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# ON THE NORMALITY OF SOME CAYLEY DIGRAPHS WITH VALENCY 2

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**Abstract.** We call a Cayley digraph $\Gamma = \mathrm{Cay}(G, S)$ normal for $G$ if $R(G)$, the right regular representation of $G$, is a normal subgroup of the full automorphism group $\mathrm{Aut}(\Gamma)$ of $\Gamma$. In this paper we determine the normality of Cayley digraphs of valency 2 on the groups of order $pq$ and also on non-abelian finite groups $G$ such that every proper subgroup of $G$ is abelian.

*Key words*: Cayley digraph, normal Cayley digraph, automorphism group.

**Abstrak.** Digraf Cayley $\Gamma = \mathrm{Cay}(G, S)$ dinyatakan normal untuk $G$ jika $R(G)$, representasi reguler kanan $G$, adalah subgrup normal dari grup automorfisma penuh $\mathrm{Aut}(\Gamma)$ dari *Gamma*. Dalam paper ini kami menentukan normalitas digraf Cayley bervalensi 2 pada grup berorde $pq$ dan juga grup hingga non-Abelian $G$ sedemikian sehingga setiap subgrup sejati dari $G$ adalah Abelian.

*Kata kunci*: Digraf Cayley, digraf Cayley normal, grup automorfisma.

## 1. Introduction

Let $G$ be a finite group, and $S$ a subset of $G$ such that $1_G \notin S$. The Cayley digraph $\Gamma$=Cay$(G, S)$ of $G$ with respect to $S$ is defined to have vertex set $V(\Gamma) = G$ and edge set $E(\Gamma) = \{(g, sg) \mid g \in G, s \in S\}$. From the definition the following obvious facts are basic for Cayley digraphs. (1) the automorphism group $\mathrm{Aut}(\Gamma)$ of $\Gamma$ contains $R(G)$, the right regular representation of $G$, as a subgroup; (2) $\Gamma$ is connected if and only if $S$ generates the group $G$; (3) $\Gamma$ is undirected if and only if $S = S^{-1}$. A Cayley digraph $\Gamma$=Cay$(G, S)$ is called normal if the subgroup $R(G)$ is

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a normal subgroup of the automorphism group of $\Gamma$.

The concept of normality of Cayley digraphs is known to be important for the study of arc-transitive graphs and half-transitive graphs (e.g. see [10]). In general, it is known to be difficult to determine the normality of Cayley digraphs. The only groups for which a complete classification of normal Cayley digraphs is available, are the cyclic groups of prime order and the groups of order twice a prime [1, 4]. Since Wang, Wang and Xu [9] obtained all normal disconnected Cayley digraphs of finite groups, we always suppose in this paper that the Cayley digraph $\mathrm{Cay}(G, S)$ is connected. That is, $S$ is a generating subset of $G$.

A subset $S$ of $G$ is said to be a CI-subset (CI stands for Cayley isomorphism) if whenever $\mathrm{Cay}(G, T)$ is isomorphic to $\mathrm{Cay}(G, S)$, there is an automorphism $\alpha$ of $G$ such that $S^\alpha = T$. (Then we call the corresponding graph $\mathrm{Cay}(G, S)$ a CI-graph.) Let $G$ be a finite group and let $S$ be a minimal generating set of $G$. Then Xu [10, Problem 6] asked the question:
(1) Are $S$ and $S \cup S^{-1}$ CI-subsets?
(2) Are the corresponding Cayley digraphs and graphs normal?

For abelian groups, Feng and Gao [5] proved that if the Sylow 2-subgroup of $G$ is cyclic, then the answers to both questions (1) and (2) are positive; otherwise, they are negative in general. More details can be found in [3, 6, 7].

Let $X$ and $Y$ be two graphs. The direct product $X \times Y$ is defined as the graph with vertex set $V(X \times Y) = V(X) \times V(Y)$ such that for any two vertices $u = [x_1, y_1]$ and $v = [x_2, y_2]$ in $V(X \times Y)$, $[u, v]$ is an edge in $X \times Y$ whenever $x_1 = x_2$ and $[y_1, y_2] \in E(Y)$ or $y_1 = y_2$ and $[x_1, x_2] \in E(X)$. Two graphs are called relatively prime if they have no nontrivial common direct factor. The lexicographic product $X[Y]$ is defined as the graph with vertex set $V(X[Y]) = V(X) \times V(Y)$ such that for any two vertices $u = [x_1, y_1]$ and $v = [x_2, y_2]$ in $V(X[Y])$, $[u, v]$ is an edge in $X[Y]$ whenever $[x_1, x_2] \in E(X)$ or $x_1 = x_2$ and $[y_1, y_2] \in E(Y)$. Let $V(Y) = \{y_1, y_2, ..., y_n\}$. Then there is a natural embedding of $nX$ in $X[Y]$, where for $1 \leq i \leq n$, the $i$th copy of $X$ is the sub-graph induced on the vertex subset $\{(x, y_i) | x \in V(X)\}$ in $X[Y]$. The deleted lexicographic product $X[Y] - nX$ is the graph obtained by deleting all the edges of (this natural embedding of) $nX$ from $X[Y]$.

In the first theorem we determine all non-normal connected Cayley digraphs of valency 2 on the groups of order $pq$, where $p$ and $q$ are prime numbers. From elementary group theory we know that up to isomorphism there are two groups of order $pq$ $(p < q)$ defined as: $G_1 = \mathbb{Z}_{pq}$, and $G_2 = \langle x, y \mid x^p = y^q = 1,\ x^{-1}yx = y^r \rangle$ where $r$, $p$, $q$ satisfy $r^p \equiv 1 \pmod q$, $r \not\equiv 1 \pmod q$, $(r, p) = 1$. If $p \nmid q - 1$, then the second case can not occur. Moreover when $p = q$, then $G$ is isomorphic to $\mathbb{Z}_{p^2}$ or $\mathbb{Z}_p \times \mathbb{Z}_p$.

**Theorem 1.1.** *Let $G$ be a group of order $pq$ where $p$, $q$ are distinct primes, and $S$ a two-element generating subset of $G$. If $\Gamma = Cay(G,S)$ is not normal, then $\Gamma = Cay(G,S) \cong C_q[2K_1]$ and $Aut(\Gamma) \cong C_2 \wr C_q$, where $C_q$ is a directed cyclic of length $q$.*

In the following theorem we determine the normality of Cayley digraphs of valency 2 on nonabelian finite groups $G$ such that every proper subgroup of $G$ is abelian. Note that Frattini subgroup $\Phi(G)$ is the intersection of maximal subgroups of group $G$.

**Theorem 1.2.** *Let $G$ be a finite minimal non-abelian group, such that $2 \nmid |G|$ and $G/\Phi(G)$ is abelian. Moreover suppose that $\Gamma = Cay(G,S)$ be a 2-valent connected Cayley digraph of $G$ with respect to $S = \{e, f\}$. Then $\Gamma$ is normal.*

## 2. **Primary Analysis**

Let $G$ be a finite group and let $\Gamma$=Cay$(G,S)$ be a connected Cayley (di)graph of valency two. Denote by $(g, z_1g, z_2z_1g, ...., z_{n-1}z_{n-2}....z_1g, z_nz_{n-1}....z_1g)$ a cycle of length $n$ in $\Gamma$, where $z_i \in S$ $(1 \leq i \leq n)$. Obviously $z_{i+l}....z_{i+1}z_i \neq 1$ $(1 \leq i \leq i+l \leq n)$, except $z_nz_{n-1}....z_1 = 1$. For simplicity we use $C_g(z_nz_{n-1}...z_1)(= C_g(1))$ to denote this cycle.
We give some results which will be used later in the proofs of Theorems 1.1 and 1.2.

**Proposition 2.1.** *[2, Proposition 2.5] Let $\Gamma = Cay(G,S)$ and $\Gamma' = Cay(G,S^\alpha)$ where $\alpha \in Aut(G)$ . Then $\Gamma$ is normal if and only if $\Gamma'$ is normal.*

**Proposition 2.2.** *[10, Proposition 1.5] Let $\Gamma = Cay(G,S)$ and $A = Aut(\Gamma)$. Then $\Gamma$ is normal if and only if $A_1 = Aut(G,S)$, where $A_1$ is the stabilizer of $1_G$ in $A$, and $Aut(G,S) = \{\alpha \in Aut(G) | S^\alpha = S\}$.*

**Proposition 2.3.** *[3, Lemma 2.2] Let $G$ be a finite group and $S = \{e, f\}$ be a two-element generating subset of $G$ not containing the identity $1_G$. Set $\Gamma = Cay(G,S)$ and $A = Aut(\Gamma)$. If $A_1^*$ denotes the subgroup of $A$ which fixes $1_G$, $e$ and $f$, then we have the following:*
*(i) $A_1^* = 1$ if and only if $\Gamma = Cay(G,S)$ is normal.*
*(ii) Let $o(e^{-1}f) = m$ be odd. Suppose that $\langle d \rangle \subseteq \langle e^{-1}f \rangle$ or $\langle e^{-1}f \rangle \subseteq \langle d \rangle$, where $d = f(e^{-1}f)^{(m-1)/2}$. Then $\Gamma = Cay(G,S)$ is normal.*
*(iii) Suppose $\alpha \in A_1^*$. If $o(e^{-1}f) = m$ is odd, then $\alpha$ fixes at least $\langle d \rangle \langle e^{-1}f \rangle$ pointwise, where $d = f(e^{-1}f)^{(m-1)/2}$.*
*(iv) If one of $e$ and $f$ has order 2, then $\Gamma = Cay(G,S)$ is normal.*

A finite group $G$ is called minimal non-abelian if $G$ is not abelian, but every proper subgroup of $G$ is abelian. Now we have the following result.

**Proposition 2.4.** *[8] Let $G$ be a finite minimal non-abelian group. Then one of the following holds:*
*(1) $G$ is a minimal non-abelian $p$-group;*
*(2) $G$ is a semi-direct product of an elementary abelian $p$-group $P$ of order $p^\alpha$ by a cyclic group $Q = \langle b\rangle$ of order $q^\beta$, where $p$, $q$ are distinct primes and the action of $b$ on $P$ is an automorphism of $P$ of order $q$.*

**Proposition 2.5.** *[2, Theorem 1.1] Let $G$ be a finite abelian group and $S$ a generating subset of $G$ not containing the identity 1. Assume $S$ satisfies the condition*

$$x, y, u, v \in S \text{ with } 1 \neq xy = uv, \text{ implies } \{x, y\} = \{u, v\}. \tag{1}$$

*Then the Cayley (di)graph $Cay(G, S)$ is normal.*

## 3. Proof of Theorem 1.1

Note that if $S = S^{-1}$, then $\Gamma$=Cay$(G, S)\cong C_n$ (the cycle of length $n$), and obviously $\Gamma$ is normal. Therefore we may let $S \neq S^{-1}$.

Now we complete the proof of Theorem 1.1. Let $G$ be a group of order $pq$, where $p$ and $q$ are primes, and $S = \{e, f\}$ be a generating subset for $G$ with $e \neq f$ and $1 \notin S$.
Assume first that $p$ does not divide $q - 1$, so $G$ is cyclic, and $G \cong \mathbb{Z}_{pq}$. In view of Proposition 2.5, we assume $S$ does not satisfy the condition (1). Thus there are (not necessarily distinct) elements $x, y, u, v$ in $S$ with $1 \neq xy = uv$ and $\{x, y\} \neq \{u, v\}$. If one of $e$ and $f$ has order 2, then by Proposition 2.3 (iv), $\Gamma$=Cay$(G, S)$ is normal. Thus $e^2 \neq 1$ and similarly $f^2 \neq 1$. If, say, $\{x, y\} = \{e, f\}$ then since $\{x, y\} \neq \{u, v\}$ we must have $\{u, v\} = \{e\}$ or $\{f\}$ but this implies that $xy \neq uv$, a contradiction. It follows that $x = y = e$, say, and $u = v = f$ and $e^2 = f^2 \neq 1$. Since $e \neq f$, one of these elements, say $e$, must have even order , so $p$, say, is equal to 2. Now $G$ is isomorphic to $\mathbb{Z}_{2q} = \langle a\rangle$. So one may let $S = \{a^i, a^{q+i}\}$ where $(i, 2q) = 1$. Since $a \mapsto a^i$, can be extended to an automorphism of $G$, so by Proposition 2.1, we can assume that $S = \{a, a^{q+1}\}$ and $\Gamma$=Cay$(G, S)\cong C_q[2K_1]$. In this case $\sigma : a^i \mapsto a^{q+i}$ $(1 \leq i < 2q, i \neq q)$ such that $\sigma(1) = (1)$, $\sigma(a^q) = (a^q)$ is not in Aut$(G, S)$ but in $A_1$. So by Proposition 2.2 $\Gamma$ is not normal. Also Aut$(\Gamma)\cong C_2 \wr C_q$.
Now let $G$ be nonabelian. Thus we may assume that $p$ divides $q - 1$ and $G$ is is isomorphic to $\langle x, y \mid x^p = y^q = 1,\ x^{-1}yx = y^r\rangle$, where $r$, $p$, and $q$ satisfy $r^p \equiv 1 \pmod q$, $r \not\equiv 1 \pmod q$, and $(r, p) = 1$. By simply checking, we can see that, the elements of order $p$ are $\{x^i y^j\}$ $(1 \leq i \leq p - 1,\ 0 \leq j \leq q - 1)$, and the elements of order $q$ are $y^i$, where $1 \leq i \leq q - 1$, thus we will see easily that $e$ and $f$ cannot be of order $q$ simultaneously. Thus we can suppose that $o(e) = p$ and $o(f) = p$ or $q$. If $o(e) = p$ and $o(f) = q$, we have $e = x^\alpha y^\beta$, $(1 \leq \alpha \leq p - 1, 0 \leq \beta \leq q - 1)$ and $f = y^\delta$, $1 \leq \delta \leq q - 1$. We claim that there is only one directed cycle of length $p$ through every vertex $g$ of $\Gamma$. We can suppose $g = 1$ because of the transitivity of

$\Gamma$.
Obviously there is a directed cycle of length $p$ through 1: $1 \mapsto e \mapsto e^2 \mapsto .... \mapsto e^{p-1} \mapsto 1$. Let $C_1(d_p d_{p-1} .... d_1)$ be another directed cycle of length $p$ through 1, where $d_i = e$ or $f$ $(1 < i < p)$. Let $n$ be the number of $e$ appearing in the product $d_p .... d_1$. Obviously $1 \leq n < p$, and $d_p .... d_1 = 1$. Therefore $x^{\alpha n} y^t = 1$ (for some integer $t$). It is clear that for any $g \in G$, $g$ can be expressed in the unique form: $x^i y^j$ $(0 \leq i < p,\ 0 \leq j < q)$. Thus $n \equiv 0 \pmod p$, a contradiction to $1 \leq n < p$.
Suppose $\sigma \in A_1^*$. Since there is only one directed cycle of length $p$ through $f$: $f \mapsto ef \mapsto e^2 f \mapsto .... \mapsto e^p f = f$, we have that $\sigma$ fixes $\Gamma_1(e)$ and $\Gamma_1(f)$ pointwise. From the connectivity of $\Gamma$ we have $\sigma = 1$, so $A_1^* = 1$. By Proposition 2.3 (i) $\Gamma$=Cay$(G, S)$ in normal.
Now suppose that $o(e) = p$, $o(f) = p$. Then $e = x^i y^j$ $(1 \leq i < p,\ 0 \leq j < q)$. Since $\sigma$, defined by:$x^i y^j \mapsto x, y \mapsto y$ can be extended to automorphism of $G$, by Proposition 2.1 one may let $e = x$. If $f = x^\alpha y^\beta$ $(1 \leq \alpha < p,\ 0 \leq \beta < q)$ so $e^{-1} f = x^{\alpha-1} y^\beta$ $(0 \leq \alpha - 1 < p,\ 0 \leq \beta < p)$. Suppose that $\alpha - 1 = 0$, therefore $e^{-1} f = y^\beta$ and $o(e^{-1} f) = q$. We can also suppose that $\langle d \rangle \nsubseteq \langle e^{-1} f \rangle$ by Proposition 2.3 (ii), where $d = f(e^{-1} f)^{(q-1)/2}$. Thus $\langle d \rangle \langle e^{-1} f \rangle = G$. By Proposition 2.3 (iii) $A_1^*$ fixes $G = \langle d \rangle \langle e^{-1} f \rangle$ and so $\Gamma$=Cay$(G, S)$ is normal. Now suppose that $\alpha - 1 > 0$, so $o(e^{-1} f) = p$, the same as before we can suppose that $\langle d \rangle \nsubseteq \langle e^{-1} f \rangle$, where $d = f(e^{-1} f)^{(p-1)/2}$. Thus $G = \langle d \rangle \langle e^{-1} f \rangle$ and $A_1^*$ fixes $G = \langle d \rangle \langle e^{-1} f \rangle$ and so $\Gamma$=Cay$(G, S)$ is normal. The result now follows.

## 4. **Proof of Theorem 1.2**

Recall that we use $A$ to denote the automorphism group of the Cayley graph $\Gamma$, and $A_1$ to denote the group of all automorphisms of $\Gamma$ that fix identity $1_G$ of $G$. Also a finite group $G$ is called minimal non-abelian if $G$ is not abelian, but every proper subgroup of $G$ is abelian.

We now prove the Theorem 1.2. By Proposition 2.4 we have two cases. First suppose that $G$ is a $p$-group ($p$ is odd), in this case we assume $G$ has the smallest order such that Cay$(G, S)$ is not a normal 2-valent connected Cayley digraph. Note that $A$=$R(G) A_1$ and $R(G) \cap A_1$=$1_G$. Let $N$ be a nontrivial minimal normal subgroup of $A$. Since $A_1$ is a 2-group, $A$ is solvable and hence $N$ is an elementary abelian $q$-group, with $q$=2 or $p$. If $N$ is transitive on $V(\Gamma)$ then $N$ is regular on $V(\Gamma)$. Thus $|N| = |G| = |R(G)|$. Therefore $R(G) = N$ and $R(G)$ is normal in $A$ which is a contradiction. Thus $N$ is not transitive on $V(\Gamma)$. Let $m$ be the length of an $N$-orbit on V($\Gamma$), then m divides $|N|$. Note that $N \lhd A$ and $A$ acts transitively on $V(\Gamma)$, so $m$ divides $|G|$. Therefore $q$=$p$ and $N < R(G)$.
Let $\Sigma = \{T_1, T_2, ..., T_{p^\alpha}\}$ be all the orbits of $N$ on $V(\Gamma)$. Consider the quotient digraph $\Gamma_N$ defined by $V(\Gamma)$=$\Sigma$ and $(T_i, T_j) \in E(\Gamma_N)$ if and only if there exist $v_i \in T_i$, $v_j \in T_j$ such that $(v_i, v_j) \in E(\Gamma_N)$. Since the quotient group $R(G)/N$ acts regularly on $\Sigma$, therefore $\Gamma_N$ must be a connected Cayley digraph of $G/N$. So $\Gamma_N \cong$ Cay$(G/N, SN/N)$ $(|S \cap N| \leq 1)$. If $|S \cap N|$=1, then either e or f has order $p$. Without loss of generality we may assume that $o(e)$=p. Now we have two cases:

(1) $o(f)$=p and (2) $o(f) \neq p$.
We know that there is a directed $p$-path through 1: $1_G \mapsto e \mapsto e^2 \mapsto ..... \mapsto e^p$. Suppose that $1_G \mapsto d_1 \mapsto d_2d_1 \mapsto .... \mapsto d_pd_{p-1}...d_1$ is another $p$-path through 1 such that $d_pd_{p-1}...d_1$=1 and $d_i$=$f$ or $e$. Let $r$, $t$ be the numbers of $e$ and $f$ appearing in $\{d_1, d_2, ..., d_p\}$ respectively. Since $G$ is a non-cyclic $p$-group, $G/\Phi(G)$ is an elementary $p$-group where $\Phi(G)$ is the Frattini subgroup of $G$, and $G/\Phi(G)$=$\langle\Phi(G)e\rangle \times \langle\Phi(G)f\rangle$. We have $\Phi(G)d_p\Phi(G)d_{p-1}...\Phi(G)d_1$=$\Phi(G)$, so $(\Phi(G)e)^r(\Phi(G)f)^t$=$\Phi(G)$, and hence $r = t = 0$ (mod $p$). Note that $r + t$=$p$ and $0 \leq r, t \leq p$, therefore we have that either $r = p$ and $t = 0$ or $r = 0$ and $t = p$. Now if $o(f)$=$p$ then $A_1^*$=$1_G$ and $\Gamma$ is normal by Proposition 2.3 (i). If $o(f) > p$, similarly there is a unique directed $p$-cycle through $f$: $f \mapsto ef \mapsto e^2f \mapsto ... \mapsto f$ and consequently $A_1^*$=$1_G$, and $\Gamma$ is normal.
Now suppose that $|S \cap N|$=0 and $G/N$ is non-abelian. By the minimality of $|G|$, Cay$(G/N, SN/N)$ is normal. Therefore $R(G)/N \lhd A/N$. Since $R(G)$ is the full preimage of $R(G)/N$ under $A \mapsto A/N$, $R(G)$ is normal in $A$, which is a contradiction. Now suppose that $G/N$ is abelian. Suppose $\sigma \in A/N$, such that $\sigma(1) = 1$, $\sigma(Ne) = Ne$, and $\sigma(Nf) = Nf$. It is easy to show that $\sigma$ fixes $\Gamma_1(e)$, and $\Gamma_1(f)$ pointwise. Since $\Gamma$ is connected, $\sigma$=1. Therefore by Proposition 2.3 (i) $\Gamma_N$ is normal, and so under $A \mapsto A/N$, $\Gamma$ is normal. This is a contradiction.

Now let $|G|$=$p^\alpha q^\beta$, where $P$ and $Q$ are Sylow $p$-subgroups and Sylow $q$-subgroup of $G$, respectively such that $P \cong \mathbb{Z}_p \times \mathbb{Z}_p \times ... \times \mathbb{Z}_p$, and $Q$ is a cyclic group. We consider the quotient group $R(G)/P$, and suppose that $\Sigma = \{B_1, B_2, ...B_n\}$ be all the orbits of $P$ on $V(\Gamma)$. $R(G)/P$ acts on $\Sigma$ regularly, therefore the quotient graph $\Gamma_P$ is isomorphic to Cay$(G/P, SP/P)$ ($|S \cap P| \leq 1$). Since $G/\Phi(G)$ is abelian with the same reason as before one can show $\Gamma$ is normal, which is a contradiction and the proof of Theorem 1.2 is complete.

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