow{(x_1,\,x_2)}$, $(x_1,\,x_2)$ is an edge of $\overrightarrow{\mathcal{P}}_{x,y}^m\}$, $i_x^{(N)^+}=\max\{i^N\overrightarrow{(x_1,\,x_2)}$,

(x_1, x_2) is an edge of $\overrightarrow{\mathcal{P}}^m_{x,y}\}$, $f_x^{(N)^+} = \min\{f^N\overrightarrow{(x_1, x_2)}, (x_1, x_2)$ is an edge of $\overrightarrow{\mathcal{P}}^m_{x,y}\}$, respectively.

Example 2.19. Consider $\overrightarrow{G} = (A, \overrightarrow{B})$ is a bipolar neutrosophic digraph, such that $X = \{x, y, a, b, c, d\}$, as shown in Fig. 5. Then, 2-step out-neighbourhood of vertices x, and y is calculated as, $\mathcal{N}_2^+(x) = \{(b, 0.2, 0.2, 0.5, -0.2, -0.3, -0.3), (d, 0.2, 0.2, 0.5, -0.2, -0.3, -0.3)\}$, $\mathcal{N}_2^+(y) = \{(b, 0.1, 0.3, 0.2, -0.2, -0.3, -0.6), (d, 0.3, 0.5, 0.6, -0.2, -0.3, -0.5)\}$.

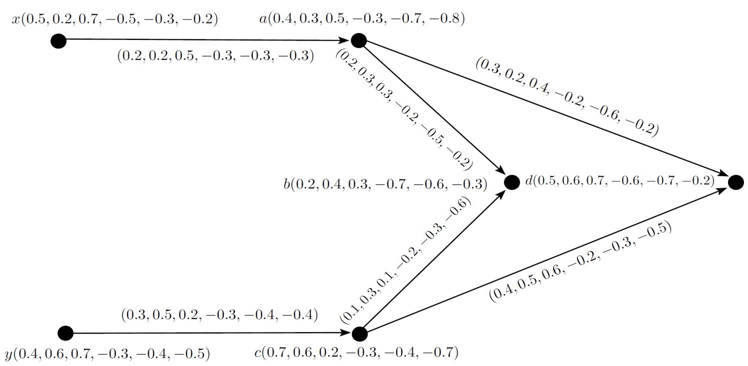

Figure 5. Bipolar neutrosophic digraph

Definition 2.20. The *bipolar neutrosophic m-step in-neighbourhood* of vertex x of a bipolar neutrosophic digraph $\overrightarrow{G} = (A, \overrightarrow{B})$ is bipolar neutrosophic set

$$\mathcal{N}_m^-(x) = (X_x^-, t_x^{(P)^-}, i_x^{(P)^-}, f_x^{(P)^-}, t_x^{(N)^-}, i_x^{(N)^-}, f_x^{(N)^-}), \quad \text{where}$$

$X_x^- = \{y|$ there exists a directed bipolar neutrosophic path of length m from y to x, $\overrightarrow{P}^m_{y,x}\}$, $t_x^{(P)^-} : X_x^- \to [0, 1]$, $i_x^{(P)^-} : X_z^- \to [0, 1]$, $f_x^{(P)^-} : X_z^- \to [0, 1]$ $t_x^{(N)^-} : X_x^- \to [-1, 0]$, $i_x^{(N)^-} : X_z^- \to [-1, 0]$, $f_x^{(N)^-} : X_z^- \to [-1, 0]$ are defined by $t_x^{(P)^-} = \min\{t^P\overrightarrow{(x_1, x_2)}, (x_1, x_2)$ is an edge of $\overrightarrow{\mathcal{P}}^m_{y,x}\}$, $i_x^{(P)^-} = \min\{i^P\overrightarrow{(x_1, x_2)}, (x_1, x_2)$ is an edge of $\overrightarrow{\mathcal{P}}^m_{y,x}\}$, $f_x^{(P)^-} = \max\{f^P\overrightarrow{(x_1, x_2)}, (x_1, x_2)$ is an edge of $\overrightarrow{\mathcal{P}}^m_{y,x}\}$, $t_x^{(N)^-} = \max\{t^N\overrightarrow{(x_1, x_2)}, (x_1, x_2)$ is an edge of $\overrightarrow{\mathcal{P}}^m_{y,x}\}$, $i_x^{(N)^-} = \max\{i^N\overrightarrow{(x_1, x_2)}, (x_1, x_2)$ is an edge of $\overrightarrow{\mathcal{P}}^m_{y,x}\}$, $f_x^{(N)^-} = \min\{f^N\overrightarrow{(x_1, x_2)}, (x_1, x_2)$ is an edge of $\overrightarrow{\mathcal{P}}^m_{y,x}\}$, respectively.

Example 2.21. Consider $\overrightarrow{G} = (A, \overrightarrow{B})$ is a bipolar neutrosophic digraph, such that, $X = \{a, b, c, d, e, f\}$, as shown in Fig. 6. Then, 2-step in-neighbourhood of vertices a, and b is calculated as, $\mathcal{N}_2^-(a) = \{(f, 0.1, 0.1, 0.5, -0.1, -0.2, -0.6), (e,$

0.3, 0.1, 0.7, −0.1, −0.2, −0.4)}, $\mathcal{N}_2^-(b) = \{(f,$ 0.1, 0.3, 0.6, −0.3, −0.4, −0.7), $(e,$ 0.4, 0.3, 0.6, −0.3, −0.4, −0.5)}.

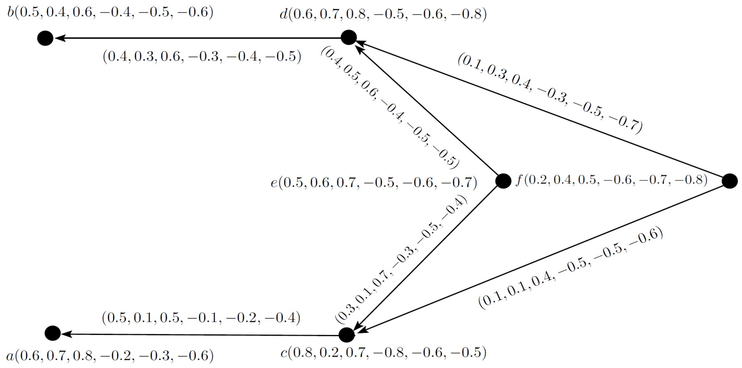

FIGURE 6. Bipolar neutrosophic digraph

Definition 2.22. Suppose $\overrightarrow{G} = (A, \overrightarrow{B})$ is a bipolar neutrosophic digraph. The *m-step bipolar neutrosophic competition graph* of bipolar neutrosophic digraph $\overrightarrow{G}$ is denoted by $\mathcal{C}_m(\overrightarrow{G}) = (A, B)$ which has same bipolar neutrosophic set of vertices as in $\overrightarrow{G}$ and has an edge between two vertices $x, y \in X$ in $\mathcal{C}_m(\overrightarrow{G})$ if and only if $(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y))$ is a non-empty bipolar neutrosophic set in $\overrightarrow{G}$. The positive truth-membership value of edge (x, y) in $\mathcal{C}_m(\overrightarrow{G})$ is $t_B^P(x, y) = [t_A^P(x) \wedge t_A^P(z)]h_1(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y))$, the positive indeterminacy-membership value of edge (x, y) in $\mathcal{C}_m(\overrightarrow{G})$ is $i_B^P(x, y) = [i_A^P(x) \wedge i_A^P(y)]h_2(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y))$, the positive falsity-membership value of edge (x, y) in $\mathcal{C}_m(\overrightarrow{G})$ is $f_B^P(x, y) = [f_A^P(x) \vee f_A^P(y)]h_3(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y))$, the negative truth-membership value of edge (x, y) in $\mathcal{C}_m(\overrightarrow{G})$ is $t_B^N(x, y) = [t_A^N(x) \vee t_A^N(z)]h_4(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y))$, the negative indeterminacy-membership value of edge (x, y) in $\mathcal{C}_m(\overrightarrow{G})$ is $i_B^N(x, y) = [i_A^N(x) \vee i_A^N(y)]h_5(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y))$, the negative falsity-membership value of edge (x, y) in $\mathcal{C}_m(\overrightarrow{G})$ is $f_B^N(x, y) = [f_A^N(x) \wedge f_A^N(y)]h_6(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y))$.

The 2−step bipolar neutrosophic competition graph is illustrated by the following example.

Example 2.23. Consider $\overrightarrow{G} = (A, \overrightarrow{B})$ is a bipolar neutrosophic digraph, such that $X = \{x, y, a, b, c, d\}$, as shown in Fig. 7. Then, $\mathcal{N}_2^+(x) = \{(b,$ 0.2, 0.2, 0.5, −0.2, −0.3, −0.3), $(d,$ 0.2, 0.2, 0.5, −0.2, −0.3, −0.3)}, $\mathcal{N}_2^+(y) = \{(b,$ 0.1, 0.3, 0.2, −0.2, −0.3, −0.6), $(d,$ 0.3, 0.5, 0.6, −0.2, −0.3, −0.5)}, there non-empty intersection

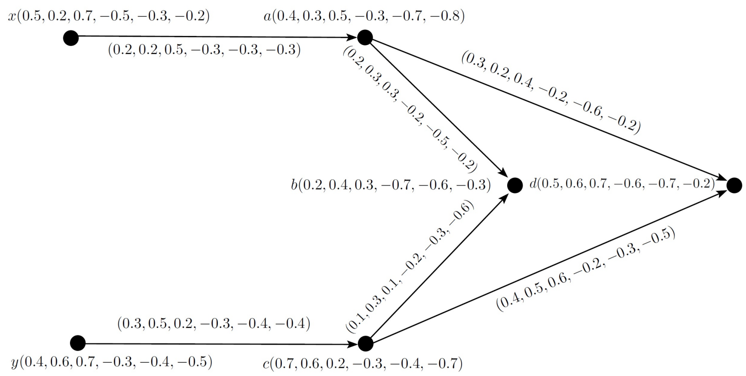

FIGURE 7. Bipolar neutrosophic digraph

is calculated as $\mathcal{N}_2^+(x) \cap \mathcal{N}_2^+(y) = \{(b,\, 0.1,\, 0.2,\, 0.5,\, -0.2,\, -0.3,\, -0.6),\, (d, 0.2,\, 0.2,\, 0.6, -0.2, -0.3, -0.5)\}$.

Thus, $t_B^P(x,\, y) = 0.08$, $i_B^P(x,\, y) = 0.04$, $f_B^P(x,\, y) = 0.35$, $t_B^N(x,\, y) = -0.06$, $i_B^N(x,\, y) = -0.06$, and $f_B^N(x,\, y) = -0.25$. Then its corresponding 2-step bipolar neutrosophic competition graph is shown in Fig. 8.

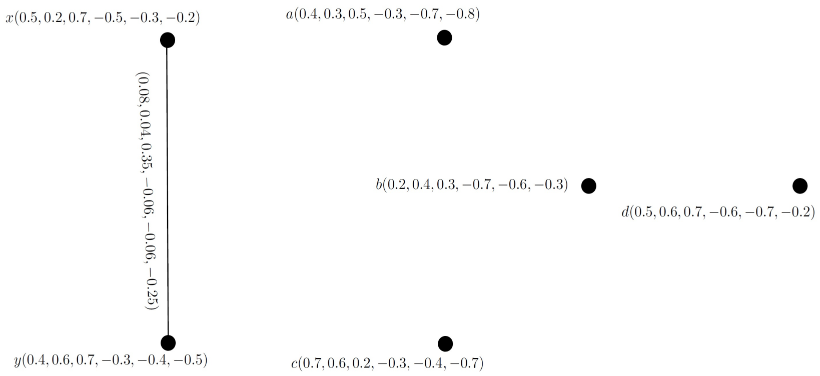

FIGURE 8. 2-step bipolar neutrosophic competition graph

If a predator x attacks one prey y, then the linkage is shown by an edge $\overrightarrow{(x, y)}$ in a bipolar neutrosophic digraph. But, if predator needs help of many other mediators x_1, x_2, $\ldots x_{m-1}$, then linkage among them is shown by bipolar neutrosophic directed path $\overrightarrow{\mathcal{P}}_{x,y}^m$ in a bipolar neutrosophic digraph. So, m-step prey in a bipolar neutrosophic digraph is represented by a vertex which is the m-step out-neighbourhood of some vertices. Now, the strength of a bipolar neutrosophic competition graphs is defined below.

Definition 2.24. Let $\overrightarrow{G} = (A, \overrightarrow{B})$ be a bipolar neutrosophic digraph. Let w be a common vertex of m-step out-neighbourhoods of vertices $x_1, x_2, \ldots, x_l$. Also, let $\overrightarrow{B_1^P}(u_1, v_1)$, $\overrightarrow{B_1^P}(u_2, v_2), \ldots, \overrightarrow{B_1^P}(u_l, v_l)$ be the minimum positive truth-membership values, $\overrightarrow{B_2^P}(u_1, v_1)$, $\overrightarrow{B_2^P}(u_2, v_2), \ldots, \overrightarrow{B_2^P}(u_l, v_l)$ be the minimum positive indeterminacy-membership values, $\overrightarrow{B_3^P}(u_1, v_1)$, $\overrightarrow{B_3^P}(u_2, v_2), \ldots, \overrightarrow{B_3^P}(u_l, v_l)$ be the maximum positive false-membership values, $\overrightarrow{B_1^N}(u_1, v_1)$, $\overrightarrow{B_1^N}(u_2, v_2), \ldots, \overrightarrow{B_1^N}(u_l, v_l)$ be the maximum negative truth-membership values, $\overrightarrow{B_2^N}(u_1, v_1)$, $\overrightarrow{B_2^N}(u_2, v_2), \ldots, \overrightarrow{B_2^N}(u_l, v_l)$ be the maximum negative indeterminacy-membership values, $\overrightarrow{B_3^N}(u_1, v_1)$, $\overrightarrow{B_3^N}(u_2, v_2), \ldots, \overrightarrow{B_3^N}(u_l, v_l)$ be the minimum negative false-membership values, of edges of the paths $\overrightarrow{\mathcal{P}}^m_{x_1,w}, \overrightarrow{\mathcal{P}}^m_{x_2,w}, \ldots, \overrightarrow{\mathcal{P}}^m_{x_l,w}$, respectively. The m-step prey $w \in X$ is *strong prey* if

$$\begin{aligned}
&\overrightarrow{B_1^P}(u_i, v_i) > 0.5, \quad \overrightarrow{B_2^P}(u_i, v_i) > 0.5, \quad \overrightarrow{B_3^P}(u_i, v_i) < 0.5,\\
&\overrightarrow{B_1^N}(u_i, v_i) < 0.5, \quad \overrightarrow{B_2^N}(u_i, v_i) < 0.5, \quad \overrightarrow{B_3^N}(u_i, v_i) < 0.5, \text{ for all } i = 1, 2, \ldots, l.
\end{aligned}$$

The strength of the prey w can be measured by the mapping $S : X \to [0, 1]$, such that:

$$\begin{aligned}
S(w) = \frac{1}{l}\Bigg\{ &\sum_{i=1}^{l}[\overrightarrow{B_1^P}(u_i, v_i)] + \sum_{i=1}^{l}[\overrightarrow{B_2^P}(u_i, v_i)] + \sum_{i=1}^{l}[\overrightarrow{B_3^P}(u_i, v_i)]\\
&- \sum_{i=1}^{l}[\overrightarrow{B_1^N}(u_i, v_i)] - \sum_{i=1}^{l}[\overrightarrow{B_2^N}(u_i, v_i)] - \sum_{i=1}^{l}[\overrightarrow{B_3^N}(u_i, v_i)]\Bigg\}.
\end{aligned}$$

Example 2.25. Consider bipolar neutrosophic digraph $\overrightarrow{G} = (A, \overrightarrow{B})$ as shown in Fig. 7, the strength of the prey b is equal to

$$\begin{aligned}
\frac{[0.2 + 0.2] + [0.2 + 0.3] + [0.5 + 0.3] - [-0.3 - 0.2] - [-0.3 - 0.5] - [-0.3 - 0.2]}{2} &= 1.75\\
&> 0.5.
\end{aligned}$$

Hence, b is strong 2-step prey.

Theorem 2.26. If a prey w of $\overrightarrow{G} = (A, \overrightarrow{B})$ is strong, then the strength of w, $S(w) > 0.5$.

PROOF. Let $\overrightarrow{G} = (A, \overrightarrow{B})$ be a bipolar neutrosophic digraph. Let w be a common vertex of m-step out-neighbourhoods of vertices $x_1, x_2, \ldots, x_l$, i.e., there exists the paths $\overrightarrow{\mathcal{P}}^m_{x_1,w}, \overrightarrow{\mathcal{P}}^m_{x_2,w}, \ldots, \overrightarrow{\mathcal{P}}^m_{x_l,w}$ in $\overrightarrow{G}$. Also, let $\overrightarrow{B_1^P}(u_1, v_1)$, $\overrightarrow{B_1^P}(u_2, v_2), \ldots, \overrightarrow{B_1^P}(u_l, v_l)$ be the minimum positive truth-membership values, $\overrightarrow{B_2^P}(u_1,v_1)$, $\overrightarrow{B_2^P}(u_2,v_2), \ldots, \overrightarrow{B_2^P}(u_l,v_l)$ be the minimum positive indeterminacy-membership values, $\overrightarrow{B_3^P}(u_1, v_1)$, $\overrightarrow{B_3^P}(u_2, v_2), \ldots, \overrightarrow{B_3^P}(u_l, v_l)$ be the maximum positive false-membership values,

$\overrightarrow{B_1^N}(u_1,v_1)$, $\overrightarrow{B_1^N}(u_2,v_2)$,..., $\overrightarrow{B_1^N}(u_l,v_l)$ be the maximum negative truth-membership values, $\overrightarrow{B_2^N}(u_1,v_1)$, $\overrightarrow{B_2^N}(u_2,v_2)$,..., $\overrightarrow{B_2^N}(u_l,v_l)$ be the maximum negative indeterminacy-membership values, $\overrightarrow{B_3^N}(u_1,v_1)$, $\overrightarrow{B_3^N}(u_2,v_2)$,..., $\overrightarrow{B_3^N}(u_l,v_l)$ be the minimum negative false-membership values, of edges of the paths $\overrightarrow{\mathcal{P}}^m_{x_1,w}$, $\overrightarrow{\mathcal{P}}^m_{x_2,w}$,..., $\overrightarrow{\mathcal{P}}^m_{x_l,w}$, respectively.
If w is strong, each edge $(u_i,\, v_i)$, $i = 1, 2, \ldots, l$ is strong. So,

$$\begin{aligned}
&\overrightarrow{B_1^P}(u_i,v_i) > 0.5, \quad \overrightarrow{B_2^P}(u_i,v_i) > 0.5, \quad \overrightarrow{B_3^P}(u_i,v_i) < 0.5,\\
&\overrightarrow{B_1^N}(u_i,v_i) < 0.5, \quad \overrightarrow{B_2^N}(u_i,v_i) < 0.5, \quad \overrightarrow{B_3^N}(u_i,v_i) < 0.5, \text{ for all } i = 1, 2, \ldots, l.
\end{aligned}$$

Now,

$$S(w) > \frac{0.5 + 0.5 + \ldots (l \quad times) + 0.5}{l} > 0.5.$$

This proves the result.

Remark: The converse of the above theorem is not true, i.e. if $S(w) > 0.5$, then all preys may not be strong. This can be explained as:
Let $S(w) > 0.5$ for a prey w in $\overrightarrow{G}$. So,

$$\begin{aligned}
S(w) = \frac{1}{l}\Bigg\{ &\sum_{i=1}^{l}[\overrightarrow{B_1^P}(u_i,v_i)] + \sum_{i=1}^{l}[\overrightarrow{B_2^P}(u_i,v_i)] + \sum_{i=1}^{l}[\overrightarrow{B_3^P}(u_i,v_i)]\\
&- \sum_{i=1}^{l}[\overrightarrow{B_1^N}(u_i,v_i)] - \sum_{i=1}^{l}[\overrightarrow{B_2^N}(u_i,v_i)] - \sum_{i=1}^{l}[\overrightarrow{B_3^N}(u_i,v_i)]\Bigg\}.
\end{aligned}$$

Hence,

$$\begin{aligned}
\Bigg\{ &\sum_{i=1}^{l}[\overrightarrow{B_1^P}(u_i,v_i)] + \sum_{i=1}^{l}[\overrightarrow{B_2^P}(u_i,v_i)] + \sum_{i=1}^{l}[\overrightarrow{B_3^P}(u_i,v_i)]\\
&- \sum_{i=1}^{l}[\overrightarrow{B_1^N}(u_i,v_i)] - \sum_{i=1}^{l}[\overrightarrow{B_2^N}(u_i,v_i)] - \sum_{i=1}^{l}[\overrightarrow{B_3^N}(u_i,v_i)]\Bigg\} > \frac{l}{2}.
\end{aligned}$$

This result does not necessarily imply that

$$\begin{aligned}
&\overrightarrow{B_1^P}(u_i,v_i) > 0.5, \quad \overrightarrow{B_2^P}(u_i,v_i) > 0.5, \quad \overrightarrow{B_3^P}(u_i,v_i) < 0.5,\\
&\overrightarrow{B_1^N}(u_i,v_i) < 0.5, \quad \overrightarrow{B_2^N}(u_i,v_i) < 0.5, \quad \overrightarrow{B_3^N}(u_i,v_i) < 0.5, \text{ for all } i = 1, 2, \ldots, l.
\end{aligned}$$

Since, all edges of the directed paths $\overrightarrow{\mathcal{P}}^m_{x_1,w}$, $\overrightarrow{\mathcal{P}}^m_{x_2,w}$,..., $\overrightarrow{\mathcal{P}}^m_{x_l,w}$, are not strong. So, the converse of the above statement is not true i.e., if $S(w) > 0.5$, the prey w of $\overrightarrow{G}$ may not be strong.

Theorem 2.27. If all preys of $\overrightarrow{G} = (A,\ \overrightarrow{B})$ are strong, then all edges of $\mathcal{C}_m(\overrightarrow{G}) = (A,\ B)$ are strong.

PROOF. Let $\overrightarrow{G} = (A,\ \overrightarrow{B})$ be a bipolar neutrosophic digraph and all preys of it are strong. Let $\mathcal{C}_m(\overrightarrow{G}) = (A,\ B)$, where,

$$\begin{aligned}
t_B^P(x,y) &= [t_A^P(x) \wedge t_A^P(z)]h_1(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)),\\
i_B^P(x,y) &= [i_A^P(x) \wedge i_A^P(y)]h_2(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)),\\
f_B^P(x,y) &= [f_A^P(x) \vee f_A^P(y)]h_3(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)),\\
t_B^N(x,y) &= [t_A^N(x) \vee t_A^N(z)]h_4(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)),\\
i_B^N(x,y) &= [i_A^N(x) \vee i_A^N(y)]h_5(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)),\\
f_B^N(x,y) &= [f_A^N(x) \wedge f_A^N(y)]h_6(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)),
\end{aligned}$$

for all edges (x, y) in $\mathcal{C}_m(\overrightarrow{G}) = (A,\ B)$. Then there arises two cases:

Case 1.: Let $\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)$ be null set. Then there does not exits any edge between x and y in $\mathcal{C}_m(\overrightarrow{G})$.

Case 2.: Let $\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)$ be non-empty. Now, clearly

$$h_1(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)) > 0.5, \quad h_2(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)) > 0.5, \quad h_3(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)) < 0.5,$$
$$h_4(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)) < 0.5, \quad h_5(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)) < 0.5, \quad h_6(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)) < 0.5,$$

in $\overrightarrow{G}$ as all preys are strong. So, the edge $(x,\ y)$, $x,\ y \in X$ in $\mathcal{C}_m(\overrightarrow{G})$ have the memberships values

$$\begin{aligned}
t_B^P(x,y) &= [t_A^P(x) \wedge t_A^P(z)]h_1(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)),\\
i_B^P(x,y) &= [i_A^P(x) \wedge i_A^P(y)]h_2(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)),\\
f_B^P(x,y) &= [f_A^P(x) \vee f_A^P(y)]h_3(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)),\\
t_B^N(x,y) &= [t_A^N(x) \vee t_A^N(z)]h_4(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)),\\
i_B^N(x,y) &= [i_A^N(x) \vee i_A^N(y)]h_5(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)),\\
f_B^N(x,y) &= [f_A^N(x) \wedge f_A^N(y)]h_6(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)),
\end{aligned}$$

and hence, all the edges are strong.

A relation is established between m-step bipolar neutrosophic competition graph of a bipolar neutrosophic digraph and bipolar neutrosophic competition graph of m-step bipolar neutrosophic digraph.

Theorem 2.28. If $\overrightarrow{G}$ is a bipolar neutrosophic digraph and $\overrightarrow{G_m}$ is the m-step bipolar neutrosophic digraph of $\overrightarrow{G}$, then $\mathcal{C}(\overrightarrow{G}_m) = \mathcal{C}_m(\overrightarrow{G})$.

PROOF. Let $\overrightarrow{G} = (A, \overrightarrow{B})$ be a bipolar neutrosophic digraph and $\overrightarrow{G_m} = (A,\ \overrightarrow{J})$ is the m-step bipolar neutrosophic digraph of $\overrightarrow{G}$. Also, let $\mathcal{C}(\overrightarrow{G}_m) = (A, J)$ and $\mathcal{C}_m(\overrightarrow{G}) = (A, B)$. It can be easily observed that bipolar neutrosophic vertex sets of these graphs are same. So, we have to show that the bipolar neutrosophic edge

sets of $\mathcal{C}(\overrightarrow{G}_m)$ and $\mathcal{C}_m(\overrightarrow{G})$ are equal.
Let (x, y) be an edge in $\mathcal{C}(\overrightarrow{G}_m)$. So, there exists bipolar neutrosophic directed edges $\overrightarrow{(x, a_1)}$, $\overrightarrow{(y, a_1)}$; $\overrightarrow{(x, a_2)}$, $\overrightarrow{(y, a_2)}$;. . . ;$\overrightarrow{(x, a_l)}$, $\overrightarrow{(y, a_l)}$, for some positive integer l in $\overrightarrow{G}_m$. Now, in $\overrightarrow{G}_m$,

$$\mathcal{N}^+(x) \cap \mathcal{N}^+(y) = \{(a_i, s_i^P, q_i^P, r_i^P, s_i^N, q_i^N, r_i^N)|i = 1, 2, \ldots, l\},$$

where,

$$\begin{aligned} s_i^P &= \overrightarrow{J}(x, a_i) \wedge \overrightarrow{J}(y, a_i), & s_i^N &= \overrightarrow{J}(x, a_i) \vee \overrightarrow{J}(y, a_i), \\ q_i^P &= \overrightarrow{J}(x, a_i) \wedge \overrightarrow{J}(y, a_i), & q_i^N &= \overrightarrow{J}(x, a_i) \vee \overrightarrow{J}(y, a_i), \\ r_i^P &= \overrightarrow{J}(x, a_i) \vee \overrightarrow{J}(y, a_i), & r_i^N &= \overrightarrow{J}(x, a_i) \wedge \overrightarrow{J}(y, a_i). \end{aligned}$$

Let

$$\begin{aligned} S^P &= \max\{s_i^P|i = 1, 2, \ldots, l\}, & S^N &= \min\{s_i^N|i = 1, 2, \ldots, l\}, \\ Q^P &= \max\{q_i^P|i = 1, 2, \ldots, l\}, & Q^N &= \min\{q_i^N|i = 1, 2, \ldots, l\}, \\ R^P &= \min\{r_i^P|i = 1, 2, \ldots, l\}, & R^N &= \max\{r_i^N|i = 1, 2, \ldots, l\}. \end{aligned}$$

Hence,

$$\begin{aligned} t_J^P(x, y) &= (t_A^P(x) \wedge t_A^P(y))h_1(\mathcal{N}^+(x) \cap \mathcal{N}^+(y)) = S^P \times t_A^P(x) \wedge t_A^P(y), \\ i_J^P(x, y) &= (i_A^P(x) \wedge i_A^P(y))h_2(\mathcal{N}^+(x) \cap \mathcal{N}^+(y)) = Q^P \times i_A^P(x) \wedge i_A^P(y), \\ f_J^P(x, y) &= (f_A^P(x) \vee f_A^P(y))h_3(\mathcal{N}^+(x) \cap \mathcal{N}^+(y)) = R^P \times f_A^P(x) \vee f_A^P(y), \\ t_J^N(x, y) &= (t_A^N(x) \vee t_A^N(y))h_4(\mathcal{N}^+(x) \cap \mathcal{N}^+(y)) = S^N \times t_A^N(x) \vee t_A^N(y), \\ i_J^N(x, y) &= (i_A^N(x) \vee i_A^N(y))h_5(\mathcal{N}^+(x) \cap \mathcal{N}^+(y)) = Q^N \times i_A^N(x) \vee i_A^N(y), \\ f_J^N(x, y) &= (f_A^N(x) \wedge f_A^N(y))h_6(\mathcal{N}^+(x) \cap \mathcal{N}^+(y)) = R^N \times f_A^N(x) \wedge f_A^N(y). \end{aligned}$$

An edge $\overrightarrow{(x, a_i)}$ exists in $\overrightarrow{G}_m$ that implies there exists a bipolar neutrosophic directed path from x to a_i of length m, $\overrightarrow{\mathcal{P}}^m_{x,a_i}$ in $\overrightarrow{G}$ and

$$\begin{aligned} \overrightarrow{J}_1^P(x, a_i) &= \min\{\overrightarrow{B}_1^P(u, v)|(u, v) \text{ is an edge in} \overrightarrow{\mathcal{P}}^m_{x,a_i}\}, \\ \overrightarrow{J}_2^P(x, a_i) &= \min\{\overrightarrow{B}_2^P(u, v)|(u, v) \text{ is an edge in} \overrightarrow{\mathcal{P}}^m_{x,a_i}\}, \\ \overrightarrow{J}_3^P(x, a_i) &= \max\{\overrightarrow{B}_3^P(u, v)|(u, v) \text{ is an edge in} \overrightarrow{\mathcal{P}}^m_{x,a_i}\}, \\ \overrightarrow{J}_1^N(x, a_i) &= \max\{\overrightarrow{B}_1^N(u, v)|(u, v) \text{ is an edge in} \overrightarrow{\mathcal{P}}^m_{x,a_i}\}, \\ \overrightarrow{J}_2^N(x, a_i) &= \max\{\overrightarrow{B}_2^N(u, v)|(u, v) \text{ is an edge in} \overrightarrow{\mathcal{P}}^m_{x,a_i}\}, \\ \overrightarrow{J}_3^N(x, a_i) &= \min\{\overrightarrow{B}_3^N(u, v)|(u, v) \text{ is an edge in} \overrightarrow{\mathcal{P}}^m_{x,a_i}\}. \end{aligned}$$

Thus, the edge (x, y) is also available in $\mathcal{C}_m(\overrightarrow{G})$. Also,

$$\begin{aligned} h_1(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)) &= S^P, \quad h_4(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)) = S^N, \\ h_2(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)) &= Q^P, \quad h_5(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)) = Q^N, \\ h_3(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)) &= R^P, \quad h_6(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)) = R^N, \end{aligned}$$

in $\overrightarrow{G}$. Hence, finally

$$\begin{aligned} t_B^P(x, y) &= [t_A^P(x) \wedge t_A^P(z)]h_1(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)) = S^P \times t_A^P(x) \wedge t_A^P(z), \\ i_B^P(x, y) &= [i_A^P(x) \wedge i_A^P(y)]h_2(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)) = Q^P \times i_A^P(x) \wedge i_A^P(y), \\ f_B^P(x, y) &= [f_A^P(x) \vee f_A^P(y)]h_3(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)) = R^P \times f_A^P(x) \vee f_A^P(y), \\ t_B^N(x, y) &= [t_A^N(x) \vee t_A^N(z)]h_4(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)) = S^N \times t_A^N(x) \vee t_A^N(z), \\ i_B^N(x, y) &= [i_A^N(x) \vee i_A^N(y)]h_5(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)) = Q^N \times i_A^N(x) \vee i_A^N(y), \\ f_B^N(x, y) &= [f_A^N(x) \wedge f_A^N(y)]h_6(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)) = R^N \times f_A^N(x) \wedge f_A^N(y). \end{aligned}$$

This proves that there exists an edge in $\mathcal{C}_m(\overrightarrow{G})$ for every edge in $\mathcal{C}(\overrightarrow{G}_m)$. Similarly, for every edge in $\mathcal{C}_m(\overrightarrow{G})$ there exists an edge in $\mathcal{C}(\overrightarrow{G}_m)$. This proves that $\mathcal{C}(\overrightarrow{G}_m) = \mathcal{C}_m(\overrightarrow{G})$.

Theorem 2.29. Let $\overrightarrow{G} = (A, \overrightarrow{B})$ be a bipolar neutrosophic digraph. If $m > |X|$ then $\mathcal{C}_m(\overrightarrow{G}) = (A, B)$ has no edge.

PROOF. Let $\overrightarrow{G} = (A, \overrightarrow{B})$ be a bipolar neutrosophic digraph and $\mathcal{C}_m(\overrightarrow{G}) = (A, B)$ be the corresponding m-step bipolar neutrosophic competition graph, where,

$$\begin{aligned} t_B^P(x, y) &= [t_A^P(x) \wedge t_A^P(z)]h_1(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)), \\ i_B^P(x, y) &= [i_A^P(x) \wedge i_A^P(y)]h_2(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)), \\ f_B^P(x, y) &= [f_A^P(x) \vee f_A^P(y)]h_3(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)), \\ t_B^N(x, y) &= [t_A^N(x) \vee t_A^N(z)]h_4(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)), \\ i_B^N(x, y) &= [i_A^N(x) \vee i_A^N(y)]h_5(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)), \\ f_B^N(x, y) &= [f_A^N(x) \wedge f_A^N(y)]h_6(\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)), \end{aligned}$$

for all edges (x, y) in $\mathcal{C}_m(\overrightarrow{G})$.
If $m > |X|$, there does not exist any directed bipolar neutrosophic path of length m in $\overrightarrow{G}$. So, $\mathcal{N}_m^+(x) \cap \mathcal{N}_m^+(y)$ ia an empty set. Hence, there does not exist any edge in $\mathcal{C}_m(\overrightarrow{G})$.

Now, m-step bipolar neutrosophic neighbouhood graphs are defines below.

Definition 2.30. The *bipolar neutrosophic m-step out-neighbourhood* of vertex x of a bipolar neutrosophic digraph $\overrightarrow{G} = (A, \overrightarrow{B})$ is bipolar neutrosophic set

$$\mathcal{N}_m(x) = (X_x,\, t_x^P,\, i_x^P,\, f_x^P,\, t_x^N,\, i_x^N,\, f_x^N), \quad \text{where}$$

$X_x = \{y|$ there exists a directed bipolar neutrosophic path of length m from x to y, $\mathcal{P}^m_{x,y}\}$, $t_x^P : X_x \to [0, 1]$, $i_x^P : X_x \to [0, 1]$, $f_x^P : X_x \to [0, 1]$, $t_x^N : X_x \to [-1, 0]$, $i_x^N : X_x \to [-1, 0]$, $f_x^N : X_x \to [-1, 0]$, are defined by $t_x^P = \min\{t^P(x_1, x_2), (x_1, x_2)$ is an edge of $\mathcal{P}^m_{x,y}\}$, $i_x^P = \min\{i^P(x_1, x_2), (x_1, x_2)$ is an edge of $\mathcal{P}^m_{x,y}\}$, $f_x^P = \max\{f^P(x_1, x_2), (x_1, x_2)$ is an edge of $\mathcal{P}^m_{x,y}\}$, $t_x^N = \max\{t^N(x_1, x_2), (x_1, x_2)$ is an edge of $\mathcal{P}^m_{x,y}\}$, $i_x^N = \max\{i^N(x_1, x_2), (x_1, x_2)$ is an edge of $\mathcal{P}^m_{x,y}\}$, $f_x^N = \min\{f^N(x_1, x_2), (x_1, x_2)$ is an edge of $\mathcal{P}^m_{x,y}\}$, respectively.

Definition 2.31. Suppose $G = (A, B)$ is a bipolar neutrosophic graph. Then *m-step bipolar neutrosophic neighbouhood graph* $\mathcal{N}_m(G)$ is defined by $\mathcal{N}_m(G) = (A, \acute{B})$ where $A = (A_1^P, A_2^P, A_3^P, A_1^N, A_2^N, A_3^N)$, $\acute{B} = (\acute{B}_1^P, \acute{B}_2^P, \acute{B}_3^P, \acute{B}_1^N, \acute{B}_2^N, \acute{B}_3^N)$, $\acute{B}_1^P : X \times X \to [0, 1]$, $\acute{B}_2^P : X \times X \to [0, 1]$, $\acute{B}_3^P : X \times X \to [0,1]$, $\acute{B}_1^N : X \times X \to [-1, 0]$, $\acute{B}_2^N : X \times X \to [-1, 0]$, and $\acute{B}_3^N : X \times X \to [-1,0]$ are such that:

$$\begin{aligned}
\acute{B}_1^P(x,y) &= A_1^P(x) \wedge A_1^P(y) h_1(\mathcal{N}_m(x) \cap \mathcal{N}_m(y)),\\
\acute{B}_2^P(x,y) &= A_2^P(x) \wedge A_2^P(y) h_2(\mathcal{N}_m(x) \cap \mathcal{N}_m(y)),\\
\acute{B}_3^P(x,y) &= A_3^P(x) \vee A_3^P(y) h_3(\mathcal{N}_m(x) \cap \mathcal{N}_m(y)),\\
\acute{B}_1^N(x,y) &= A_1^N(x) \vee A_1^N(y) h_4(\mathcal{N}_m(x) \cap \mathcal{N}_m(y)),\\
\acute{B}_2^N(x,y) &= A_2^N(x) \vee A_2^N(y) h_5(\mathcal{N}_m(x) \cap \mathcal{N}_m(y)),\\
\acute{B}_3^N(x,y) &= A_3^N(x) \wedge A_3^N(y) h_6(\mathcal{N}_m(x) \cap \mathcal{N}_m(y)), \quad \text{respectively.}
\end{aligned}$$

Definition 2.32. [?]Consider a bipolar neutrosophic graph $G = (A, B)$, where $A = (A_1^P, A_2^P, A_3^P, A_1^N, A_2^N, A_3^N)$, and $B = (B_1^P, B_2^P, B_3^P, B_1^N, B_2^N, B_3^N)$ then, an edge (x, y), $x, y \in X$ is called *independent strong* if

$$\begin{aligned}
&\frac{1}{2}[A_1^P(x) \wedge A_1^P(y)] < B_1^P(x,y), \quad \frac{1}{2}[A_1^N(x) \vee A_1^N(y)] > B_1^N(x,y),\\
&\frac{1}{2}[A_2^P(x) \wedge A_2^P(y)] < B_2^P(x,y), \quad \frac{1}{2}[A_2^N(x) \vee A_2^N(y)] > B_2^N(x,y),\\
&\frac{1}{2}[A_3^P(x) \vee A_3^P(y)] > B_3^P(x,y), \quad \frac{1}{2}[A_3^N(x) \wedge A_3^N(y)] < B_3^N(x,y).
\end{aligned}$$

Otherwise, it is called *weak*.

Theorem 2.33. If all the edges of bipolar neutrosophic digraph $\overrightarrow{G} = (A, \overrightarrow{B})$ are independent strong, then all the edges of $\mathcal{C}_m(\overrightarrow{G})$ are independent strong.

PROOF. Suppose $\overrightarrow{G} = (A, \overrightarrow{B})$ is a bipolar neutrosophic digraph and $\mathcal{C}_m(\overrightarrow{G}) = (A, B)$ is corresponding m-step bipolar neutrosophic competition graph. Since all the edges of $\overrightarrow{G}$ are independent strong, then

$h_1(\mathcal{N}^+_m(x)\cap\mathcal{N}^+_m(y))>0.5,\quad h_2(\mathcal{N}^+_m(x)\cap\mathcal{N}^+_m(y))>0.5,\quad h_3(\mathcal{N}^+_m(w)\cap\mathcal{N}^+_m(z))<0.5,$
$h_4(\mathcal{N}^+_m(x)\cap\mathcal{N}^+_m(y))<0.5,\quad h_5(\mathcal{N}^+_m(x)\cap\mathcal{N}^+_m(y))<0.5,\quad h_6(\mathbb{N}^+_m(w)\cap\mathbb{N}^+_m(z))<0.5.$

Then,
$t^P_B(x,y)=(t^P_A(x)\wedge t^P_A(y))h_1(\mathcal{N}^+_m(x)\cap\mathcal{N}^+_m(y))$ or, $t^P_B(x,y)>0.5(t^P_A(x)\wedge t^P_A(y))$ or, $\frac{t^P_B(x,y)}{(t^P_A(x)\wedge t^P_A(y))}>0.5$, $i^P_B(x,y)=(i^P_A(x)\wedge i^P_A(y))h_2(\mathcal{N}^+_m(x)\cap\mathcal{N}^+_m(y))$ or, $i^P_B(x,y)>0.5(i^P_A(x)\wedge i^P_A(y))$ or, $\frac{i^P_B(x,y)}{(i^P_A(x)\wedge i^P_A(y))}>0.5$, $f^P_B(x,y)=(f^P_A(x)\vee f^P_A(y))h_3(\mathcal{N}^+_m(x)\cap\mathcal{N}^+_m(y))$ or, $f^P_B(x,y)<0.5(f^P_A(x)\vee f^P_A(y))$ or, $\frac{f^P_B(x,y)}{(f^P_A(x)\vee f^P_A(y))}<0.5$, $t^N_B(x,y)=(t^N_A(x)\vee t^N_A(y))h_4(\mathcal{N}^+_m(x)\cap\mathcal{N}^+_m(y))$ or, $t^N_B(x,y)<0.5(t^N_A(x)\vee t^N_A(y))$ or, $\frac{t^N_B(x,y)}{(t^N_A(x)\vee t^N_A(y))}<0.5$, $i^N_B(x,y)=(i^N_A(x)\vee i^N_A(y))h_5(\mathcal{N}^+_m(x)\cap\mathcal{N}^+_m(y))$ or, $i^N_B(x,y)<0.5(i^N_A(x)\vee i^N_A(y))$ or, $\frac{i^N_B(x,y)}{(i^N_A(x)\vee i^N_A(y))}<0.5$, $f^N_B(x,y)=(f^N_A(x)\wedge f^N_A(y))h_6(\mathcal{N}^+_m(x)\cap\mathcal{N}^+_m(y))$ or, $f^N_B(x,y)<0.5(f^N_A(x)\wedge f^N_A(y))$ or, $\frac{f^N_B(x,y)}{(f^N_A(x)\wedge f^N_A(y))}<0.5$.

Hence, the edge $(x,\ y)$ is independent strong in $\mathcal{C}_m(\overrightarrow{G})$. Since, $(x,\ y)$ is taken to be arbitrary edge of $\mathcal{C}_m(\overrightarrow{G})$, thus all the edges of $\mathcal{C}_m(\overrightarrow{G})$ are independent strong.

3. Application

Sports are very important, every society has its own special kinds of sports. The proper end of sports is bodily health arid physical fitness. Sports and games have now come to stay in our civilization as an essential feature of human activity, and their object is not merely fun, they also instill the sprit of discipline and team-work. Sports like cricket, hockey and foot ball are popular because of the sprit of team work which they inspire. This no doubt true. The discipline that gained in playing up sports is invaluable in later life. It makes for a life of co-operation and team work which could be used for building up a great society and a nation. Key components of sports are goals, rules, challenge, and interaction. Sports generally involve mental or physical stimulation, and often both. Many sports help develop practical skills, serve as a form of exercise, or otherwise perform an educational, simulational, or psychological role etc. Many sports require special equipment and dedicated playing fields, leading to the involvement of a community much larger than the group of players. A city or town may set aside such resources for the organization of sports leagues, like, tabletop games, board games, etc. All these types of sports are called local sports. These sports can be extended to provisional level sports. After provisional level sports there are national sports. A national sport is a sport or game that is considered to be an intrinsic part of the culture of a nation. Every nation has different sports, such as, baseball is known as national sports in the United States, cricket is in England, and hockey is in Pakistan, etc. After, national level of sports there are international level of sports. International sport is a sport in which the participants represent different countries. The most

well-known international sports event is the Olympic Games, FIFA World Cup and the Paralympic Games.
Consider the set consisting of three countries $\{C_1, C_2, C_3\}$ and also consider the set of players {(Abigail, 0.9, 0.8, 0.5, −0.6, −0.5, −0.2), (Alex, 0.6, 0.3, 0.4, −0.2, −0.4, −0.4),(Amelia, 0.8, 0.7, 0.2, −0.7, −0.8, −0.5), (Agatha, 0.9, 0.8, 0.5, −0.6, −0.5, −0.2), (Angela, 0.9, 0.8, 0.5, −0.6, −0.5, −0.2), (Belinda, 0.9, 0.8, 0.5, −0.6, −0.5, −0.2), (Ann, 0.5, 0.3, 0.5, −0.5, −0.3, −0.2), (Arlene, 0.8, 0.8, 0.9, −0.8, −0.9, −0.8), (Bella, 0.6, 0.4, 0.9, −0.6, −0.7, −0.5), (Anne, 0.9, 0.7, 0.8, −0.8, −0.8, −0.8), (April, 0.5, 0.3, 0.5, −0.5, −0.3, −0.2), (Abbey, 0.5, 0.3, 0.5, −0.5, −0.3, −0.2)}, which are taking part in their local, provisional, national, and international level games, as shown in Fig. 9. The positive degree of membership $t^P(x)$ of each player represent the percentage of hardwork towards to achieve the success in particular game, $i^P(x)$ and $f^P(x)$ represent the indeterminacy and falsity in this percentage. The negative degree of membership $t^N(x)$ represents the percentage that the player faces failure in the achievement of success in a particular game, $i^N(x)$ and $f^N(x)$ represent the indeterminacy and falsity in this percentage. The positive degree of membership $t^P(x)$ of each directed edge between player and local, provisional, national and international level games represent the percentage of having stamina for that level of sports in international game, $i^P(x)$ and $f^P(x)$ represent the indeterminacy and falsity in this percentage. The negative degree of membership $t^N(x)$ of each directed edge between player and local, provisional, national and international level games represent the percentage of having no stamina for that level of sports in international game, $i^N(x)$ and $f^N(x)$ represent the indeterminacy and falsity in this percentage.
Thus, 4-step bipolar neutrosophic competition graph can be used in order to find the best results. There 4-step bipolar neutrosophic out-neighbourhoods is calculated in Table 2.

Table 2. 4-Step bipolar neutrosophic out-neighbourhoods

$x \in X$	$\mathcal{N}_4^+(x)$
Abigail	{(International games, 0.2, 0.2, 0.6, −0.1, −0.3, −0.4)}
Alex	{(International games, 0.4, 0.2, 0.6, −0.1, −0.2, −0.7)}
Amelia	{(International games, 0.5, 0.5, 0.6, −0.2, −0.2, −0.8)}

Therefore, $\mathcal{N}_4^+(Abigail) \cap \mathcal{N}_4^+(Alex) = \{(International\ games$, 0.2, 0.2, 0.6, −0.1, −0.2, −0.7)}, $\mathcal{N}_4^+(Abigail) \cap \mathcal{N}_4^+(Amelia) = \{(International\ games$, 0.2, 0.2, 0.6, −0.1, −0.2, −0.8)}, and $\mathcal{N}_4^+(Alex) \cap \mathcal{N}_4^+(Amelia) = \{(International\ games$, 0.4, 0.2, 0.6, −0.1, −0.2, −0.8)}. Further, $h(\mathcal{N}_4^+(Abigail) \cap \mathcal{N}_4^+(Alex))$ = (0.2, 0.2, 0.6, 0.2, 0.2, 0.6), $h(\mathcal{N}_4^+(Abigail) \cap \mathcal{N}_4^+(Amelia))$ = (0.2, 0.2, 0.6, 0.2, 0.2, 0.6), and $h(\mathcal{N}_4^+(Amelia) \cap \mathcal{N}_4^+(Alex))$ = (0.4, 0.2, 0.6, 0.4, 0.2, 0.6). Thus, we obtain 4-step bipolar neutrosophic competition graph, as shown in Fig. 10.

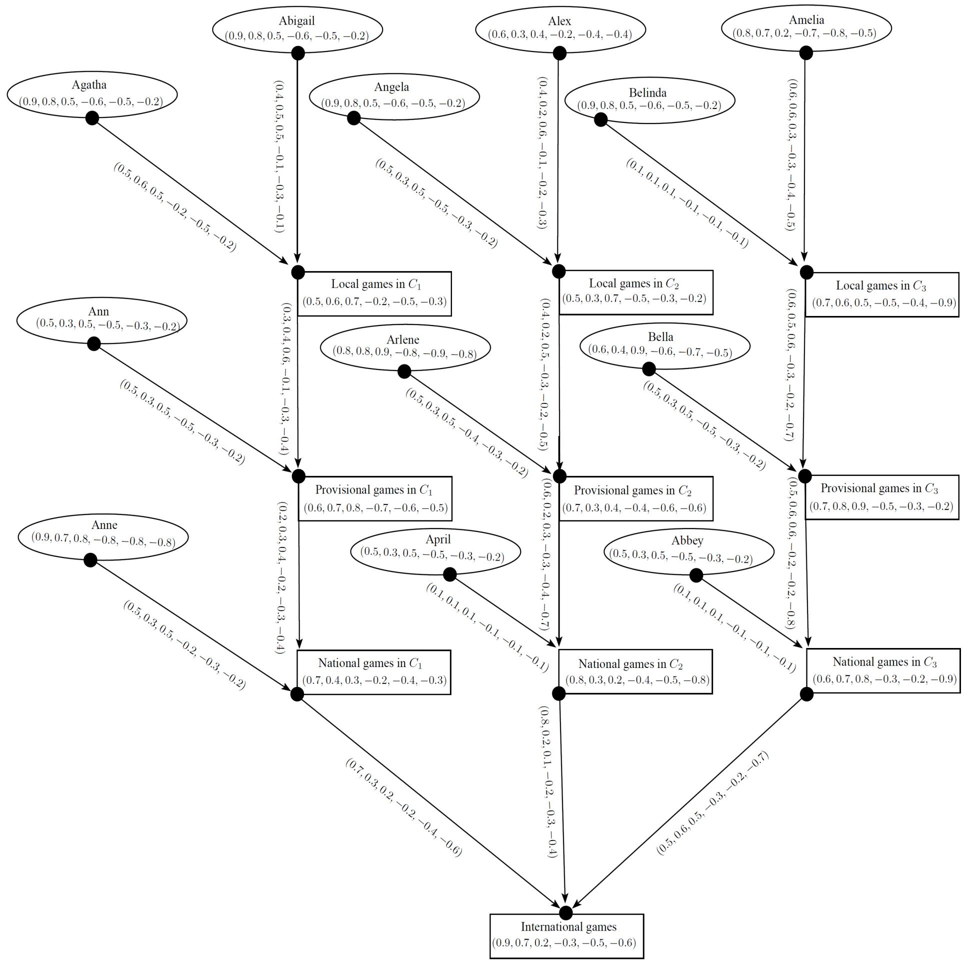

FIGURE 9. Bipolar neutrosophic digraph

TABLE 3. Strength of competition of applicants for international games

(x, y)	$T(x, y)$	$S(x, y)$
(Abigail, Alex)	$(0.12, 0.06, 0.30, -0.04, -0.08, -0.24)$	1.04
(Abigail, Amelia)	$(0.16, 0.14, 0.30, -0.12, -0.10, -0.30)$	1
(Alex, Amelia)	$(0.24, 0.06, 0.24, -0.08, -0.08, -0.30)$	1.24

The strength to compete the others players with respect hardwork in order to achieve success is calculated in Table 3. In Table 3, $T(x,\, y)$ represents the value of strength of competition between players x and y with respect to hardwork to

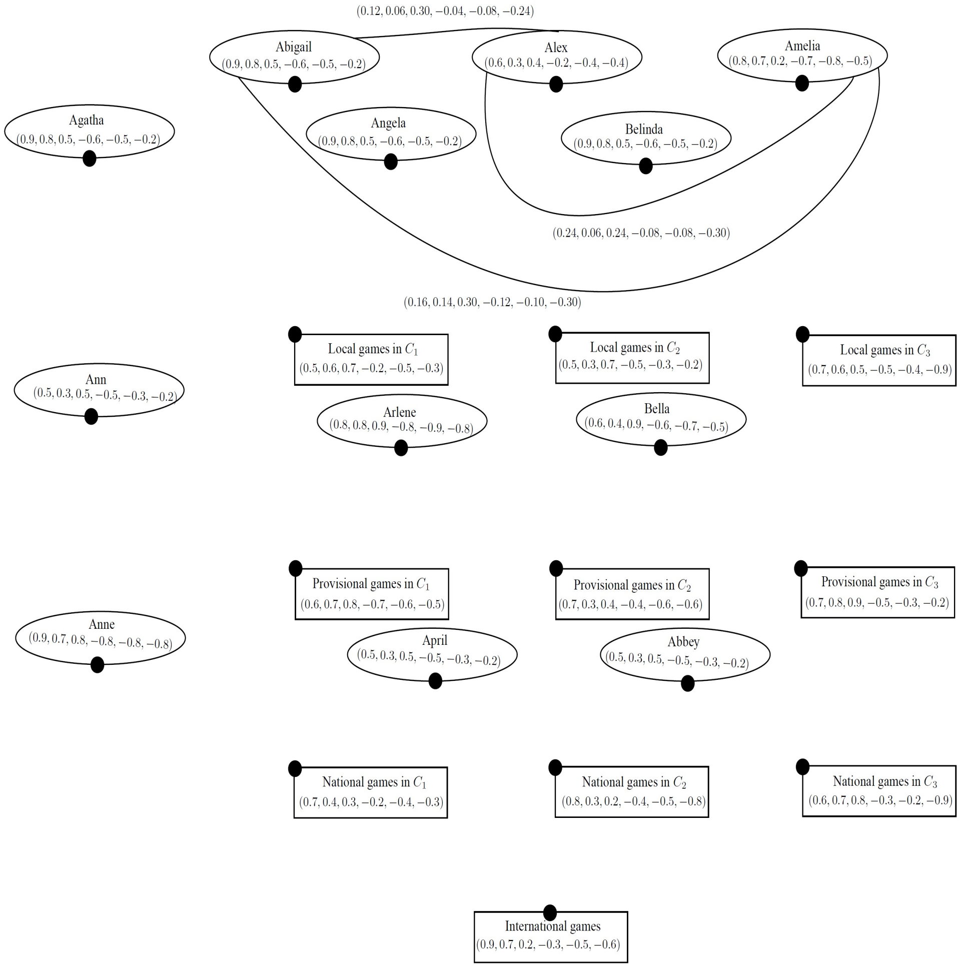

Figure 10. 4-Step bipolar neutrosophic competition graph

achieve the success in particular game. From Table 3, it is clear that the strength of competition between Alex and Amelia to achieve the success in particular game in international level is 1.24, while strength of competition between between Abigail and Amelia is 1, and strength of competition between between Abigail and Alex is 1.04. It is also clear from the Table 3, that Alex and Amelia are strongest contestants, as the strength of competition between them has the largest value than the other contestants.

We now elaborate this method with the help of an algorithm.

Algorithm

Step 1.: Input the positive truth, indeterminacy and falsity-memberships values and negative truth, indeterminacy and falsity-memberships values for set of r applicants.

Step 2.: If for any two distinct vertices x_i and x_j, $t^P(x_ix_j) > 0$, $i^P(x_ix_j) > 0$, $f^P(x_ix_j) > 0$, $t^N(x_ix_j) < 0$, $i^N(x_ix_j) < 0$, $f^N(x_ix_j) < 0$, then

$$(x_j, t^P(x_ix_j), i^P(x_ix_j), f^P(x_ix_j), t^N(x_ix_j), i^N(x_ix_j), f^N(x_ix_j)) \in \mathcal{N}_m^+(x_i).$$

Step 3.: Repeat step 2 for all vertices x_i and x_j to calculate m-step bipolar neutrosophic-out-neighbourhoods $\mathcal{N}_m^+(x_i)$.

Step 4.: Calculate $\mathcal{N}_m^+(x_i) \cap \mathcal{N}_m^+(x_j)$ for each pair of distinct vertices x_i and x_j.

Step 5.: Calculate $h[\mathcal{N}_m^+(x_i) \cap \mathcal{N}_m^+(x_j)]$.

Step 6.: If $\mathcal{N}_m^+(x_i) \cap \mathcal{N}_m^+(x_j) \neq \emptyset$ then draw an edge x_ix_j.

Step 7.: Repeat step 6 for all pair of distinct vertices.

Step 8.: Assign membership values to each edge x_ix_j using the conditions

$$t^P(x_ix_j) = (x_i \wedge x_j)h_1[\mathcal{N}_m^+(x_i) \cap \mathcal{N}_m^+(x_j)] \quad t^N(x_ix_j) = (x_i \vee x_j)h_4[\mathcal{N}_m^+(x_i) \cap \mathcal{N}_m^+(x_j)]$$

$$i^P(x_ix_j) = (x_i \wedge x_j)h_2[\mathcal{N}_m^+(x_i) \cap \mathcal{N}_m^+(x_j)] \quad i^N(x_ix_j) = (x_i \vee x_j)h_5[\mathcal{N}_m^+(x_i) \cap \mathcal{N}_m^+(x_j)]$$

$$f^P(x_ix_j) = (x_i \vee x_j)h_3[\mathcal{N}_m^+(x_i) \cap \mathcal{N}_m^+(x_j)] \quad f^N(x_ix_j) = (x_i \wedge x_j)h_6[\mathcal{N}_m^+(x_i) \cap \mathcal{N}_m^+(x_j)].$$

Step 10.: Calculate $S(x, y)$, the strength of competition between players x and y.

$$S(x, y) = t^P(x, y) - (i^P(x, y) + f^P(x, y)) + 1 + t^N(x, y) - (i^N(x, y) + f^N(x, y)).$$

Step 11.: Maximum value of $S(x, y)$ gives that x and y are strongest players than the others.

4. Concluding Remarks

Graph theory is an enjoyable playground for the research of proof techniques in discrete mathematics. There are many applications of graph theory in different fields. We have introduced the concepts of the bipolar neutrosophic competition graphs. We have described an application of m-step bipolar neutrosophic competition graphs in different level of games with the help of an algorithm. We aim to extend our research work to (1) Bipolar fuzzy rough graphs; (2) Bipolar fuzzy rough hypergraphs, (3) Bipolar fuzzy rough neutrosophic graphs, and (4) Decision support systems based on bipolar neutrosophic graphs.

Acknowledgement. The authors are highly thankful to Executive Editor and the referees for their valuable comments and suggestions.

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