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Coprime Degree of a Finite Group

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Abstract. The coprime probability of a finite group has been defined, and the coprime probability of a p-group for any prime number p has also been obtained. In this paper, we refine the concept by introducing the coprime degree of a finite group. We calculate the coprime degrees of cyclic groups, nilpotent groups, dihedral groups, and general quaternion groups.

Key words and Phrases: coprime degree, cyclic groups, nilpotent groups, dihedral groups, general quaternion groups.

1. INTRODUCTION

The concept of group probability was initially proposed by Miller [1]. The commutativity degree of a finite group G, denoted as $P(G)$, is defined as the probability that any two randomly selected elements in G commute. Following Miller's foundational work, numerous researchers have explored this topic, leading to significant advancements. For example, Erdos and Turan [2] investigated the commutativity degrees of symmetric groups, while Gustafson [3] adopted an alternative approach by demonstrating that the commutativity degree of G equals the number of conjugacy classes divided by the order of G. Through further analysis, [3] and [4] established that the maximum commutativity degree for a finite non-abelian group G is $P(G) \leq \frac{5}{8}$. In addition, various upper and lower bounds for different probabilistic measures have been derived. For instance, Pournaki and Sobhan [5] determined a lower bound for the commutativity degree, and R. Kanti & Kumar [6] examined the probability that the commutator of two group elements equals a specific element, known as the g-commutativity degree. Recent work by Moradipour et al. [7] has precisely determined the commutativity degrees for several important

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group families, including generalized quaternion groups, dihedral groups, semidihedral groups, and quasi-dihedral groups. Subsequent studies have further explored the commutativity degrees of groups and their extensions, as documented in [8, 9]. These investigations remain fundamentally connected to the core concept of group commutativity degree.

The coprime degree concept was developed as an extension of the group commutativity degree. In [10], Rhani et al. introduce the notion of coprime probability of a finite group. Given two elements of the group such a notion measures the chance of their orders to be coprime. The researchers initially determined probability measures for the p-groups and certain dihedral groups. Subsequently, Zulkifli and Ali [11] conducted comprehensive investigations of coprime degree in their 2019 work, focusing specifically on non-abelian metabelian groups.

In this paper, we refine the definition of coprime probability by considering unordered pairs of the group instead. The proposed formulation of coprime probability is considered more natural and reduces repetitions since, in fact, the orders of two group elements being coprime are free of their ordering. Moreover, this definition is consistent with the coprime graph of a group, which is not a directed graph and therefore does not distinguish between edges (x, y) and edges (y, x). With this new definition, we then determine the number for cyclic groups, nilpotent groups, dihedral groups, and generalized quaternion groups.

2. MAIN RESULTS

First, we provide some definitions and elementary properties in group theory. Let G be a group. The cardinality of G, denoted by $|G|$, is called the *order* of G. For $a \in G$, the smallest positive integer n satisfying $a^n = e$ is called the *order* of a, denoted by $o(a)$. It is well known that when G is finite, $o(a)$ divides $|G|$, for all $a \in G$. For a prime number p, a finite group G is said to be a *p-group* if $|G|$ is a positive power of p. Hence, in a p-group every element has order a non-negative power of p.

Let S be a subset of G. We say that S *generates* G, denoted with $G = \langle S \rangle$, if every element in G can be written as a product of a finite number of elements in S or their inverse. In this case, the set S is called a *generator* of G. Furthermore, when $|S| = 1$, we say that G is a *cyclic* group. Other properties of a group can be found in, for example, [12] or [13].

Next, we have two classes of finite groups, namely dihedral groups and generalized quaternion groups [14]. For an integer $n \geq 3$, the *dihedral group* D_{2n} is a group of order $2n$ generated by two elements x, y that satisfy the relations $x^n = y^2 = (xy)^2 = e$. The last relation may also be expressed as $yxy^{-1} = x^{-1}$.

The dihedral group D_{2n} is the symmetry group on regular n-gon. For an integer $n \geq 2$, the *generalized quaternion group* Q_{4n} is a group of order $4n$ generated by two elements x, y that satisfy the relations $x^n = y^2, y^{-1}xy = x^{-1}$.

We recall some structural notions that will be used throughout the paper. Two groups G and H are said to be *isomorphic*, written $G \cong H$, if there exists

a bijective homomorphism between them. In this case, the groups share identical algebraic structure and group-theoretic properties, such as element orders being preserved.

If $G_1, G_2, \ldots, G_m$ are groups, their *direct product* $G_1 \times \cdots \times G_m$ is the group whose elements are tuples $(g_1, \ldots, g_m)$, $g_i \in G_i$, for all $i = 1, 2, \ldots, m$ with componentwise operation. The order of an element in a direct product satisfies

$$o((g_1, \ldots, g_m)) = \operatorname{lcm}(o(g_1), \ldots, o(g_m)).$$

In particular, by the fundamental theorem of finite abelian groups, a finite cyclic group and, more generally, a finite nilpotent group can be decomposed as a direct product of its Sylow p-subgroups (see, e.g., Dummit and Foote [12] or Isaacs [13]). These structural facts play a key role in our computation of coprime degrees.

Two positive integers m, n are *coprime* if their greatest common divisor, denoted by $\gcd(m, n)$, is 1. A pair of coprime numbers does not have a common prime factor. Two elements in G are *coprime* if their orders are coprime.

Given a finite group G, the set of all ordered pairs of coprime elements in G is denoted as

$$\operatorname{copp}(G) = \{(x, y) \in G \times G \mid \gcd(o(x), o(y)) = 1\}.$$

Rhani et al. [10] defines the *coprime probability* of a group G as

$$P_{copr}(G) = \frac{|\operatorname{copp}(G)|}{|G|^2}.$$

They also established that if G is a p-group of order p^n, $n \geq 1$, then

$$|\operatorname{copp}(G)| = 2p^n - 1, \tag{1}$$

hence

$$P_{copr}(G) = \frac{2p^n - 1}{p^{2n}}. \tag{2}$$

We first comment on the definition of coprime probability. Notice that $(x, x) \in \operatorname{copp}(G)$ if and only if $x = e$. Therefore, all pairs of the same elements do not factor in determining the chance of obtaining a coprime pair of elements of G. We think it is more natural to consider instead the coprimeness of unordered pairs of elements, that is to take into account 2-subsets of G. We propose the following definition.

Definition 2.1. *Let G be a finite group. The* coprime degree *of G is the proportion of unordered pairs of elements of G whose orders are coprime, defined by*

$$\deg_{cop}(G) = \frac{|\{\{x, y\} \subseteq G \mid x \neq y, \gcd(o(x), o(y)) = 1\}|}{|\{\{x, y\} \subseteq G \mid x \neq y\}|}.$$

We can relate the coprime degree to the coprime probability. Let x, y be two distinct elements of G. Whenever x and y are coprime, they appear twice, as (x, y) and (y, x), in $\operatorname{copp}(G)$, but they appear once as an unordered pair of coprime elements in G. Also, $\operatorname{copp}(G)$ contains exactly one pair of the same elements, namely (e, e). Therefore, the number of unordered pairs of coprime elements in G equals

$\frac{|\operatorname{copp}(G)|-1}{2}$. The number of unordered pairs of elements in G is $\frac{|G|(|G|-1)}{2}$. This establishes

$$\deg_{\text{cop}}(G)=\frac{|\operatorname{copp}(G)|-1}{|G|(|G|-1)}=\frac{|G|}{|G|-1}P_{copr}(G)-\frac{1}{|G|(|G|-1)}.$$

In order to calculate $\deg_{cop}(G)$ we will utilize $\operatorname{copp}(G)$.

We will need the following lemma for the rest of our discussion.

Lemma 2.2. *Let G be a finite group, H be any p-group of order p^k, and $K=G\times H$. If $\gcd(|G|,p)=1$, then $|\operatorname{copp}(K)|=q\left(2p^k-1\right)$, where $q=|\operatorname{copp}(G)|$.*

Proof. Consider the partition of $\operatorname{copp}(K)$ into

$$U=\{(u,v)\in\operatorname{copp}(K)\mid u=(x,e_H),\ \text{for some } x\in G\}$$

and

$$V=\{(u,v)\in\operatorname{copp}(K)\mid u=(x,y),\ \text{for some } x\in G, e_H\neq y\in H\}.$$

Let $u=(x,e_H)$ and $v=(x',y)$, for some $x,x'\in G$, $y\in H$. Since $o(u)=o(x)$ and $o(v)=o((x',y))=\operatorname{lcm}(o(x'),o(y))=o(x')\cdot o(y)$, it follows that $(u,v)\in\operatorname{copp}(K)$ if and only if $\gcd(o(x),o(x'))=1$ if and only if $(x,x')\in\operatorname{copp}(G)$. Hence, $U=\cup_{y\in H}\{((x,e_H),(x',y))\mid(x,x')\in\operatorname{copp}(G)\}$. Therefore, $|U|=|H|\,|\operatorname{copp}(G)|=p^k\cdot q$.

Next, let $u=(x,y)$ and $v=(x',y')$, for some $x,x'\in G$, $y,y'\in H$, $y\neq e_H$. Then $o(y)=p^i$, for some $i>0$, so $p|o(u)$. It follows that $(u,v)\in\operatorname{copp}(K)$ if and only if $\gcd(o(x),o(x'))=1$ and $p\not|o(y')$ if and only if $\gcd(o(x),o(x'))=1$ and $y'=e_H$. Hence, $V=\cup_{y\in H,y\neq e_H}\{((x,y),(x',e_H))\mid(x,x')\in\operatorname{copp}(G)\}$. Therefore $|V|=|H-1|\,|\operatorname{copp}(G)|=\left(p^k-1\right)\cdot q$.

We conclude that $|\operatorname{copp}(K)|=\left(2p^k-1\right)q$. □

We are now ready to calculate the coprime degrees of some special groups. It is clear that we only need to determine the number of coprime pairs of each group.

2.1. **Cyclic and nilpotent groups.**

Let n be a positive integer larger than 1. It is well-known that a cyclic group of order n is isomorphic to the additive group $\mathbb{Z}_n$. Moreover, if $n=p_1^{e_1}p_2^{e_2}\cdots p_m^{e_m}$ is the prime factorization of n, then $\mathbb{Z}_n\cong\mathbb{Z}_{p_1^{e_1}}\times\mathbb{Z}_{p_2^{e_2}}\times\cdots\times\mathbb{Z}_{p_m^{e_m}}$.

Theorem 2.3. *Let G be a cyclic group of order n and $n=p_1^{e_1}p_2^{e_2}\cdots p_m^{e_m}$ be the prime factorization of n. Then,*

$$|\operatorname{copp}(G)|=(2p_1^{e_1}-1)(2p_2^{e_2}-1)\cdots(2p_m^{e_m}-1),\tag{3}$$

and therefore

$$\deg_{cop}(G)=\frac{\prod_{i=1}^{m}(2p_i^{e_i}-1)-1}{n(n-1)}.\tag{4}$$

Proof. It suffices to prove the case of $G = Z_{p_1^{e_1}} \times \mathbb{Z}_{p_2^{e_2}} \times \cdots \times \mathbb{Z}_{p_m^{e_m}}$. We use induction on the number m of prime factors of n.

If $m = 1$, then G is a p_1-group, and the result follows from [10].

Now, assume $m > 1$ and the result is true for any cyclic group with $m - 1$ prime factors. Write $G = H \times \mathbb{Z}_{p_m^{e_m}}$, where $H = Z_{p_1^{e_1}} \times \mathbb{Z}_{p_2^{e_2}} \times \cdots \times \mathbb{Z}_{p_{m-1}^{e_{m-1}}}$.

Since $|H|$ and p_m are coprime, it follows from Lemma 2.2 that $|\operatorname{copp}(G)| = (2p_m^{e_m} - 1)\,|\operatorname{copp}(H)|$. By the induction hypothesis, $|\operatorname{copp}(H)| = (2p_1^{e_1} - 1)(2p_2^{e_2} - 1)\cdots(2p_{m-1}^{e_{m-1}} - 1)$ and (5) and (6) follow immediately. □

Nilpotent groups are generalizations of cyclic groups. By Theorem 8.15 in [13], a finite nilpotent group is characterized by having unique Sylow p-subgroups for each prime factor p of its order, and the group is a direct product of those Sylow subgroups. Since every Sylow subgroup is a p-group, it follows that a nilpotent group is a direct product of p-groups.

Using the same proof as in Theorem 2.3, we obtain the following. In particular, let G be a finite nilpotent group of order $n = p_1^{e_1} p_2^{e_2} \cdots p_m^{e_m}$, where this is the prime factorization of n. According to [13], the group G can be expressed as a direct product $G = S_{p_1} \times S_{p_2} \times \cdots \times S_{p_m}$, where S_{p_i} denotes the Sylow p_i-subgroup of G of order $p_i^{e_i}$ for $i = 1, 2, \cdots, m$. Theorem 2.4 can then be obtained by mathematical induction on m, the number of different prime factor of n, following steps similar to those used in the proof of Theorem 2.3.

Theorem 2.4. *Let G be a nilpotent group of order n and $n = p_1^{e_1} p_2^{e_2} \cdots p_m^{e_m}$ be the prime factorization of n. Then*

$$|\operatorname{copp}(G)| = (2p_1^{e_1} - 1)(2p_2^{e_2} - 1)\cdots(2p_m^{e_m} - 1), \tag{5}$$

and therefore

$$\deg_{cop}(G) = \frac{\prod_{i=1}^{m}(2p_i^{e_i} - 1) - 1}{n(n-1)}. \tag{6}$$

Before we proceed with other classes of groups, let us observe the order of elements in a cyclic group.

Let G be a cyclic group of order n generated by an element a. Let $a^k \in G$, for some integer k, $1 \le k \le n - 1$. Then

$$o(a^k) = \frac{\operatorname{lcm}(n,k)}{k} = \frac{n}{\gcd(n,k)}.$$

Let s be a divisor of n such that s^l is the highest power of s that divides n. For s not to divide $o(a^k)$, a necessary and sufficient condition is that s^l divides $\gcd(n,k)$, hence s^l divides k. The number of k for which $o(a^k)$ is coprime with s is therefore $\dfrac{n}{s^l}$.

2.2. **Dihedral groups.**

Let n be an integer, $n \ge 3$ and let D_{2n} be the dihedral group of order $2n$. Let D_{2n} be generated by a and b that satisfy $a^n = b^2 = (ab)^2 = e$, see [14]. Letting

$H = \langle a \rangle$, the group D_{2n} are partitioned into two cosets H and Hb. Notice that all elements in Hb are of order 2.

To determine $|\operatorname{copp}(D_{2n})|$, let $n = p_1^{e_1} p_2^{e_2} \cdots p_m^{e_m}$ be the prime factorization of n. Next, we partition $\operatorname{copp}(D_{2n})$ into four subsets:

(A) $U_1 = \{(u, v) \in \operatorname{copp}(D_{2n}) \mid u, v \in H\}$,
(B) $U_2 = \{(u, v) \in \operatorname{copp}(D_{2n}) \mid u \in H, v \in Hb\}$,
(C) $U_3 = \{(u, v) \in \operatorname{copp}(D_{2n}) \mid u \in Hb, v \in H\}$, and
(D) $U_4 = \{(u, v) \in \operatorname{copp}(D_{2n}) \mid u, v \in Hb\}$.

First notice that $U_1 = \operatorname{copp}(H)$ and since H is a cyclic group of order n, we have $|U_1| = \prod_{i=1}^{m} (2p_i^{e_i} - 1)$. Next, since $o(x) = 2$, for all $x \in Hb$, we have $U_4 = \emptyset$. Notice also that $|U_2| = |U_3|$.

To determine $|U_2|$ we need to consider two cases: 2 divides n, and 2 does not divide n.

Assume now that 2 divides n and, without loss of generality, let $p_1 = 2$. Then $(u, v) \in U_2$ if and only if $\gcd(o(u), 2) = 1$. The number of elements in H whose order is coprime with 2 is exactly $\frac{n}{2^{e_1}}$. Since $|Hb| = n$, it follows that $|U_2| = \frac{n^2}{2^{e_1}}$.

Assume now that 2 does not divide n, i.e., all prime factors of n are odd. Then $(u, v) \in U_2$, for all $u \in H$ and $v \in Hb$, therefore $|U_2| = n^2$.

Adding the number of elements in those four subsets, we obtain $|\operatorname{copp}(D_{2n})|$ that leads to the following theorem.

Theorem 2.5.

$$|\operatorname{copp}(D_{2n})| = \begin{cases} \displaystyle\prod_{i=1}^{m} (2p_i^{e_i} - 1) + \frac{n^2}{2^{e_1 - 1}} & \text{if 2 divides } n, \\ \displaystyle\prod_{i=1}^{m} (2p_i^{e_i} - 1) + 2n^2 & \text{if 2 does not divide } n, \end{cases} \tag{7}$$

and therefore

$$\deg_{cop}(D_{2n}) = \begin{cases} \dfrac{\displaystyle\prod_{i=1}^{m} (2p_i^{e_i} - 1) + \frac{n^2}{2^{e_1 - 1}} - 1}{2n(2n-1)} & \text{if 2 divides } n, \\ \dfrac{\displaystyle\prod_{i=1}^{m} (2p_i^{e_i} - 1) + 2n^2 - 1}{2n(2n-1)} & \text{if 2 does not divide } n. \end{cases} \tag{8}$$

Example. Consider the dihedral group D_{72} of order 72, where $n = 2^2 \cdot 3^2$. The rotation subgroup $\langle a \rangle$ is cyclic of order 36, while all reflections have order 2. Using Theorem 2.5 with prime factorization $n = p_1^{e_1} p_2^{e_2}$, where $p_1 = 2$, $e_1 = 2$, and

$p_2 = 3$, $e_2 = 2$, we obtain

$$\prod_{i=1}^{2}(2p_i^{e_i} - 1) = (2 \cdot 2^2 - 1)(2 \cdot 3^2 - 1) = 7 \cdot 17 = 119.$$

Since 2 divides n, we have

$$\frac{n^2}{2^{e_1-1}} = \frac{36^2}{2} = 648.$$

Therefore,

$$|\mathrm{copp}(D_{72})| = 119 + 648 = 767.$$

Hence, the coprime degree of D_{72} is

$$\deg_{\mathrm{cop}}(D_{72}) = \frac{767 - 1}{72 \cdot 71} = \frac{766}{5112} = \frac{383}{2556}.$$

2.3. **General quaternion groups.**

Let n be an integer, $n \geq 2$ and let Q_{4n} be the generalized quaternion group of order $4n$. Let Q_{4n} be generated by a and b that satisfy $a^n = b^2, b^{-1}ab = a^{-1}$, see [14]. From these relations, we will deduce some order properties of elements in Q_{4n}.

First, notice that b^2 and a^n cannot be the identity, else $|Q_{4n}| = 2n$. For nonnegative integer i we have $b^{-1}a^ib = a^{-i}$. In particular, taking $i = n$ leads to $b^{-1}b^2b = b^{-1}a^nb = a^{-n} = b^{-2}$, hence $b^4 = e$. Since neither $b = e$ nor $b^2 = e$, we have $o(b) = 4$. We then have $a^{2n} = e$. Since $|Q_{4n}| = 4n$ and $o(b) = 4$, the order of a is at least n, but since $a^n \neq e$, it must be $o(a) = 2n$.

As in the case of dihedral groups, we partition Q_{4n} into two cosets $H = \langle a \rangle$ and Hb. Next, we partition $\mathrm{copp}(Q_{4n})$ into four subsets with respect to membership of elements in the pair $(u, v) \in \mathrm{copp}(Q_{4n})$ in H or Hb. Let $2n = \prod_{i=1}^{m} p_i^{e_i}$ be the prime factorization of $2n$ with $p_1 = 2$. Proceeding as in the case of dihedral groups, i.e. by partioning the set $\mathrm{copp}(Q_{4n})$ into 4 subsets, we obtain the following theorem.

Theorem 2.6.

$$|\,\mathrm{copp}(Q_{4n})| = \prod_{i=1}^{m}(2p_i^{e_i} - 1) + \frac{4n^2}{2^{e_1-1}}, \tag{9}$$

and therefore

$$\deg_{cop}(Q_{4n}) = \frac{\prod_{i=1}^{m}(2p_i^{e_i} - 1) + \frac{4n^2}{2^{e_1-1}} - 1}{4n(4n-1)}. \tag{10}$$

3. CONCLUDING REMARKS

In this paper, we introduced the notion of the coprime degree of a finite group as a refinement of the coprime probability previously studied in the literature. Unlike coprime probability, which is defined using ordered pairs and includes trivial repetitions such as (e, e), the coprime degree measures the proportion of unordered pairs of distinct elements whose orders are coprime. This formulation is more natural from both combinatorial and graph-theoretic perspectives, particularly in connection with coprime graphs of groups.

Our results show that, for large classes of groups such as cyclic and nilpotent groups, the coprime degree of a group depends only on the prime factorization of the group order. In contrast, for non-abelian groups such as dihedral and generalized quaternion groups, additional structural features influence the coprime degree.

Possible directions for future research include the study of coprime degree for other families of groups, such as symmetric groups or finite solvable groups, as well as investigating extremal problems and asymptotic behavior of coprime degree. Another interesting problem is to explore deeper connections between coprime degree and other graph invariants associated with finite groups.

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