# DECOMPOSITION OF COMPLETE GRAPHS AND COMPLETE BIPARTITE GRAPHS INTO COPIES OF $P_{n}^{3}$ OR $S_{2}\left(P_{n}^{3}\right)$ AND HARMONIOUS LABELING OF $K_{2}+P_{n}$ 

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#### Abstract

In this paper, the graphs $P_{n}^{3}$ and $S_{2}\left(P_{n}^{3}\right)$ are shown to admit an $\alpha$ valuation, where $P_{n}^{3}$ is the graph obtained from the path $P_{n}$ by joining all the pairs of vertices $u, v$ of $P_{n}$ with $d(u, v)=3$ and $S_{2}\left(P_{n}^{3}\right)$ is the graph obtained from $P_{n}^{3}$ by merging the centre of the star $S_{n_{1}}$ and that of the star $S_{n_{2}}$ respectively at the two unique 2-degree vertex of $P_{n_{3}}$ (the origin and terminus of the path $P_{n}$ contained in $\left.P_{n}^{3}\right)$. It follows from the significant theorems due to Rosa [1967] and EI-Zanati and Vanden Eynden [1996] that the complete graphs $K_{2 c q+1}$ or the complete bipartite graphs $K_{m q, n q}$ can be cyclically decomposed into the copies of $P_{n}^{3}$ or copies of $S_{2}\left(P_{n}^{3}\right)$, where $c, m, n$ are arbitrary positive integer and $q$ denotes either $\left|E\left(P_{n}^{3}\right)\right|$ or $\left|E\left(S_{2}\left(P_{n}^{3}\right)\right)\right|$. Further, it is shown that join of complete graph $K_{2}$ and path $P_{n}$, denoted $K_{2}+P_{n}$, for $n \geq 1$ is harmonious graph.


Key words: $\alpha$-labeling, harmonious labeling, $P_{n}^{3}$ graphs, join, path.

