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On Graceful Labeling of Some Power Graphs

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Abstract. Power graphs of groups and semigroups constitute a significant class of graphs in algebraic graph theory, bridging algebraic structures and graph theoretic concepts. Further, labeling of graphs, particularly those endowed with an algebraic structure as their vertex set, is a vibrant area of research. This article contributes to this burgeoning field by exploring new classes of finite groups whose power graphs admit or forbid an important type of labeling called graceful labeling.

Key words and Phrases: Power graph, graceful labeling, finite group.

1. INTRODUCTION

Rosa [1] defined a β-valuation of a graph G with n edges as an injective mapping $\Phi : V(G) \to \{0, 1, 2, \ldots, n\}$ such that, when the label $|\Phi(x) - \Phi(y)|$ is assigned to the edge $e = xy$, the resulting edge labels are distinct. The term graceful labeling for this valuation was later coined by Golomb [2], and this terminology has since become the standard in the literature.

The problem of classifying graphs as graceful or otherwise has sparked extensive research endeavors by numerous researchers, yielding a substantial body of work in this fascinating area. A detailed overview of various labeling techniques, including the graceful labeling, has been presented in [3].

The concept of directed power graphs was introduced in [4] for groups and was subsequently extended to semigroups in [5, 6]. "The directed power graph $\vec{P}(G)$ of a group G is the graph with vertex set $V(\vec{P}(G)) = G$ and edge set $E(\vec{P}(G)) = \{(u, v) : u \neq v \text{ and } v \text{ is a power of } u\}$. The underlying undirected power graph $P(G)$ is the graph with vertex set G in which two distinct vertices are adjacent if and only if one is a power of the other."

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In [7, 8, 9], the authors carried out a detailed study of undirected power graphs, revealing many of their structural and combinatorial properties. Subsequent works have investigated several graph-theoretic parameters associated with power graphs and their variants, such as diameter, domination number, coding properties, and metric dimensions; see, for example, [10, 11, 12, 13].

These developments have naturally led to the study of graceful labelings of power graphs of finite groups. Abawajy et al. [14] provided an extensive survey of results on power graphs available in the literature. They also discussed various conjectures, open questions, and research problems proposed by other authors, including the problem of identifying groups and semigroups whose undirected power graphs admit graceful labelings. In 2022, the authors in [15] established a graceful labeling of the power graph of the group $\mathbb{Z}_2^{k-1} \times \mathbb{Z}_4$.

In this paper, we investigate power graphs of certain finite groups and classify them as graceful or non-graceful. Our results further illustrate the connection between algebraic structures and graph labeling theory. Throughout the paper, all power graphs considered are undirected.

For completeness, we recall some well-known results that will be used later.

Theorem 1.1 ([1]). *"If G is an Eulerian graph with n edges, where $n \equiv 1 \pmod 4$ or $n \equiv 2 \pmod 4$, then there does not exist a β-valuation of the graph G."*

Theorem 1.2 ([2]). *"If $n > 4$, then the complete graph K_n can not be graceful."*

Theorem 1.3 ([7]). *"Let G be a finite group. Then $P(G)$ is complete if and only if G is a cyclic group of order 1 or p^m, for some prime number p and some $m \in \mathbb{N}$."*

2. MAIN RESULTS

The key results of our study are presented below. We begin by establishing the gracefulness of the finite abelian group $\mathbb{Z}_k^n$. Subsequent results address other classes of finite groups and their associated power graphs.

Theorem 2.1. *The power graph of $\mathbb{Z}_k^n$ is graceful for $k = 2, 3$ and for all $n \in \mathbb{N}$.*

Proof. For $k = 2$, the power graph of $\mathbb{Z}_k^n$ consists of $2^n - 1$ vertices of order two and a single vertex of order one, namely the identity element e. Further, e is adjacent to all other vertices, and no edges exist among the remaining vertices. We assign labels to the vertices as mentioned in the following table:

Thus, the induced edge labels are $1, 2, 3, \ldots, 2^n - 1$; consequently, $P(\mathbb{Z}_k^n)$ is graceful in this case.

For $k = 3$, every non-identity vertex of $P(\mathbb{Z}_k^n)$ has order 3. Moreover, the structure of the power graph of $\mathbb{Z}_k^n$ coincides with the windmill graph $W_d\left(\frac{3^n - 1}{2}, 3\right)$.

Table 1. A vertex labeling of the power graph of $\mathbb{Z}_2^n$

Element(s)	Order	Assigned vertex label(s)
e	1	0
$\prod_{i=1}^{n} {x_i}^{\alpha_i}$, where $\alpha_i \in \{0,1\}$ for all $1 \le i \le n$, and at least one α_i is non-zero	2	$1, 2, 3, \dots, 2^n - 1$ (in arbitrary order)

As an illustration, consider the power graph of the group $\mathbb{Z}_3^2 =< x > \times < y >$ shown in Figure 1. It is isomorphic to the windmill graph $W_d(4,3)$.

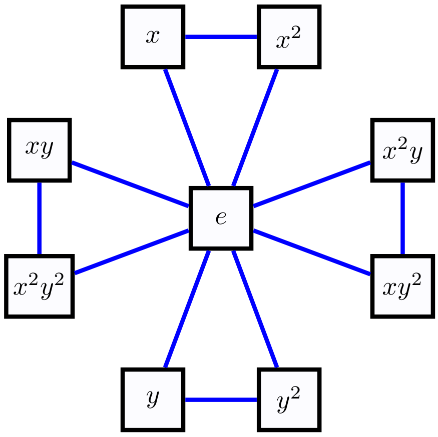

Figure 1. The power graph of $\mathbb{Z}_3^2 \cong W_d(4,3)$

It can be readily verified that, for $n \in \mathbb{N}$, $\dfrac{3^n - 1}{2} \equiv 0$ or $1 \pmod 4$.
Hence, the desired result follows from the fact that the windmill graph $W_d(m,3)$ is graceful whenever $m \equiv 0$ or $1 \pmod 4$ (see [16]).

$\square$

Theorem 2.2. *Let G be a finite non-trivial group of order $n \le 7$. Then the power graph of G is graceful unless $n = 5$ or $n = 7$.*

Proof. We proceed by considering the following cases.

Case I: $n \in \{2,3\}$.

In this case, $G \cong \mathbb{Z}_n$, and by Theorem 1.3 we have $P(G) \cong K_n$. It can be readily checked that $P(G)$ is graceful.

Case II: $n = 4$.

Up to isomorphism, there are two groups of order four: the cyclic group $\mathbb{Z}_4$ and the Klein four-group $\mathbb{Z}_2 \times \mathbb{Z}_2$. Hence, either $G \cong \mathbb{Z}_4$ or $G \cong \mathbb{Z}_2 \times \mathbb{Z}_2$.
If $G \cong \mathbb{Z}_4$, then $P(G)$ is graceful, as shown in [15].
If $G \cong \mathbb{Z}_2 \times \mathbb{Z}_2$, then by Theorem 2.1, the power graph of G is graceful.

Case III: $n = 6$.

Then G must be isomorphic to either $\mathbb{Z}_6$ or D_3 (the dihedral group of order 6), since these are the only two groups of order 6 up to isomorphism.

Subcase A: $G \cong \mathbb{Z}_6$.

Let $G = \{e, a, a^2, a^3, a^4, a^5\}$. Define a function $\Phi : V(P(G)) \to \{0, 1, 2, \ldots, 13\}$ by

$$\Phi(e) = 0,\ \Phi(a) = 1,\ \Phi(a^2) = 6,\ \Phi(a^3) = 11,\ \Phi(a^4) = 13,\ \Phi(a^5) = 9.$$

Then Φ is a graceful labeling of the power graph of G (see Figure 2).

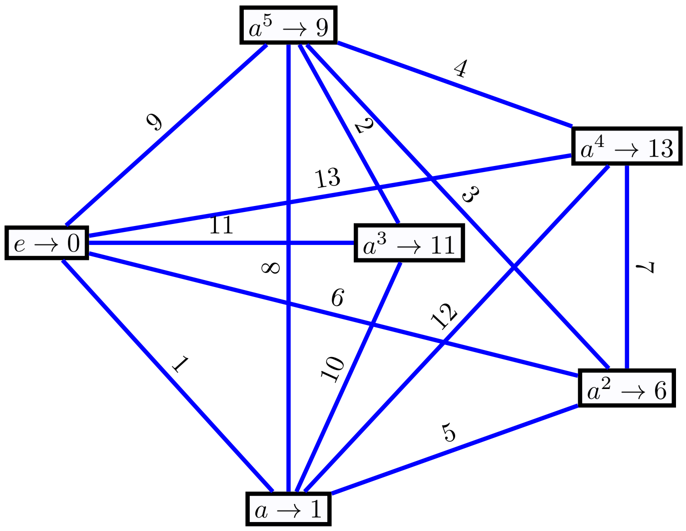

FIGURE 2. A graceful labeling of the power graph of $G \cong \mathbb{Z}_6$

Subcase B: $G \cong D_3$.

Let us take

$$G = \langle x, y \mid x^2 = y^3 = e,\ yx = xy^{-1} \rangle.$$

Define a function $\Psi : V(P(G)) \to \{0, 1, 2, \ldots, 6\}$ by

$$\Psi(e) = 0,\ \Psi(x) = 4,\ \Psi(xy) = 5,\ \Psi(xy^2) = 6,\ \Psi(y) = 1,\ \Psi(y^2) = 3.$$

Then Ψ provides a graceful labeling of the power graph of G, as illustrated in Figure 3.

Case IV: $n \in \{5, 7\}$.

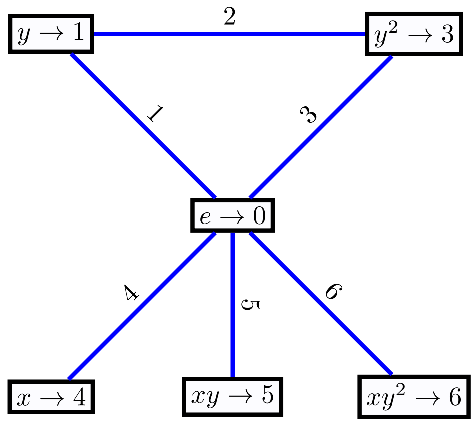

FIGURE 3. $P(G \cong D_3)$ with graceful vertex and edge labeling.

In this case, G has prime order and hence is cyclic. By Theorem 1.3, $P(G)$ is a complete graph, and therefore, by Theorem 1.2, it is not graceful.

Consequently, all non-trivial groups of order at most 7 yield graceful power graphs, except for orders 5 and 7.

□

Theorem 2.3. *Let G be a finite group of order $n \in \{8, 9\}$. Then the power graph of G is graceful unless G is isomorphic to the cyclic group $\mathbb{Z}_n$.*

Proof. We establish the theorem by analyzing the possible group structures corresponding to the two values of n separately.

Case I: $n = 8$.

In this case, G is isomorphic to one of the following five groups:

$$\mathbb{Z}_8,\ D_4,\ Q_8,\ \mathbb{Z}_2 \times \mathbb{Z}_4,\ \text{and } \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2,$$

where D_4 denotes the dihedral group of order 8 and Q_8 denotes the quaternion group of order 8. We consider the following subcases.

Subcase A: If $G \cong \mathbb{Z}_8$, then the result follows from Theorems 1.2 and 1.3.

Subcase B: If $G \cong D_4$, let

$$G = \langle x, y \mid x^2 = y^4 = e,\ yx = xy^{-1} \rangle.$$

The vertex and edge labeling of $P(G)$ shown in Figure 4 is graceful, which establishes the result in this case.

Subcase C: If $G \cong \mathbb{Z}_2 \times \mathbb{Z}_4$, then it has been shown in [15] that the power graph $P(G)$ admits a graceful labeling.

Subcase D: If $G \cong \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$, then $P(G)$ is graceful by Theorem 2.1.

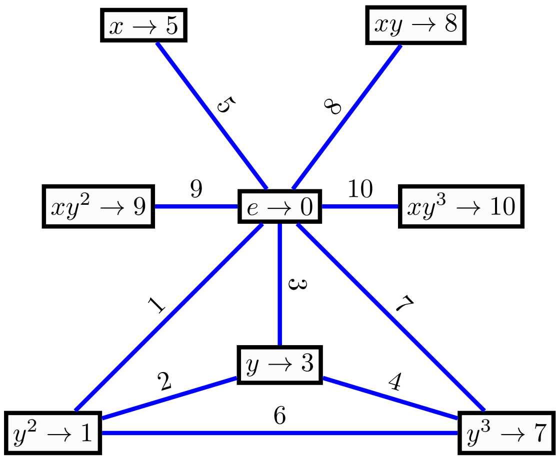

FIGURE 4. Gracefully labeled power graph of $G \cong D_4$

Subcase E: If $G \cong Q_8$, let

$$G = \langle x, y \mid x^4 = e,\ y^2 = x^2,\ y^{-1}xy = x^{-1} \rangle.$$

Define $\Omega : V(P(G)) \to \{0, 1, 2, \ldots, 16\}$ by

$$\Omega(e) = 0,\ \Omega(x) = 5,\ \Omega(x^2) = 1,\ \Omega(x^3) = 8,\ \Omega(y) = 12,$$

$$\Omega(x^2y) = 14,\ \Omega(xy) = 10,\ \Omega(x^3y) = 16.$$

The vertex labeling defined by Ω together with the induced edge labeling of $P(G)$ is illustrated in Figure 5. This labeling is graceful, and hence the theorem holds in this case.

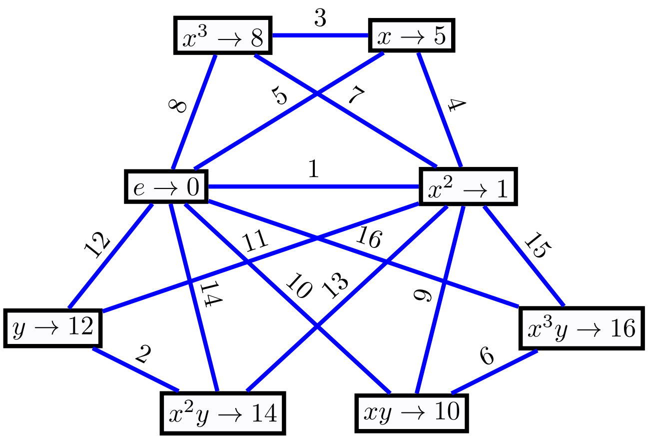

FIGURE 5. Graceful labeling on the power graph associated with $G \cong Q_8$

Case II: $n = 9$.

Since there are only two groups of order 9 up to isomorphism, namely $\mathbb{Z}_9$ and $\mathbb{Z}_3 \times \mathbb{Z}_3$, it follows that G is isomorphic to one of these two groups.

If $G \cong \mathbb{Z}_9$, then proceeding as in Subcase A of Case I, the power graph $P(G)$ is not graceful. On the other hand, if $G \cong \mathbb{Z}_3 \times \mathbb{Z}_3$, then by Theorem 2.1, the power graph $P(G)$ is graceful.

The result now follows by combining the conclusions of Case I and Case II. $\square$

Theorem 2.4. *If p and q are odd prime numbers such that $pq \equiv 5$ or $7 \pmod 8$, then the power graph of $\mathbb{Z}_{pq}$ is not graceful.*

Proof. Let $\mathbb{Z}_{pq} = \langle x \rangle$. The distinct vertices of the power graph $P(\mathbb{Z}_{pq})$, together with their corresponding degrees, are listed in Table 2.

TABLE 2. Vertex orders and corresponding degrees in $P(\mathbb{Z}_{pq})$

Vertex(class)	**Order**	**Degree**
e	1	$pq - 1$
x^{qi}, $i = 1, 2, \dots p - 1$	p	$q(p-1)$
x^{pi}, $i = 1, 2, \dots q - 1$	q	$p(q-1)$
x^{i}, $i = 1, 2, \dots pq$ with $(i, pq) = 1$	pq	$pq - 1$

The total number of edges of the power graph $P(\mathbb{Z}_{pq})$ is given by

$$\begin{aligned}
|E(P(\mathbb{Z}_{pq}))| &= \frac{1}{2} \sum_{v \in V(P(\mathbb{Z}_{pq}))} \deg(v) \\
&= \frac{(pq-1)(\varphi(pq)+1) + q(p-1)(p-1) + p(q-1)(q-1)}{2} \\
&= \frac{(pq-1)pq}{2} - (p-1)(q-1),
\end{aligned}$$

where φ denotes Euler's totient function.

Since p and q are odd primes, both $p-1$ and $q-1$ are even. Consequently,

$$(p-1)(q-1) \equiv 0 \pmod 4.$$

We now consider the following cases.

Case 1: If $pq \equiv 5 \pmod 8$, then from

$$|E(P(\mathbb{Z}_{pq}))| = \frac{(pq-1)pq}{2} - (p-1)(q-1),$$

it follows that

$$|E(P(\mathbb{Z}_{pq}))| \equiv 2 \pmod 4.$$

Case 2: If $pq \equiv 7 \pmod 8$, the same expression yields

$$|E(P(\mathbb{Z}_{pq}))| \equiv 1 \pmod 4.$$

Moreover, Table 2 shows that every vertex of $P(\mathbb{Z}_{pq})$ has even degree. Hence $P(\mathbb{Z}_{pq})$ is an Eulerian graph. Consequently, by Theorem 1.1, the power graph of $\mathbb{Z}_{pq}$ cannot be graceful. $\square$

Theorem 2.5. *Let*
$G = \mathbb{Z}_2 \ltimes \mathbb{Z}_3^n = \left\langle x, y_1, \ldots, y_n \;\middle|\; x^2 = y_j^3 = e,\ xy_j = y_j^{-1}x,\ y_jy_k = y_ky_j,\ j,k = 1, \ldots, n \right\rangle,$
where $n \geq 1$. Then the power graph $P(G)$ is graceful.

Proof. The following observations about the group G are noteworthy:

- The group has $2 \cdot 3^n$ elements, of which only the identity element e has order one.
- There are 3^n elements of the form
$$x \prod_{j=1}^{n} y_j^{\alpha_j} \quad (\alpha_j = 0, 1, 2;\ j = 1, \ldots, n),$$
each of order 2. We denote these elements by g_s, where $1 \leq s \leq 3^n$.
- Apart from the identity, there are $3^n - 1$ elements of the form
$$\prod_{j=1}^{n} y_j^{\alpha_j} \quad (\alpha_j = 0, 1, 2;\ j = 1, \ldots, n),$$
where the exponents α_j's are not all zero simultaneously. These elements have order 3 and can be partitioned into $\frac{3^n-1}{2}$ pairs $\{h_t, h_t^2\}$, $1 \leq t \leq \frac{3^n-1}{2}$, where each pair consists of an element and its square.
- Consequently, the number of cyclic subgroups of order 3 is
$$\frac{3^n - 1}{\varphi(3)} = \frac{3^n - 1}{2},$$
and these subgroups are precisely $\{\, e, h_t, h_t^2 \,\}, 1 \leq t \leq \frac{3^n-1}{2}$.

The components of the power graph $P(G)$ are described as follows.

- The vertex set of the graph is $V(P(G)) = \mathbb{Z}_2 \ltimes \mathbb{Z}_3^n$.
- For each element g_s of order 2, the inclusion $\langle e \rangle \subset \langle g_s \rangle, 1 \leq s \leq 3^n$, gives rise to 3^n edges.

- In each cyclic subgroup of order 3,
$$\langle h_t\rangle = \langle h_t^2\rangle = \{e, h_t, h_t^2\}, \qquad 1 \le t \le \frac{3^n-1}{2},$$
the relations
$$\langle e\rangle \subset \langle h_t\rangle, \quad \langle e\rangle \subset \langle h_t^2\rangle, \quad \langle h_t\rangle = \langle h_t^2\rangle,$$
imply that the pairs $\{e, h_t\}$, $\{e, h_t^2\}$ and $\{h_t, h_t^2\}$ are adjacent in $P(G)$. Hence these subgroups contribute $\dfrac{3(3^n-1)}{2}$ edges in total.
- There are no other edges in $P(G)$ beyond those described above.
- Hence the total number of edges of $P(G)$ is
$$|E(P(G))| = \frac{5\cdot 3^n - 3}{2}.$$

Consequently, the structure of the power graph of the finite non-abelian group G can be written as
$$P(G) \cong K_1 + \left(3^n K_1 \cup \frac{3^n-1}{2} K_2\right).$$

We define a labeling
$$\mho : V(P(G)) \to \left\{0, 1, 2, \ldots, \frac{5\cdot 3^n - 3}{2}\right\}$$
as follows:
$$\mho(e) = 0,$$
$$\mho(h_t) = t, \qquad t = 1, 2, \ldots, \frac{3^n-1}{2},$$
$$\mho(g_s) = \frac{3^n-1}{2} + s, \qquad s = 1, 2, \ldots, 3^n - 1,$$
$$\mho(g_{3^n}) = \begin{cases} \dfrac{9\cdot 3^n - 5}{4}, & \text{if } n \text{ is even} \\[2ex] \dfrac{7\cdot 3^n - 5}{4}, & \text{if } n \text{ is odd,} \end{cases}$$
$$\mho(h_t^2) = \begin{cases} \dfrac{9\cdot 3^n - 5}{4} + \dfrac{t}{2}, & \text{if } \dfrac{3^n-1}{2} - t \text{ is even} \\[2ex] \dfrac{7\cdot 3^n - 5}{4} + \dfrac{t}{2}, & \text{if } \dfrac{3^n-1}{2} - t \text{ is odd.} \end{cases}$$

Table 3 and Table 4 display the edge labels induced by $\mho$ for even and odd values of n, respectively.

TABLE 3. Induced edge labeling in $P(\mathbb{Z}_2 \ltimes \mathbb{Z}_3^n)$ for even values of n

Edge(s) Connecting		**Induced Edge Label(s)**
Vertex u	**Vertex v**	$\lvert \mho(u) - \mho(v) \rvert$
e	h_t, $t = 1, 2, \ldots, \frac{3^n - 1}{2}$	$1, 2, \ldots, \frac{3^n - 1}{2}$
e	g_s, $s = 1, 2, \ldots, 3^n - 1$	$\frac{3^n + 1}{2}, \frac{3^n + 3}{2}, \ldots, \frac{3 \cdot 3^n - 3}{2}$
h_t, $t = \frac{3^n - 3}{2}, \frac{3^n - 7}{2}, \ldots, 3, 1$	h_t^2	$\frac{3 \cdot 3^n - 1}{2}, \frac{3 \cdot 3^n + 1}{2}, \ldots, \frac{7 \cdot 3^n - 7}{4}$
e	h_t^2, $t = 1, 3, \ldots, \frac{3^n - 3}{2}$	$\frac{7 \cdot 3^n - 3}{4}, \frac{7 \cdot 3^n + 1}{4}, \ldots, 2 \cdot 3^n - 2$
h_t, $t = \frac{3^n - 1}{2}, \frac{3^n - 5}{2}, \ldots, 4, 2$	h_t^2	$2 \cdot 3^n - 1, 2 \cdot 3^n, \ldots, \frac{9 \cdot 3^n - 9}{4}$
e	g_s, $s = 3^n$	$\frac{9 \cdot 3^n - 5}{4}$
e	h_t^2, $t = 2, 4, \ldots, \frac{3^n - 1}{2}$	$\frac{9 \cdot 3^n - 1}{4}, \frac{9 \cdot 3^n + 3}{4}, \ldots, \frac{5 \cdot 3^n - 3}{2}$

It is evident from the two tables that the induced edge labels are

$$1, 2, \ldots, \frac{5 \cdot 3^n - 3}{2}, \qquad \text{for all } n \geq 1.$$

Consequently, the labeling $\mho$ is a graceful labeling of the power graph $P(\mathbb{Z}_2 \ltimes \mathbb{Z}_3^n)$, which completes the proof.

$\square$

The labeling scheme described above is illustrated in Figure 6, which represents the power graph of the group

$$G = \mathbb{Z}_2 \ltimes \mathbb{Z}_3^2 = \langle x, y_1, y_2 \mid x^2 = y_1^3 = y_2^3 = e,\ xy_j = y_j^{-1}x\ (j = 1, 2),\ y_1y_2 = y_2y_1 \rangle.$$

The figure displays both the graceful vertex labeling and the induced edge labeling. The structure of this power graph is

$$P(G) \cong K_1 + (9K_1 \cup 4K_2).$$

Table 4. Induced edge labeling in $P(\mathbb{Z}_2 \ltimes \mathbb{Z}_3^n)$ for odd values of n

Edge(s) Connecting		**Induced Edge Label**
Vertex u	**Vertex v**	$\lvert\mho(u)-\mho(v)\rvert$
e	h_t, $t=1,2,\dots,\frac{3^n-1}{2}$	$1,2,\dots,\frac{3^n-1}{2}$
e	g_s, $s=1,2,\dots,3^n-1$	$\frac{3^n+1}{2},\frac{3^n+3}{2},\dots,\frac{3\cdot 3^n-3}{2}$
h_t, $t=\frac{3^n-3}{2},\frac{3^n-7}{2},\dots,4,2$	h_t^2	$\frac{3\cdot 3^n-1}{2},\frac{3\cdot 3^n+1}{2},\dots,\frac{7\cdot 3^n-9}{4}$
e	g_s, $s=3^n$	$\frac{7\cdot 3^n-5}{4}$
e	h_t^2, $t=2,4,\dots,\frac{3^n-3}{2}$	$\frac{7\cdot 3^n-1}{4},\frac{7\cdot 3^n+3}{4},\dots,2\cdot 3^n-2$
h_t, $t=\frac{3^n-1}{2},\frac{3^n-5}{2},\dots,3,1$	h_t^2	$2\cdot 3^n-1,2\cdot 3^n,\dots,\frac{9\cdot 3^n-7}{4}$
e	h_t^2, $t=1,3,\dots,\frac{3^n-1}{2}$	$\frac{9\cdot 3^n-3}{4},\frac{9\cdot 3^n+1}{4},\dots,\frac{5\cdot 3^n-3}{2}$

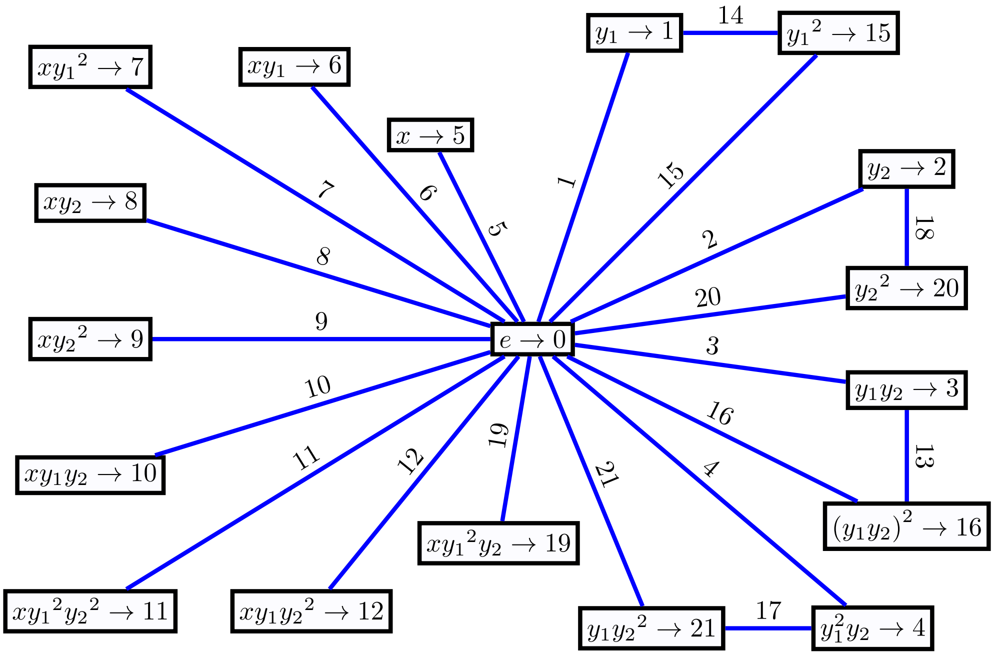

Figure 6. Gracefully labelled power graph $P(\mathbb{Z}_2 \ltimes \mathbb{Z}_3^2)$

3. CONCLUDING REMARKS

In this paper, we have investigated the graceful labeling of power graphs associated with finite groups. Our study has led to a classification of these graphs as either graceful or non-graceful. In particular, we analyzed power graphs of finite groups of order up to 9.

Furthermore, we identified two distinct classes of finite groups: one consisting of cyclic groups whose power graphs do not admit graceful labeling, and the other comprising certain non-abelian groups whose power graphs are graceful. These results highlight the influence of algebraic properties of groups on graph labeling behavior.

We hope that this perspective will stimulate further research on labeling problems in algebraic graph theory and encourage the study of power graphs of larger and more general classes of groups.

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Declarations. The authors declare that there are no conflicts of interest.

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