J. Indones. Math. Soc.
Vol. 31, No. 02 (2025), pp. 1–8.

Signal Processing Of The One-Factor Mean-Reverting Model In Energy System

Getut Pramesti[1*] and Ristu Saptono[2]

[1]Department of Mathematics Education , Universitas Sebelas Maret, Indonesia.
getutpramesti@staff.uns.ac.id
[2]Department of Informatics, Universitas Sebelas Maret, Indonesia.
ristu.saptono@staff.uns.ac.id

Abstract. One of the models that can be considered in the energy system is the one-factor mean-reverting process. We propose the one-factor mean-reverting model with sinusoidal signal processing involved. The frequency component of the model can be estimated with a high-frequency scheme. The estimation of the frequency component is believed to produce a precise estimate. This is because the high-frequency scheme has the potential to handle possible non-linear coefficient cases in a unified way, that is, $nh \to \infty$, and $nh^2 \to 0$. This paper shows that the frequency component estimator in the one-factor mean-reverting model is strongly consistent with the rate convergence, namely $\sqrt{(nh)^3}$. It is also can be shown that the estimator has a normal approximation with a mean of 0 and variance $\frac{1}{6}(1+\theta^2)$. We applied the proposed model to the energy systems data.

Key words and Phrases: one-factor, mean-reverting, frequency, signals, high-frequency

Kata kunci: satu-faktor, mean-reverting, frekuensi, sinyal, high-frequency

1. INTRODUCTION

Diffusion process contain drift and diffusion coefficients. The stochastic diffusion process has two type depend on its drift coefficients, namely homogeneous diffusion process and non-homogeneous diffusion process. The first model whose drift coefficient is a constant or not depends on time, while the second model whose drift coefficient depends on the time variable.

The one-factor mean-reverting model is a part of the non-homogeneous diffusion process. The mean-reversion process is an interesting part of a stochastic

---

*Corresponding author

2020 Mathematics Subject Classification: 92E24, 35B40, 62M10
Received: 21-02-2025, accepted: 12-03-2025.

differential equation since the process can be regarded as an energy's path, which is the pattern will always tend to its mean. [1] shows the digital signal process can be used in approaching the periodicity term of the seasonal variation of the energy system time series data. The theoretical background became an important thing in proposing a forecast energy model, i.e., in the case of the one-factor mean-reverting model. Moreover, we can apply the proposed model to the systems.

This study will provide the consistency of the least-square of the frequency component of the signal processing, and also the asymptotic normality of the estimates. The numerical study will be examined to support the theoretical findings. Therefore, we do literature research on signal processing [2], the harmonic sinusoidal function [3], and the previous research of [1] which provides the theoretical background of the sinusoidal signal processing of the drift. The proposed model will be applied to the energy systems. The electric power and energy systems model is accommodated in [4].

Consider $\{X_t\}$ is a stochastic process that has a following equation

$$dX_t = -\lambda X_t dt + \sigma dw_t \tag{1}$$

with

$$\log C_t = \mu_t + X_t, \tag{2}$$

where μ_t is a deterministic function of t, $t \in (0, \infty)$.

If we denote

$$Y_t := \log C_t, \tag{3}$$

then we can find the relation of the one-factor mean-reverting model and Ornstein–Uhlenbeck process. Observe that

$$\begin{aligned} d(Y_t - \mu_t) &= -\lambda(Y_t - \mu_t)dt + \sigma dw_t \\ dY_t - d\mu_t &= (-\lambda Y_t + \lambda\mu_t)\, dt + \sigma dw_t \\ dY_t &= \left(-\lambda Y_t + \lambda\mu_t + \frac{d\mu_t}{dt}\right) dt + \sigma dw_t. \end{aligned}$$

Assume that

$$b_t(\beta) := \lambda\mu_t(\tilde{\theta}) + \frac{d\mu_t(\tilde{\theta})}{dt}, \tag{4}$$

with $\beta = (\lambda, \tilde{\theta})$, for any parameter $\tilde{\theta}$ of the deterministic function μ_t; then we obtain

$$dY_t = (-\lambda Y_t + b_t(\beta))\, dt + \sigma dw_t. \tag{5}$$

From (5), we can find the relation between the diffusion process, the Gompertz process, and the one-factor mean-reverting model. If $b_t(\beta) = 0$ then the expression (5) can be defined as a diffusion process. If $b_t(\beta) = c$, where c is a constant then the expression (5) can be classified as a time-homogeneous Gompertz diffusion process (see e.g., [5, 6] for references). If $b_t(\beta)$ is defined as (4) then the expression (5) can be called as the one-factor mean-reverting model. This model is part of a time-inhomogeneous Ornstein–Uhlenbeck process (see e.g., [7] for reference).

Discrete-time sinusoidal signals can be expressed as (see i.e., [2] for reference)

$$x(n) = A\cos(\omega n + \vartheta), \tag{6}$$

where

- n is an integer variable,
- A is the amplitude of the sinusoidal,
- ω is the frequency component,
- ϑ is the phase of the sinusoidal.

If we took continuous-time sinusoidal signals, namely

$$\mu_t(\beta) =: \sin(\theta t),$$

with $t \in (0, \infty)$, then we have

$$b_t(\beta) = \lambda \sin(\theta t) + \theta \cos(\theta t),$$

and the one-factor mean-reverting process with sinusoidal signal as follow

$$dY_t = (-\lambda Y_t + \lambda \sin(\theta t) + \theta \cos(\theta t))\, dt + dw_t. \tag{7}$$

For $\lambda = 1$, then we have

$$dY_t = (-Y_t + \sin(\theta t) + \theta \cos(\theta t))\, dt + dw_t. \tag{8}$$

Further,

$$\int_0^t dY_t = \int_0^t (-Y_s + \sin(\theta s) + \theta\cos(\theta s))\, ds + \int_0^t dw_s$$
$$Y_t = Y_0 + \int_0^t (-Y_s + \sin(\theta s) + \theta\cos(\theta s))\, ds + w_t$$

defined on an underlying complete filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in\mathbb{R}_+}, \mathbb{P})$ with $\mathcal{F}_t = \sigma(w_s : s \leq t)$, the parameter spaces $\Theta \subset (0, \infty)$ are bounded convex domains. The true parameter value is denoted by $\theta_0 \in \Theta$.

Using Euler–Maruyama approach, we have

$$Y_{t_j} = Y_{t_{j-1}} + \int_{t_{j-1}}^{t_j} (-Y_s + \sin(\theta s) + \theta\cos(\theta s))\, ds + \Delta_j w, \tag{9}$$

with $\Delta_j w = w_{t_j} - w_{t_{j-1}}$.

Moreover, we define $\Delta_j Y := Y_{t_j} - Y_{t_{j-1}}$, with

$$\Delta_j Y \overset{\mathbb{P}_{\theta_0}}{=} \int_{t_{j-1}}^{t_j} (-Y_s + \sin(\theta s) + \theta\cos(\theta s))\, ds + \Delta_j w. \tag{10}$$

We apply least-squares estimation (LSE) for the parameters of (8) based on discrete time observations, that is, we concentrate on the situation where the diffusion process is observed at discrete times $0 \equiv t_0 < t_1 < \cdots < t_n$, where $t_j^n = t_j = jh$, with $j \leq n$ and for some non-random discrete instant time step $h := h_n$ (see. e.g., [8] for reference of the time step),

$$h \to 0, \tag{11}$$

such that for $n \to \infty$,

$$T := T^n = nh, \quad T^n \to \infty, \tag{12}$$

and

$$nh^2 \to 0. \tag{13}$$

We define the LSE of θ as

$$\hat{\theta}_n \in \underset{\theta \in \overline{\Theta}}{\operatorname{argmin}} \quad \mathcal{Q}_n(\theta), \tag{14}$$

where

$$\mathcal{Q}_n(\theta) = \frac{1}{h} \sum_{j=1}^{n} \left[\Delta_j Y - \left(-Y_{t_{j-1}} + \sin(\theta t_{j-1}) + \theta \cos(\theta t_{j-1})\right) h\right]^2,$$

with $\Delta_j Y = Y_{t_j} - Y_{t_{j-1}}$.

2. MAIN RESULTS

In this section, first we provide the consistency of the estimate.

2.1. The consistency of the LSE of θ.

First of all, we define

$$\mathcal{G}_n(\theta) = \frac{1}{T^n} \left[\mathcal{Q}_n(\theta) - \mathcal{Q}_n(\theta_0)\right] \tag{15}$$

Using Lemma 4.1 of [3] we show the estimator is a strongly consistent.

Observe that

$$\begin{aligned} \mathcal{G}_n(\theta) &= \frac{1}{T^n} \left[\mathcal{Q}_n(\theta) - \mathcal{Q}_n(\theta_0)\right] \\ &= \frac{1}{n} \sum_{j=1}^{n} \left[g_{j-1}(\theta_0) - g_{j-1}(\theta)\right]^2 + \frac{1}{T^n} \sum_{j=1}^{n} \left[g_{j-1}(\theta_0) - g_{j-1}(\theta)\right] \Delta_j w + o_p(h), \end{aligned}$$

with

$$g_{j-1}(\theta) = -Y_{t_{j-1}} + \sin(\theta t_{j-1}) + \theta \cos(\theta t_{j-1}). \tag{16}$$

Moreover, we find the following expression

$$\mathcal{G}_n(\theta) = \mathcal{G}^n_{1,j}(\theta) + \mathcal{G}^n_{2,j}(\theta) + o_p(h), \tag{17}$$

where

$$\mathcal{G}^n_{1,j}(\theta) = \frac{1}{n} \sum_{j=1}^{n} \Big\{ \sin(\theta_0 t_{j-1}) - \sin(\theta t_{j-1}) + \theta_0 \cos(\theta_0 t_{j-1}) - \theta \cos(\theta t_{j-1}) \Big\}^2, \tag{18}$$

$$\mathcal{G}^n_{2,j}(\theta) = \frac{2\sigma}{T^n}\sum_{j=1}^{n}\Big\{\sin(\theta_0 t_{j-1}) - \sin(\theta t_{j-1}) + \theta_0\cos(\theta_0 t_{j-1}) - \theta\cos(\theta t_{j-1})\Big\}\Delta_j w. \tag{19}$$

From (18) and (19), we have

$$\begin{aligned}
\mathcal{G}^n_{1,j}(\theta) &= \frac{1}{n}\sum_{j=1}^{n}\Big\{\left[\sin(\theta t_{j-1}) - \sin(\theta_0 t_{j-1})\right]^2 + \left[\theta_0\cos(\theta_0 t_{j-1}) - \theta\cos(\theta t_{j-1})\right]^2 \\
&+ 2\left[\sin(\theta t_{j-1}) - \sin(\theta_0 t_{j-1})\right]\left[\theta_0\cos(\theta_0 t_{j-1}) - \theta\cos(\theta t_{j-1})\right]\Big\} \\
&= 1 + \frac{1}{2}(\theta^2 + \theta_0^2) + o\left(\frac{1}{T^n}\right).
\end{aligned}$$

Clearly, we obtain the fact

$$\lim_{T^n\to\infty}\inf_{\theta} \quad \mathcal{G}^n_{1,j}(\theta) > 0. \tag{20}$$

Whenever for (19),

$$\lim_{T^n\to\infty}\sup_{\theta} \quad \mathcal{G}^n_{2,j}(\theta) = 0. \tag{21}$$

Therefore, based on (20), (21), and Lemma 4.1 of [3] we get

$$\hat{\theta} \xrightarrow{a.s.} \theta \quad as \quad T^n \to \infty.$$

2.2. **The asymptotic normality of the LSE of θ.**

Now, we will provide the proof of asymptotic normality of the estimate. Using Taylor's approach, we have

$$\begin{aligned}
\mathcal{Q}_n(\theta) &= \mathcal{Q}_n(\theta_0) + \partial\mathcal{Q}_n(\theta^*)(\theta - \theta_0) \\
\partial\mathcal{Q}_n(\theta) &= \partial\mathcal{Q}_n(\theta_0) + \partial^2\mathcal{Q}_n(\theta^*)(\theta - \theta_0)
\end{aligned}$$

θ^* is the point between θ and θ_0. Because of (14) then $\partial\mathcal{Q}_n(\theta) = 0$. Therefore

$$\begin{aligned}
0 &= \partial\mathcal{Q}_n(\theta_0) + \partial^2_{\theta_0^2}\mathcal{Q}_n(\theta^*)(\theta - \theta_0) \\
\partial_{\theta_0}\mathcal{Q}_n(\theta_0) &= -\partial^2_{\theta_0^2}\mathcal{Q}_n(\theta^*)(\theta - \theta_0)
\end{aligned}$$

By Lemma 3.12 of [1] for the right hand side of the above equation, then we have

$$\frac{1}{\sqrt{(T^n)^3}}\partial_{\theta_0}\mathcal{Q}_n(\theta_0) = \left\{-\frac{1}{\sqrt{(T^n)^3}}\partial^2_{\theta_0^2}\mathcal{Q}_n(\theta_0)\frac{1}{\sqrt{(T^n)^3}}\right\}\sqrt{(T^n)^3}(\theta - \theta_0). \tag{22}$$

We will proceed the left hand side and the first of the right hand side of the equation (22) to provide the following Theorem prove.

Theorem 2.1. *We have*

$$\sqrt{(T^n)^3}(\theta - \theta_0) \xrightarrow{\mathcal{L}} \mathcal{N}\left(0, \frac{1}{6}(1 + \theta_0^2)\right) \quad as \quad T^n \to \infty.$$

PROOF.

We will prove the Theorem 2.1 with the show the following expressions.

$$-\frac{1}{\sqrt{(T^n)^3}}\partial^2_{\theta_0^2}\mathcal{Q}_n(\theta_0)\frac{1}{\sqrt{(T^n)^3}} \xrightarrow{p} \frac{1}{6}(1+\theta_0^2), \tag{23}$$

and

$$\frac{1}{\sqrt{(T^n)^3}}\partial\mathcal{Q}_n(\theta_0) \xrightarrow{\mathcal{L}} \mathcal{N}\left(0, \frac{1}{6}(1+\theta_0^2)\right). \tag{24}$$

First, we will take a look (23).

If $\mathbb{E}_0^{\mathcal{F}_{t_{j-1}}}$ denotes the expectation operator under $\mathbb{P}_{\theta_0}$ conditional on $\mathcal{F}_{t_{j-1}}$, then by applying Lemma 9 of [9], we can obtain

$$\begin{aligned}
&\sum_{j=1}^{n}\mathbb{E}_0^{\mathcal{F}_{t_{j-1}}}\left[\frac{h}{(T^n)^3}\left[t_{j-1}\cos(\theta_0 t_{j-1}) - \theta_0 t_{j-1}\sin(\theta_0 t_{j-1})\right]^2\right] \\
&= \sum_{j=1}^{n}\mathbb{E}_0\left[\frac{h}{(T^n)^3}\left[t_{j-1}\cos(\theta_0 t_{j-1}) - \theta_0 t_{j-1}\sin(\theta_0 t_{j-1})\right]^2 |\mathcal{F}_{t_{j-1}}\right] \\
&= \frac{h}{(T^n)^3}\sum_{j=1}^{n}\left[t_{j-1}\cos(\theta_0 t_{j-1}) - \theta_0 t_{j-1}\sin(\theta_0 t_{j-1})\right]^2 \\
&= \frac{h}{(T^n)^3}\sum_{j=1}^{n}\left[t_{j-1}^2\cos^2(\theta_0 t_{j-1}) + \theta_0^2 t_{j-1}^2\sin^2(\theta_0 t_{j-1}) - 2t_{j-1}^2\theta_0\cos(\theta_0 t_{j-1})\sin(\theta_0 t_{j-1})\right]
\end{aligned}$$

Clearly, we can easy find the expression above using Lemma 3.1 and Corollary 3.2 of [1]. Hence

$$\sum_{j=1}^{n}\mathbb{E}_0^{\mathcal{F}_{t_{j-1}}}\left[\frac{h}{(T^n)^3}\left[t_{j-1}\cos(\theta_0 t_{j-1}) - \theta_0 t_{j-1}\sin(\theta_0 t_{j-1})\right]^2\right] = \frac{1}{6}+\frac{\theta_0^2}{6}+o\left(\frac{1}{T^n}\right) - 2\theta_0 o\left(\frac{1}{T^n}\right)$$

Therefore

$$\sum_{j=1}^{n}\mathbb{E}_0^{\mathcal{F}_{t_{j-1}}}\left[\frac{h}{(T^n)^3}\left[t_{j-1}\cos(\theta_0 t_{j-1}) - \theta_0 t_{j-1}\sin(\theta_0 t_{j-1})\right]^2\right] \xrightarrow{p} \frac{1}{6}(1+\theta_0^2). \tag{25}$$

Similarly with (25), we can obtain the following

$$\sum_{j=1}^{n}\mathbb{E}_0^{\mathcal{F}_{t_{j-1}}}\left[\left(\frac{h}{(T^n)^3}\left[t_{j-1}\cos(\theta_0 t_{j-1}) - \theta_0 t_{j-1}\sin(\theta_0 t_{j-1})\right]^2\right)\right] \xrightarrow{p} 0. \tag{26}$$

Next, we will observe (24).

$$\sum_{j=1}^{n} \mathbb{E}_0^{\mathcal{F}_{t_{j-1}}} \left[\left(\frac{1}{\sqrt{(T^n)^3}} \partial_{\theta_0} g_{j-1}(\theta_0) \Delta_j w \right)^2 \right]$$

$$= \sum_{j=1}^{n} \mathbb{E}_0 \left[\frac{1}{(T^n)^3} \left(t_{j-1} \cos(\theta_0 t_{j-1}) - \theta_0 t_{j-1} \sin(\theta_0 t_{j-1}) \right)^2 | \mathcal{F}_{t_{j-1}} \right] h = \frac{1}{6} \left(1 + \theta_0^2 \right)$$

3. ESTIMATING FREQUENCY COMPONENT IN APPLIANCES ENERGY DATASET

The proposed model was applied to real data for the energy consumption of light fixtures in one Belgium household. These data are available at [10] and the relevant paper at [11]. The dataset is at 10 min for about 4.5 months. The dataset contains 19737 energy uses in the household with a ten-minute sampling rate over a period from January 11, to May 27, 2016. We estimate the frequency component of (8) of appliances energy use (in Wh). All calculations in the empirical results have been performed in the R program.

We visualize the appliances energy of the dataset [10] with the proposed model (8).

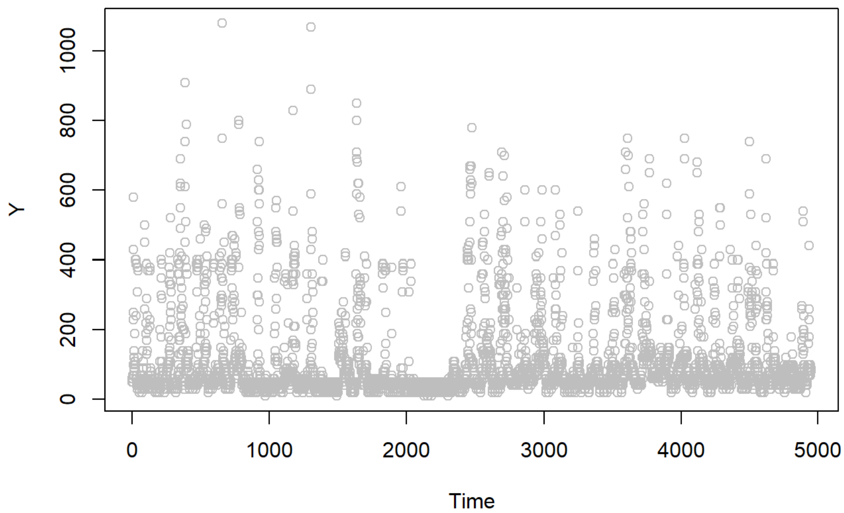

FIGURE 1. Ten-minute sampling rate of appliances energy (Y) over a period from January 1, to February 18, 2016. Time in hours.

We choose the considered period, namely:

- $T^n = 45, T^n h = 4.5$ for period January 11, 2016 to January 14, 2016,

- $T^n = 100, T^n h = 2.5$ for period January 11, 2016 to February 8, 2016,
- $T^n = 150, T^n h = 1.5$ for period January 11, 2016 to April 24, 2016,
- $T^n = 165, T^n h = 1.4$ for period January 11, 2016 to May 25, 2016.

TABLE 1. The performance of $\hat{\theta}_n$ of (8) for considered T^n and $T^n h$. The Root Mean Squares Error (RMSE) of the model is given.

T^n	$T^n h$	$\hat{\theta}_n$	RMSE
45	4.5	0.27	176.87
100	2.5	0.36	152.59
150	1.5	0.34	144.92
165	1.4	0.27	140.44

From Table 1, we can see that the estimate get better for larger T^n and smaller $T^n h$; the RMSE of the model seems getting smaller.

4. CONCLUDING REMARKS

The sinusoidal signal processing of the one-factor mean-reverting model can be considered in energy system modeling. For a simple rate of reversion $\lambda = 1$, and sinusoidal signal $sin(\theta t)$ in the one-factor mean-reverting model (5) tend to Normal distribution with a mean 0, and variance $\frac{1}{6}(1 + \theta^2)$.

In the future, we can extend (5) for general mean reversion λ, and various sinusoidal signal processing form $\mu_t(\tilde{\theta})$, i.e., $\mu_t(\tilde{\theta}) = \sum_{k=1}^{K} [A_k \sin(\theta_k t) + B_k \cos(\theta_k t)]$, with $\tilde{\theta} = (\mathbf{A}, \mathbf{B}, \theta)$.

Data availability statement The data that support the findings of this study are available in UCI Machine Learning Repository at [https://archive.ics.uci.edu/ml/machine-learning-databases/00374/]. These data were derived from the following resources available in the public domain: [https://archive.ics.uci.edu/ml/datasets/Appliances+energy+prediction]

REFERENCES

[1] G. Pramesti, "Parameter least-squares estimation for time-inhomogeneous ornstein–uhlenbeck process," *Monte Carlo Methods and Applications*, vol. 29, no. 1, pp. 1–32, 2023. https://doi.org/10.1515/mcma-2022-2127.

[2] J. G. Proakis, *Digital signal processing: principles algorithms and applications*. Pearson Education India, 2001.

[3] D. Kundu and S. Nandi, *Statistical signal processing: frequency estimation*. Springer Science & Business Media, 2012. https://dokumen.pub/statistical-signal-processing-frequency-estimation-2nd-ed-9789811562792-9789811562808.html.

[4] M. E. El-Hawary, *Advances in electric power and energy: Forecasting in electric power systems.* John Wiley & Sons, Incorporated, 2017.

[5] S. Kiremitciyan, A. Goncu, and T. U. Kuzubas, "A Comparison of Stochastic Models of Natural Gas Consumption," Working Papers 2014/10, Bogazici University, Department of Economics, Oct. 2014. https://ideas.repec.org/p/bou/wpaper/2014-10.html.

[6] R. Gutiérrez, R. Gutiérrez-Sanchez, *et al.*, "A stochastic gompertz model with logarithmic therapy functions: parameters estimation," *Applied Mathematics and Computation*, vol. 219, no. 8, pp. 3729–3739, 2012. https://doi.org/10.1016/j.amc.2012.09.074.

[7] G. Pramesti, "The least-squares estimator of sinusoidal signal of diffusion process for discrete observations," *J. Math. Comput. Sci.*, vol. 11, no. 5, pp. 6433–6443, 2021. https://scik.org/index.php/jmcs/article/view/6401.

[8] S. M. Iacus *et al.*, *Simulation and inference for stochastic differential equations: with R examples*, vol. 486. Springer, 2008. https://doi.org/10.1007/978-0-387-75839-8.

[9] V. Genon-Catalot and J. Jacod, "On the estimation of the diffusion coefficient for multi-dimensional diffusion processes," in *Annales de l'IHP Probabilités et statistiques*, vol. 29, pp. 119–151, 1993. https://www.numdam.org/item/AIHPB_1993__29_1_119_0.pdf.

[10] "Appliances energy prediction," 2017. https://archive.ics.uci.edu/ml/datasets/Appliances+energy+prediction.

[11] L. M. Candanedo, V. Feldheim, and D. Deramaix, "Data driven prediction models of energy use of appliances in a low-energy house," *Energy and buildings*, vol. 140, pp. 81–97, 2017. https://doi.org/10.1016/j.enbuild.2017.01.083.