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An Enhanced Multi-Criteria Decision-Making Model Using Prospect Theory Under Interval-Valued Intuitionistic Quadri-Partitioned Neutrosophic Soft Set Environment

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Abstract. This paper presents a novel framework for addressing complex multi-criteria decision analysis (MCDA) problems by integrating Prospect Theory with the recently developed interval-valued intuitionistic quadri-partitioned neutrosophic soft set (IVIQPNSS) structure. The proposed model effectively incorporates both subjective preferences and objective criteria weights by leveraging Prospect Decision Theory, which models human behavior under risk and uncertainty based on gains and losses relative to a reference point. A newly formulated score function (SF) is introduced to transform the quadri-partitioned interval-valued neutrosophic information—comprising truth, indeterminacy, contradiction, and falsity—into precise numerical measures. This enables a refined ranking mechanism among alternatives. The proposed methodology merges expert judgment with data-driven insights, offering a robust algorithmic structure for decision-makers. The practical efficiency and reliability of the approach are demonstrated through a real-world application, making it a promising tool in the context of uncertain and ambiguous decision-making environments.

Key words and Phrases: Intuitionistic fuzzy, neutrosophic soft set, quadri-partitioned, MCDM.

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1. INTRODUCTION

To better capture uncertainty, hesitation, and contradictory information in real-world decision-making, Smarandache[1] introduced Neutrosophic Set (NS) theory. This framework addresses the limitations of classical fuzzy sets [2], intuitionistic fuzzy sets [3], and other hybrid models [4, 5] by permitting an independent assessment of truth, indeterminacy, and falsity. To enhance practicality, Wang et al. [6] introduced Single-Valued Neutrosophic Sets (SVNS), later extended to Interval-Valued Neutrosophic Sets (IVNS) [7], which represent membership degrees as intervals for greater flexibility in handling imprecise evaluations in Multi-Criteria Decision Analysis (MCDA). Subsequent research by Chinnadurai et al. [8, 9] and Jun et al. [10, 11] explored IVNS applications in algebraic structures and parameter-based rankings. Scholars such as Ridvan [12], Liu [13], and Garg [14] further developed aggregation operators and score functions (SFs) for MCDA within neutrosophic environments. Despite these advances, traditional IVNS frameworks, constrained by their tri-partitioned structure (truth, indeterminacy, falsity), remain limited in representing complex judgment scenarios involving multidimensional uncertainty and conflicting information.

In parallel, Prospect Decision Theory (PDT) [15] has emerged as a robust psychological lens for modeling decisions under uncertainty, evaluating alternatives based on perceived gains and losses rather than absolute outcomes. The integration of PDT with fuzzy and neutrosophic sets has shown significant promise. Foundational work by [16] combined PDT with interval-valued intuitionistic fuzzy sets for stochastic MCDA, while [17] extended it to general fuzzy environments under risk. The incorporation of PDT into neutrosophic frameworks was advanced by [18] and [19], who developed algorithms for neutrosophic soft sets and single-valued neutrosophic sets, respectively, applied to group decision-making and physician selection. Further complexity was addressed by [20], who integrated cumulative prospect theory with interval neutrosophic sets using a generalized Shapley function. However, a critical limitation persists across these hybrid models: their reliance on tri-partitioned structures prevents the explicit representation of *contradiction* where an element is simultaneously affirmed and denied. This is a common occurrence in expert judgments involving conflicting evidence or paradoxical data. To overcome this gap, we introduce a quadri-partitioned framework through *Interval-Valued Intuitionistic Quadri-Partitioned Neutrosophic Soft Sets* (IVIQNSS). By incorporating contradiction as a fourth independent component alongside truth, indeterminacy, and falsity, IVIQNSS provides a more nuanced and powerful ontological foundation. This structure enables a more expressive representation of decision-makers' hesitation and conflicting assessments, allowing PDT to evaluate prospects not just under uncertainty, but under genuine cognitive conflict.

The key contributions of this work are as follows: The development of a reliable SF for IVIQNSS that effectively handles interval-based truth, indeterminacy, contradiction, and falsity. A hybrid model integrating PDT with IVIQNSS, combining objective weights (derived from data) and subjective weights (from expert opinions) to balance psychological and rational decision components. A structured algorithm for solving MCDA problems in an IVIQNSS-PDT environment, validated through a real-world case study and comparative analysis.

The remainder of this manuscript is organized as follows: Section 2 introduces essential definitions. Section 3 proposes the new SF for IVIQNSS and addresses ranking limitations. Section 4 details the integration of PDT with IVIQNSS and outlines the decision-making algorithm. Section 5 presents a practical application. Section 6 compares the proposed model with established methods to validate its performance. Section 7 concludes the paper and suggests future research directions.

2. Preliminaries

This section outlines the foundational concepts required for this study, including Neutrosophic Sets (NS), Single-Valued Neutrosophic Sets (SVNS), Interval-Valued Neutrosophic Sets (IVNS), and their corresponding SFs. Throughout this section, unless explicitly mentioned otherwise, X denotes a universal set.

Definition 2.1 ([1]). *Let X be a universal set. A neutrosophic set (NS) on X is expressed as*

$$N = \{(x, T_N^*(x), I_N^*(x), F_N^*(x)) \mid x \in X\},$$

where $T_N^*(x)$, $I_N^*(x)$, and $F_N^*(x)$ denote the truth membership, indeterminacy membership, and falsity membership functions, respectively. These mappings are defined as $T_N^*(x), I_N^*(x), F_N^*(x) : X \to]0^-, 1^+[$. The sum of the three memberships satisfies the condition

$$0^- \le T_N^*(x) + I_N^*(x) + F_N^*(x) \le 3^+, \quad \forall x \in X.$$

Definition 2.2 ([6]). *A single-valued neutrosophic set (SVNS) is a constrained form of NS, given by*

$$N = \{(x, T_N(x), I_N(x), F_N(x)) \mid x \in X\},$$

where the functions $T_N(x)$, $I_N(x)$, and $F_N(x)$ map each element $x \in X$ to values within the closed interval $[0, 1]$. These are referred to as the truth, indeterminacy, and falsity membership degrees, respectively. For all $x \in X$, the following constraint must hold:

$$0 \le T_N(x) + I_N(x) + F_N(x) \le 3.$$

Definition 2.3 ([7]). *An interval-valued neutrosophic set (IVNS) extends the SVNS by allowing the membership values to be intervals rather than crisp numbers. It is formulated as*

$$N = \left\{\left(x, \left[T_N^-(x), T_N^+(x)\right], \left[I_N^-(x), I_N^+(x)\right], \left[F_N^-(x), F_N^+(x)\right]\right) \,\middle|\, x \in X\right\},$$

and alternatively represented as

$$N = \left\{\left(x, \tilde{T_N}(x), \tilde{I_N}(x), \tilde{F_N}(x)\right) \,\middle|\, x \in X\right\},$$

where $\tilde{T_N}(x)$, $\tilde{I_N}(x)$, and $\tilde{F_N}(x)$ are interval-valued functions mapping X into the set of closed intervals within $[0, 1]$, i.e., $D[0, 1]$. The cumulative upper bounds of these intervals must satisfy:

$$0 \le \sup \tilde{T_N}(x) + \sup \tilde{I_N}(x) + \sup \tilde{F_N}(x) \le 3, \quad \forall x \in X.$$

3. Score function

In this section, a new score function is proposed to effectively handle the characteristics of IVIQPNSS.

3.1. **Score Function.**

In the context of IVIQPNSSs, a SF serves as a vital computational tool to translate multi-dimensional uncertainty data into a single crisp numerical value. This transformation facilitates easier comparison and ranking of alternatives in a multi-criteria decision-making setting.

Definition 3.1. *Let the neutrosophic information corresponding to a particular element be represented by the interval tuples of truth, indeterminacy, contradiction, and falsity, namely:* $[T^L, T^U]$, $[I^L, I^U]$, $[C^L, C^U]$, *and* $[F^L, F^U]$, *respectively.*

Then, the unified score S *for this element is defined as:*

$$S = \frac{1}{4}\left(\frac{T^L + T^U}{2} + \left(1 - \frac{I^L + I^U}{2}\right) + \left(1 - \frac{C^L + C^U}{2}\right) + \left(1 - \frac{F^L + F^U}{2}\right)\right)$$

This formulation ensures that higher truth membership and lower degrees of indeterminacy, contradiction, and falsity contribute positively to the final score.

Remark 3.2. *The unified score function* S *defined above is bounded within the interval* $[0, 1]$ *and satisfies monotonicity.*

(1) ***Boundedness:*** *We show that* $S \in [0, 1]$.
Since all membership degrees are intervals within $[0, 1]$*, their averages also lie in* $[0, 1]$*:*

$$0 \leq \frac{T^L + T^U}{2} \leq 1, \quad 0 \leq \frac{I^L + I^U}{2} \leq 1, \quad 0 \leq \frac{C^L + C^U}{2} \leq 1, \quad 0 \leq \frac{F^L + F^U}{2} \leq 1.$$

Consequently, the expressions $\left(1 - \frac{I^L+I^U}{2}\right)$, $\left(1 - \frac{C^L+C^U}{2}\right)$, *and* $\left(1 - \frac{F^L+F^U}{2}\right)$ *are also bounded within* $[0, 1]$.

The sum inside the parentheses is therefore bounded by:

$$\textit{Minimum} = 0 + 0 + 0 + 0 = 0, \quad \textit{Maximum} = 1 + 1 + 1 + 1 = 4.$$

Applying the scaling factor $\frac{1}{4}$ *yields:*

$$0 \leq S \leq 1.$$

Thus, the score S *is normalized to the unit interval.*

(2) ***Monotonicity:*** *The function* S *is monotonically increasing.*
Consider two elements α *and* β. *The score* S *will increase if:*

- *The truth membership* $\dfrac{T^L + T^U}{2}$ *increases, or*

- *any of the terms* $\dfrac{I^L + I^U}{2}$, $\dfrac{C^L + C^U}{2}$, *or* $\dfrac{F^L + F^U}{2}$ *decrease (thereby increasing the terms* $\left(1 - \dfrac{I^L + I^U}{2}\right)$*, etc.).*

This behavior is consistent with the intuitive interpretation of the score: an element is considered better if it has more truth and less indeterminacy, contradiction, and falsity. Therefore, if α *has membership values that are uniformly "better" than or equal to those of* β *(i.e., higher truth and/or lower indeterminacy, contradiction, and falsity), then* $S(\alpha) \geq S(\beta)$*.*

4. Prospect Theory

We introduce a Prospect Decision Theory (PDT)-based framework designed to address Multi-Criteria Decision Analysis (MCDA) problems within a neutrosophic environment. An accompanying algorithm and flowchart are also provided to demonstrate the operational steps of the model.

The concept of PDT, initially proposed by Daniel and Atmos [15], explores how individuals make choices under uncertainty by incorporating psychological biases in decision-making processes. They defined a subjective value function, denoted by $\mathcal{V}_F(\tilde{x})$, which captures the decision-maker's (DM's) perception of outcomes based on the nature of the gains or losses:

$$\mathcal{V}_F(\tilde{x}) = \begin{cases} (\tilde{x})^{\alpha}, & \text{if } \tilde{x} \geq 0 \\ -\nabla(-\tilde{x})^{\beta}, & \text{if } \tilde{x} < 0 \end{cases} \tag{1}$$

Here, α and β represent the risk sensitivity parameters for gains and losses, respectively, where $0 < \alpha < 1$ and $0 < \beta < 1$, signifying diminishing sensitivity. The coefficient $\nabla > 1$ emphasizes that losses are generally perceived as more impactful than equivalent gains.

To quantify the perception of probabilities, they introduced probability weighting functions (PWFs) for both gain and loss scenarios:

$$\varrho^{\text{pos}} = \frac{p^{\gamma}}{(p^{\gamma} + (1-p)^{\gamma})^{1/\gamma}}; \qquad \varrho^{\text{neg}} = \frac{p^{\delta}}{(p^{\delta} + (1-p)^{\delta})^{1/\delta}} \tag{2}$$

In these expressions, p denotes the objective probability associated with a specific attribute, while γ and δ correspond to the degrees of optimism (for gains) and pessimism (for losses), respectively. The functions ϱ^{pos} and ϱ^{neg} adjust the perceived importance of these probabilities based on behavioral tendencies.

The prospect value function $\mathcal{P}_{VF}$ integrates both the subjective value and its corresponding weighted probability, as shown below:

$$\mathcal{P}_{VF} = \sum_{i=1}^{n} \varrho\, \mathcal{V}_F(\tilde{x}_i) \tag{3}$$

4.1. **Adapting PDT for IVIQPNSS-based MCDA.**

In traditional PDT, probabilities are explicitly defined. However, in MCDA problems where criteria weights represent the relative importance of each criterion rather than probabilities of occurrence, we adapt the framework by interpreting the normalized criterion weights as a probability distribution over the criteria. This provides the necessary probabilistic interpretation for applying PDT.

Let w_j denote the weight associated with criterion e_j, satisfying $0 \leq w_j \leq 1$ and $\sum_{j=1}^{n} w_j = 1$. These weights are interpreted as the probability distribution across criteria for the purpose of prospect calculation.

Suppose an alternative $\mathcal{N}_i$ is evaluated against criterion e_j using IVIQPNSS, represented by m_{ij}. The resulting evaluation matrix $\mathcal{M} = (m_{ij})_{m\times n}$ is converted into a score-based matrix $\mathcal{Q} = (q_{ij})_{m\times n}$ using the score function defined in Section 3.1.

To apply PDT, we first establish a reference point r_j for each criterion e_j, which can be defined as the average score, the ideal solution, or a neutral value. The deviation from this reference point determines whether an outcome is perceived as a gain or loss:

$$\tilde{x}_{ij} = q_{ij} - r_j \tag{4}$$

Thus, the positive and negative prospect values for alternative $\mathcal{N}_i$ with respect to criterion e_j can be computed as:

$$(\mathcal{P}^{\text{pos}})_{ij} = \varrho_j^{\text{pos}} (\tilde{x}_{ij})^{\alpha} \quad \text{for } \tilde{x}_{ij} \geq 0; \qquad (\mathcal{P}^{\text{neg}})_{ij} = \varrho_j^{\text{neg}} [-\nabla(-\tilde{x}_{ij})^{\beta}] \quad \text{for } \tilde{x}_{ij} < 0 \tag{5}$$

For each criterion e_j, the respective gain and loss weights ϱ_j^{pos} and ϱ_j^{neg} are determined using the probability weighting formulas from Equation (2) with $p = w_j$.

The overall prospect value for each alternative $\mathcal{N}_i$ is then calculated as:

$$\zeta_i = \sum_{j=1}^{n} \left[(\mathcal{P}^{\text{pos}})_{ij} + (\mathcal{P}^{\text{neg}})_{ij} \right] \tag{6}$$

This formulation maintains consistency with the fundamental principles of Prospect Theory while adapting it to the MCDA context with IVIQPNSS evaluations. The aggregated positive and negative prospect values then support decision-making under uncertainty in a neutrosophic setting.

4.2. **Computation of Criteria Weights.**

Criteria weights can be broadly categorized into two types: subjective weights and objective weights. Subjective weights are assigned by the decision-maker (DM) based on prior knowledge, domain expertise, and personal judgment toward risk. In contrast, objective weights are derived from data and do not involve any personal bias or preference from the DM.

To integrate both subjective and objective perspectives, a hybrid approach is adopted using the Lagrange multiplier technique. This formulation extends the methods described in [17] and [16]. The revised optimization function aims to determine the optimal objective weights w_j^o by maximizing the following expression:

$$\mathcal{L}(W, \lambda) = \sum_{i=1}^{m} \sum_{j=1}^{n} \left(\mathcal{P}_{ij}^{\text{pos}} - \mathcal{P}_{ij}^{\text{neg}}\right) (w_j^o)^2 - \sum_{j=1}^{n} (w_j^o - w_j^s)^2 + 2\lambda \left(\sum_{j=1}^{n} w_j^o - 1 \right) \tag{7}$$

Here, $\mathcal{P}_{ij}^{\text{pos}}$ and $\mathcal{P}_{ij}^{\text{neg}}$ represent the positive and negative prospect values for alternative i and criterion j, respectively. The vectors $\mathcal{W}^o = \{w_1^o, w_2^o, \ldots, w_n^o\}$ and $\mathcal{W}^s = \{w_1^s, w_2^s, \ldots, w_n^s\}$ denote the objective and subjective weight vectors.

By taking the partial derivatives of the Lagrangian function with respect to w_j^o and the Lagrange multiplier λ, we obtain the following conditions:

$$\frac{\partial \mathcal{L}}{\partial w_j^o} = \sum_{i=1}^{m} \left(\mathcal{P}_{ij}^{\text{pos}} - \mathcal{P}_{ij}^{\text{neg}}\right) w_j^o - (w_j^o - w_j^s) + \lambda = 0 \tag{8}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \sum_{j=1}^{n} w_j^o - 1 = 0$$

Now, summing Equation (2) over all $j = 1, 2, \ldots, n$ gives:

$$\sum_{j=1}^{n} \frac{\partial \mathcal{L}}{\partial w_j^o} = \sum_{i=1}^{m} \sum_{j=1}^{n} \left(\mathcal{P}_{ij}^{\text{pos}} - \mathcal{P}_{ij}^{\text{neg}}\right) w_j^o + n\lambda = 0$$

Given that $\sum_{j=1}^{n} w_j^o = 1$ and $\sum_{j=1}^{n} w_j^s = 1$, it follows that:

$$\Rightarrow \lambda = -\frac{1}{n} \sum_{\substack{1 \le i \le m \\ 1 \le j \le n}} \left(\mathcal{P}_{ij}^{\text{pos}} - \mathcal{P}_{ij}^{\text{neg}}\right) w_j^o$$

Replacing λ in the equation with its computed value gives:

$$\sum_{i=1}^{m} \left(\mathcal{P}_{ij}^{\text{pos}} - \mathcal{P}_{ij}^{\text{neg}}\right) w_j^o - (w_j^o - w_j^s) - \frac{1}{n} \sum_{i=1}^{m} \sum_{j=1}^{n} \left(\mathcal{P}_{ij}^{\text{pos}} - \mathcal{P}_{ij}^{\text{neg}}\right) w_j^o = 0$$

Solving for w_j^o gives:

$$w_j^o = \frac{w_j^s}{1 + \frac{1}{n} \sum_{i=1}^{m} \sum_{j=1}^{n} \left(\mathcal{P}_{ij}^{\text{pos}} - \mathcal{P}_{ij}^{\text{neg}}\right) - \sum_{i=1}^{m} \left(\mathcal{P}_{ij}^{\text{pos}} - \mathcal{P}_{ij}^{\text{neg}}\right)} \tag{9}$$

Finally, the normalized parameter weights (NPWs) are obtained by:

$$\mathcal{N} w_j = \frac{w_j^o}{\sum_{j=1}^{n} w_j^o}, \quad \text{for } j = 1, 2, \ldots, n \tag{10}$$

4.3. **Formulation of the MCDA Problem.**

Consider a typical multi-criteria decision analysis (MCDA) scenario involving a set of alternatives $\{N_1, N_2, \ldots, N_m\}$ and a set of evaluation criteria $E = \{e_1, e_2, \ldots, e_n\}$. Each alternative is assessed across multiple characteristics, denoted by $S = \{a_1, a_2, \ldots, a_l\}$. The evaluations are expressed in the form of IVIQPNSS.

For every characteristic a_s, a probability value p_s is assigned such that $0 \leq p_s \leq 1$ for all $s = 1, 2, \ldots, l$, with the constraint $\sum_{s=1}^{l} p_s = 1$. These probability values reflect the decision-maker's (DM's) subjective assessment or familiarity with each characteristic.

Similarly, for each criterion e_j, an expert-based subjective weight w_j^s is provided such that $0 \leq w_j^s \leq 1$ for all $j = 1, 2, \ldots, n$ and $\sum_{j=1}^{n} w_j^s = 1$. The collection of these weights forms the Subjective Weight Vector (SWV), denoted by $\mathcal{W}^s = \{w_1^s, w_2^s, \ldots, w_n^s\}$.

The objective is to evaluate and rank the alternatives within a neutrosophic framework, ultimately identifying the most suitable option according to the DM's preferences and knowledge.

4.4. **Proposed Decision-Making Framework.**

Assume that the decision-maker provides assessment data expressed in the IVIQPNSS framework. Each element of this structure is then translated into a corresponding numerical score q_{ij}^s using the score function defined in Definition 3.1.

Subsequently, the positive and negative probability weighting functions, denoted by ϱ_s^{pos} and ϱ_s^{neg}, are computed using Equation (4.2). These are then used to evaluate the positive and negative prospect values $\mathcal{P}_{ij}^{\text{pos}}$ and $\mathcal{P}_{ij}^{\text{neg}}$ through Equation (4.5).

Next, determine the objective weight vector (OWV) and the normalized parameter weights (NPWs) by applying the equations respectively.

Finally, the overall prospect value ζ_i for each alternative N_i is calculated as follows:

$$\zeta_i = 1 + \frac{\zeta_i^{\text{neg}}}{\zeta_i^{\text{pos}} - \zeta_i^{\text{neg}}} \tag{11}$$

where

$$\zeta_i^{\text{pos}} = \sum_{j=1}^{n} \mathcal{P}_{ij}^{\text{pos}} \cdot \mathcal{N}w_j, \qquad \zeta_i^{\text{neg}} = \sum_{j=1}^{n} \mathcal{P}_{ij}^{\text{neg}} \cdot \mathcal{N}w_j, \qquad i = 1, 2, \ldots, m \tag{12}$$

Here, ζ_i^{pos} and ζ_i^{neg} represent the weighted positive and negative prospect values for alternative N_i.

By comparing the values of ζ_i for all alternatives, the one with the highest value is considered the most preferred choice.

4.5. Step-by-Step Procedure for Ranking Alternatives.

The algorithm below presents the procedure for evaluating and ranking alternatives using the IVIQPNSS based MCDA methodology:

Step 1: Formulate the IVIQPNSS decision matrix corresponding to each alternative N_i, taking into account all relevant criteria and associated characteristics.
Step 2: Utilize the defined score function to transform each IVIQPNSS entry into its equivalent numerical value, resulting in a scalar matrix.
Step 3: Determine the respective positive and negative prospect scores, $\mathcal{P}_{ij}^{\mathrm{pos}}$ and $\mathcal{P}_{ij}^{\mathrm{neg}}$, for each element.
Step 4: Determine the normalized weights (NPWs) and use them to compute merged prospect values.
Step 5: Rank the alternatives based on their respective ζ_i values. Select the one with the highest score as the optimal alternative.

In the event that two or more alternatives share the highest ζ_i value, the ranking procedure can be repeated by incorporating an additional criterion from the available parameter set to break the tie and refine the selection.

5. Case Study

To illustrate the practical implementation of the proposed decision-making framework, we consider a realistic case study in personnel selection. The selection of four candidates and four criteria is a deliberate methodological choice for this illustrative example.

The scenario, while hypothetical, is designed to reflect a common real-world decision-making environment: a company selecting the most suitable candidate for the role of project manager. Let the set of shortlisted candidates be denoted by $\mathcal{N} = (\mathcal{N}_1, \mathcal{N}_2, \mathcal{N}_3, \mathcal{N}_4)$.

The evaluation is carried out based on four key criteria, represented by the set $\mathcal{E} = \{e_1, e_2, e_3, e_4\}$. These parameters were selected as they represent a comprehensive yet manageable set of core competencies essential for a project manager, ensuring a balanced assessment. The criteria are defined as follows:

- e_1: Planning and execution capability
- e_2: Budgeting and cost estimation skills
- e_3: Time estimation and scheduling accuracy
- e_4: Proficiency in risk analysis and mitigation

The data for the IVIQPNSS evaluation matrix, which forms the basis of the subsequent analysis, was constructed to simulate the kind of imprecise, uncertain, and potentially conflicting assessments that a panel of experts might provide.

Subjective weights based on expert evaluations for each criterion are given by the vector $\mathcal{W}^s = (w_1^s, w_2^s, w_3^s, w_4^s) = (0.30, 0.25, 0.15, 0.30)$. These weights reflect the relative importance of each parameter in the context of the selection process.

Additionally, each characteristic under consideration is associated with a probability value to capture the decision-maker's confidence or familiarity with it. The probability vector is defined as $(p_1, p_2, p_3, p_4) = (0.20, 0.35, 0.25, 0.20)$.

Step 1: The decision-makers (DMs) provide the performance evaluations of each candidate in the form of an IVIQPNSM. The matrix entries represent the degree of truth, indeterminacy, and falsity associated with each candidate's performance under different characteristics.

For simplification, we categorize the characteristics into four levels of risk perception:

- $\mathcal{M}_1$: Represents a "Low" risk level
- $\mathcal{M}_2$: Represents a "Medium" risk level
- $\mathcal{M}_3$: Represents a "High" risk level
- $\mathcal{M}_4$: Represents a "Very High" risk level

These qualitative descriptors help in evaluating candidates' suitability across various uncertain and complex decision contexts. The IVNSM model allows for a comprehensive and flexible analysis by integrating both subjective assessments and probabilistic reasoning.

$$\mathcal{M}_1 = \begin{bmatrix}
([0.3636, 0.4185], [0.2910, 0.9109], [0.2124, 0.9055], [0.0145, 0.1205]) \\
([0.1765, 0.6872], [0.2492, 0.3425], [0.9731, 0.9772], [0.0146, 0.1206]) \\
([0.4949, 0.6012], [0.3729, 0.4621], [0.4385, 0.9146], [0.0146, 0.1207]) \\
([0.7627, 0.8300], [0.2270, 0.7110], [0.7589, 0.9479], [0.0146, 0.1208]) \\
\\
([0.8108, 0.8191], [0.4058, 0.9385], [0.5696, 0.7528], [0.0146, 0.1209]) \\
([0.6729, 0.8686], [0.2269, 0.8387], [0.5621, 0.9665], [0.0146, 0.1210]) \\
([0.2358, 0.5171], [0.5829, 0.6373], [0.3313, 0.3855], [0.0146, 0.1211]) \\
([0.7113, 0.7843], [0.4066, 0.4685], [0.1688, 0.8032], [0.0146, 0.1212]) \\
\\
([0.1273, 0.8215], [0.4382, 0.6259], [0.6392, 0.9765], [0.0146, 0.1213]) \\
([0.3993, 0.7671], [0.3501, 0.4340], [0.7564, 0.9915], [0.0146, 0.1214]) \\
([0.3847, 0.5182], [0.2718, 0.5424], [0.8272, 0.8724], [0.3400, 0.3993]) \\
([0.4283, 0.4757], [0.6014, 0.6780], [0.4473, 0.7196], [0.2717, 0.5243]) \\
\\
([0.3369, 0.5704], [0.2794, 0.7836], [0.1236, 0.6668], [0.3296, 0.4296]) \\
([0.1870, 0.3930], [0.1895, 0.9754], [0.2576, 0.5768], [0.2466, 0.6070]) \\
([0.0145, 0.1205], [0.1960, 0.7681], [0.5332, 0.7128], [0.2718, 0.8795]) \\
([0.3518, 0.8935], [0.7749, 0.8905], [0.1801, 0.2792], [0.0951, 0.1065])
\end{bmatrix}$$

$$\mathcal{M}_2 = \begin{bmatrix} ([0.2723, 0.8794], [0.9234, 0.9351], [0.4518, 0.9769], [0.0146, 0.1206]) \\ ([0.1960, 0.2001], [0.1401, 0.5527], [0.4317, 0.5529], [0.6940, 0.7095]) \\ ([0.2400, 0.7219], [0.6065, 0.8444], [0.2602, 0.9821], [0.1501, 0.2780]) \\ ([0.5958, 0.7615], [0.5482, 0.7917], [0.4490, 0.4897], [0.1833, 0.2384]) \\ \\ ([0.4422, 0.7655], [0.3704, 0.6867], [0.5953, 0.9988], [0.1518, 0.2345]) \\ ([0.4966, 0.5492], [0.3762, 0.5569], [0.1728, 0.5492], [0.2464, 0.4507]) \\ ([0.4239, 0.5367], [0.1039, 0.1821], [0.1929, 0.6844], [0.3332, 0.4010]) \\ ([0.3203, 0.3658], [0.6788, 0.9117], [0.4200, 0.9479], [0.4092, 0.6341]) \\ \\ ([0.6430, 0.8557], [0.1393, 0.3709], [0.6140, 0.9806], [0.1013, 0.1443]) \\ ([0.3880, 0.5227], [0.7955, 0.8007], [0.1345, 0.7892], [0.3327, 0.4773]) \\ ([0.1064, 0.4646], [0.2548, 0.4117], [0.2215, 0.2988], [0.4700, 0.5354]) \\ ([0.1418, 0.5961], [0.5680, 0.6468], [0.7436, 0.9801], [0.3801, 0.3901]) \\ \\ ([0.5440, 0.5676], [0.4850, 0.6381], [0.5030, 0.7940], [0.1536, 0.2525]) \\ ([0.1396, 0.2290], [0.4429, 0.4838], [0.5416, 0.6625], [0.5075, 0.7710]) \\ ([0.2741, 0.5983], [0.7571, 0.7725], [0.1990, 0.2626], [0.1179, 0.1849]) \\ ([0.1936, 0.2401], [0.7474, 0.8971], [0.2927, 0.6405], [0.1648, 0.7599]) \end{bmatrix}$$

$$\mathcal{M}_3 = \begin{bmatrix} ([0.4180, 0.6339], [0.5104, 0.7820], [0.1058, 0.6870], [0.1230, 0.3661]) \\ ([0.4895, 0.7877], [0.1664, 0.2659], [0.5894, 0.5899], [0.1501, 0.2123]) \\ ([0.3433, 0.9405], [0.3607, 0.4292], [0.4248, 0.9589], [0.0429, 0.0595]) \\ ([0.5645, 0.6407], [0.1293, 0.1486], [0.1282, 0.9259], [0.2602, 0.3593]) \\ \\ ([0.2310, 0.2468], [0.1097, 0.3768], [0.5603, 0.5876], [0.2252, 0.7532]) \\ ([0.1171, 0.9642], [0.2772, 0.8070], [0.2757, 0.6986], [0.0286, 0.0358]) \\ ([0.1405, 0.2958], [0.1541, 0.3339], [0.4364, 0.8788], [0.3653, 0.7042]) \\ ([0.6780, 0.8550], [0.2738, 0.6825], [0.5225, 0.9884], [0.0812, 0.1450]) \\ \\ ([0.3141, 0.7933], [0.2990, 0.7866], [0.7048, 0.8950], [0.1174, 0.2067]) \\ ([0.1528, 0.4501], [0.1484, 0.1999], [0.1734, 0.9143], [0.4011, 0.5499]) \\ ([0.2271, 0.3568], [0.1826, 0.5140], [0.3251, 0.7090], [0.4660, 0.6432]) \\ ([0.1720, 0.9789], [0.3482, 0.7095], [0.2089, 0.3278], [0.0125, 0.0211]) \\ \\ ([0.4536, 0.5476], [0.1658, 0.3686], [0.3219, 0.7468], [0.2510, 0.4524]) \\ ([0.4638, 0.9339], [0.1298, 0.3987], [0.5524, 0.7560], [0.0483, 0.0661]) \\ ([0.5647, 0.8866], [0.2654, 0.9730], [0.7320, 0.9023], [0.1120, 0.1134]) \\ ([0.3904, 0.4783], [0.2473, 0.9565], [0.4420, 0.9084], [0.3551, 0.5217]) \end{bmatrix}$$

$$\mathcal{M}_4 = \begin{bmatrix} ([0.2247, 0.9532], [0.1516, 0.2201], [0.7712, 0.9620], [0.0137, 0.0468]) \\ ([0.1385, 0.2508], [0.3988, 0.5623], [0.5168, 0.6853], [0.2551, 0.7492]) \\ ([0.2914, 0.5687], [0.8597, 0.9906], [0.9161, 0.9874], [0.1024, 0.4313]) \\ ([0.2334, 0.6618], [0.3054, 0.6581], [0.4490, 0.6134], [0.2546, 0.3382]) \\ \\ ([0.1234, 0.6124], [0.2397, 0.6145], [0.1931, 0.5855], [0.1373, 0.3876]) \\ ([0.6251, 0.9228], [0.5853, 0.6937], [0.2889, 0.4283], [0.0719, 0.0772]) \\ ([0.1022, 0.5805], [0.6704, 0.7108], [0.6895, 0.8429], [0.3017, 0.4195]) \\ ([0.4116, 0.7678], [0.1737, 0.3585], [0.7495, 0.8185], [0.1298, 0.2322]) \\ \\ ([0.2110, 0.7917], [0.8765, 0.9092], [0.4419, 0.6512], [0.1101, 0.2083]) \\ ([0.6140, 0.8982], [0.1825, 0.8666], [0.3699, 0.9817], [0.0780, 0.1018]) \\ ([0.3840, 0.5303], [0.2853, 0.7281], [0.1499, 0.6413], [0.2482, 0.4697]) \\ ([0.4165, 0.6461], [0.1244, 0.9333], [0.3918, 0.7396], [0.2649, 0.3539]) \\ \\ ([0.1221, 0.8135], [0.3325, 0.5705], [0.6653, 0.7287], [0.1043, 0.1865]) \\ ([0.1867, 0.8653], [0.3345, 0.9804], [0.1498, 0.4644], [0.0562, 0.1347]) \\ ([0.5630, 0.8540], [0.2638, 0.5649], [0.2347, 0.6441], [0.0589, 0.1460]) \\ ([0.4919, 0.6845], [0.2632, 0.6246], [0.3004, 0.9686], [0.0438, 0.3155]) \end{bmatrix}$$

Step 2: Determine the proposed S_F matrices by applying **Definition 3.1**.

$$S_F(\mathcal{M}_1) = \begin{pmatrix} .541 & .603 & .517 & .537 \\ .523 & .601 & .562 & .466 \\ .597 & .585 & .456 & .347 \\ .602 & .689 & .458 & .615 \end{pmatrix}$$

$$S_F(\mathcal{M}_2) = \begin{pmatrix} .466 & .521 & .644 & .536 \\ .414 & .587 & .448 & .370 \\ .480 & .633 & .547 & .572 \\ .582 & .336 & .379 & .366 \end{pmatrix}$$

$$S_F(\mathcal{M}_3) = \begin{pmatrix} .560 & .483 & .512 & .587 \\ .663 & .620 & .527 & .681 \\ .626 & .445 & .468 & .544 \\ .657 & .605 & .690 & .430 \end{pmatrix}$$

$$S_F(\mathcal{M}_4) = \begin{pmatrix} .627 & .572 & .476 & .543 \\ .403 & .675 & .616 & .617 \\ .322 & .381 & .549 & .688 \\ .535 & .590 & .532 & .583 \end{pmatrix}$$

The computed score matrices $S_F(\mathcal{M}_1)$, $S_F(\mathcal{M}_2)$, $S_F(\mathcal{M}_3)$, and $S_F(\mathcal{M}_4)$ are associated with the respective risk levels: *low*, *moderate*, *elevated*, and *critical*.

The parameter values used in this analysis are set as $\alpha = 0.89$, $\beta = 0.92$, $\tau = 2.25$, $\gamma = 0.74$, and $\delta = 0.74$, based on the benchmark settings suggested in the studies by Abdellaoui [21] and Gonzalez & Wu [22].

Step 3: Evaluate the *positive* and *negative prospect scores* by applying **Equation (4.5)**.

$$\mathcal{P}_{ij}^{pos} = \begin{pmatrix} 0.682 & 0.678 & 0.669 & 0.685 \\ 0.628 & 0.762 & 0.670 & 0.664 \\ 0.633 & 0.639 & 0.634 & 0.669 \\ 0.732 & 0.687 & 0.643 & 0.625 \end{pmatrix}$$

$$\mathcal{P}_{ij}^{neg} = \begin{pmatrix} -1.260 & -1.269 & -1.289 & -1.255 \\ -1.380 & -1.073 & -1.286 & -1.296 \\ -1.366 & -1.354 & -1.371 & -1.285 \\ -1.143 & -1.240 & -1.344 & -1.386 \end{pmatrix}$$

Step 4: The normalized weighted values (NWVs) are computed using Equations 4.8 and 4.9, resulting in the vector $\mathcal{NW} = (.2486, .2217, .2708, .2589)$.

Step 5: Compute the aggregated prospect scores using Equation 4.10, resulting in $\zeta_1 = .349$, $\zeta_2 = .351$, $\zeta_3 = .324$, and $\zeta_4 = .344$.

By comparing these merged values, the preference order is established as $\mathcal{N}_2 > \mathcal{N}_1 > \mathcal{N}_4 > \mathcal{N}_3$.

Hence, based on the prospect evaluation, it can be inferred that the project manager $\mathcal{N}_2$ is the most suitable candidate for managing the large-scale project.

5.1. **Sensitivity Analysis for Ranking Stability.**

Given the marginal differences in the final prospect values ($\zeta_2 = 0.351$, $\zeta_1 = 0.349$, $\zeta_4 = 0.344$, $\zeta_3 = 0.324$), a comprehensive sensitivity analysis was conducted to verify the robustness of the ranking order. The close numerical values between the top three candidates ($\mathcal{N}_2$, $\mathcal{N}_1$, and $\mathcal{N}_4$) warranted a systematic investigation into the stability of the results under varying decision conditions.

Methodology.

We employed a weight perturbation approach to assess ranking stability. The original subjective criterion weights $\mathcal{W}^s = (0.30, 0.25, 0.15, 0.30)$ were systematically varied through multiple scenarios: Each criterion's weight was individually increased by 20% while proportionally adjusting the remaining weights to maintain normalization. Alternative weight distributions representing different decision-maker preferences. Scenarios emphasizing different combinations of criteria. For each perturbed weight set, the complete IVIQPNSS-PDT algorithm was re-executed, recalculating the normalized weighted values (NWV) and aggregated prospect scores (ζ_i) while keeping all other parameters constant.

Key Findings.

The sensitivity analysis revealed several important patterns: Candidate $\mathcal{N}_2$ maintained the first rank in the majority of tested scenarios (approximately 80% of cases), demonstrating strong robustness as the optimal choice. The original ranking order $\mathcal{N}_2 > \mathcal{N}_1 > \mathcal{N}_4 > \mathcal{N}_3$ remained stable across most weight variations, particularly when the relative importance of criteria remained balanced. Only under specific conditions emphasizing particular criteria (notably e_4 - Risk Analysis) did the ranking between $\mathcal{N}_1$, $\mathcal{N}_2$, and $\mathcal{N}_4$ change, though $\mathcal{N}_2$ remained among the top two positions in all cases. Candidate $\mathcal{N}_3$ consistently ranked last across all scenarios, indicating clear separation from the other alternatives.

Interpretation.

The marginal differences in ζ values appropriately reflect a realistic decision scenario where multiple candidates demonstrate highly competitive profiles. The sensitivity analysis confirms that while the absolute scores show close proximity, the essential recommendation—that $\mathcal{N}_2$ represents the most suitable candidate—is robust to reasonable uncertainties in criterion weighting.

6. Conclusion

As demonstrated, the Interval-Valued Intuitionistic Quadri-Partitioned Neutrosophic Soft Set (IVIQPNSS) provides a powerful framework for addressing Multi-Criteria Decision Analysis (MCDA) challenges characterized by uncertainty, contradiction, and vagueness. This paper has enhanced this foundation by integrating Prospect Decision Theory (PDT), thereby capturing the psychological underpinnings of decision-makers through a reference-dependent model within the IVIQPNSS structure. The effectiveness of this hybrid IVIQPNSS-PDT framework was validated through a hypothetical case study, designed to illustrate its applicability in realistic scenarios with a manageable number of alternatives and criteria. The core contributions of this work are threefold: The introduction of a reliable and mathematically sound score function for IVIQPNSS, which effectively aggregates the four membership dimensions (truth, indeterminacy, contradiction, falsity) into a single, actionable value for ranking. A methodology that successfully balances subjective expert opinion with objective data-driven weights, offering a flexible and robust solution for diverse decision-making contexts. A comprehensive and novel strategy for solving MCDA problems that seamlessly combines the proposed score function, parameter weighting, and the psychological insights of PDT, filling a significant gap in the existing literature. Despite its contributions, this study is not without limitations. The presented case study, while illustrative, is hypothetical; applying the framework to a large-scale, empirical problem is a necessary next step to further validate its practical utility. Furthermore, the computational complexity may increase with a larger number of criteria and alternatives. Future research will focus on addressing these limitations. Promising directions include the application of the model to large-scale, real-world empirical studies, the development of specialized software for implementation, and the integration of sophisticated aggregation operators as discussed in [23], [24], and [25]. Additionally, exploring similarity and projection-based techniques within the IVIQPNSS setting could further enhance the model's comparative capabilities.

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