J. Indones. Math. Soc.
Vol. 31, No. 02 (2025), pp. 1–8.

"Corporate" And "Community" *Takāful*

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Abstract. In this paper, we compare different characterizations of the *takāful* organization. We propose two different characterizations with one being based on conventional firm theory from microeconomics ("corporate" *takāful*) and another being based on the mutual/cooperative insurance literature ("community" *takāful*). We find that both characterizations imply different strategies due to different objectives and operational conditions. We also find that if participants in a community *takāful* organization are altruistic, those overseeing the organization must make sure that participants do not spend more than they have when paying for claims made by the community.

Key words and Phrases: *Takāful*, Mutual Insurance, Altruism, Adverse Selection, Social Welfare

1. INTRODUCTION

The actuarial sciences and financial mathematics literature is well-established and contributes valuable insights to the insurance industry from those related to the pricing of various insurance aspects to insurance strategy itself. However, many question the applicability of these insights to the Islamic concept of insurance mainly referred to as *takāful*. According to Fatwa No. 21/DSN-MUI/X/2001 issued by Indonesia's DSN MUI, takāful is (roughly translated) a mutual effort to protect and aid on the basis of doing good, which is a definition acknowledged in the literature such as in El-Gamal [1] and Khan [2]. However, El-Gamal [1] notes that the implementation of *takāful* seems to have deviated from its spirit as there is a noticeable lack of mutuality in the operation of significant *takāful* operators, which Kassim [3] comments as possibly being a necessary evil.

The implication of this perspective is that *takāful* organizations should not be modeled as profit-maximizing firms such as in Rothschild & Stiglitz [4] and Khan

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2020 Mathematics Subject Classification: 91B06, 91B41, 91B43
Received: 10-09-2024, accepted: 12-11-2024.

[2] or even perfectly competitive insurance markets as in Aase [5]. They should instead be modeled as mutual/cooperative insurance organizations which has its own literature, contemporary examples of which include Ambrus et al. [6] and Levantesi & Piscopo [7]. While Albrecht & Huggenberger [8] notes that this might not lead to a fundamental difference, a significant amount of papers on mutual/cooperative insurance modeling portray such organizations as placing more emphasis on the human aspect instead of financial strategies, such as in Charness & Genicot [9] and Bourlès & Rouchier [10]. We therefore model both organizational forms and investigate whether there would be any differences in operational strategy.

2. CORPORATE *TAKĀFUL*

In this section, we discuss the implications of our characterization of a "corporate" *takāful* manager/operator's optimization problem. We first characterize "corporate" *takāful* as a firm that wishes to maximize its profit π. The following are also assumed regarding corporate *takāful*:

- The *takāful* organization is a price-taker so its contributions (i.e. premiums) p are (mostly) exogenously determined
- The *takāful* organization is risk-neutral such that its objective function can simply be represented by its profit function
- The general operating costs of the organization scale with the number of participants (i.e. policyholders) n and assumed to take the form $\frac{1}{2}n^2$
- The *takāful* organization knows that there are two types of participants; high-risk participants with a probability q_H of making a claim of size X_H and low-risk participants with a probability q_L of making a claim of size X_L
- The *takāful* organization cannot tell the difference between the two types and charges them the same contribution amount
- The *takāful* organization only knows that a portion β of its participants are high-risk
- The *takāful* organization can screen for high-risk participants to reduce the proportion of such participants at a cost of c which, as is common for economic models, is assumed to be increasing at an increasing rate as it justifies the need for an optimal level of c; it is more specifically assumed that $\beta(c) = e^{-c}$

The manager of the takāful organization therefore only has control over how many participants to service n and the amount of resources c it wishes to dedicate to screening for high-risk participants. The associated maximization problem is:

$$\max_{n,c} \quad \pi = np - (1-\beta(c))nq_L X_L - \beta(c)nq_H X_H - \frac{1}{2}n^2 - c \tag{1}$$

which incorporates features of insurance/*takāful* operators used in Rothschild & Stiglitz [4] and Khan [2].

Proposition 2.1. *The optimal capacity* n^* *of the takāful organization is (i) positively affected by the level of contributions (ii) negatively affected by the expected claims payable to low-risk participants (iii) not affected by the expected claims payable to high-risk participants (iv) only positive if an implicit loading factor is charged as determined by* $p - q_L X_L > 2$.

Proof. The manager/operator of the corporate *takāful* solves the profit maximization problem in (1), resulting in the following first-order conditions:

$$\frac{\partial \pi}{\partial n} = p - (1 - \beta(c))q_L X_L - \beta(c)q_H X_H - n = 0$$

$$\frac{\partial \pi}{\partial c} = \beta'(c)nq_L X_L - \beta'(c)nq_H X_H - 1 = 0$$

The optimal level of profit for the corporate *takāful* organization is therefore characterized by the two conditions:

$$n = p - q_L X_L + \beta(c)(q_L X_L - q_H X_H) \tag{2}$$

$$\beta'(c)n = \frac{1}{q_L X_L - q_H X_H} \tag{3}$$

By substituting the second condition into the first and making use of the fact that $\beta'(c) = -\beta(c)$, the following expression can be obtained:

$$n = p - q_L X_L - \frac{1}{n}$$

Multiplying both sides by n and taking the positive root of the resulting quadratic equation results in a closed-form expression for the optimal capacity for corporate *takāful*:

$$n^* = \frac{p - q_L X_L + \sqrt{(p - q_L X_L)^2 - 4}}{2} \tag{4}$$

which mathematically expresses our first proposition. □

The first two parts of this first proposition are to be expected as they simply follow from applying firm theory from microeconomics to *takāful* modeling; the selling price of a good (in this case the contributions of the policyholders) increases the optimal quantity to be produced whereas the cost of production (which in this case the expected claims payable to low-risk policyholders is a component of) decreases the optimal quantity to be produced. The fourth is arguably not that surprising either as it follows from the classic analysis of the adverse selection problem in Rothschild & Stiglitz [4]; it is to be expected that in order to meet all claims, an insurance company must charge more than the expected value of claims made by low-risk policyholders if the company cannot perfectly price discriminate between low-risk and high-risk policyholders. What is highly surprising is the third part of this proposition, namely that the model implies that the expected value of claims made by high-risk participants has no effect on the organization's optimal capacity. One should expect results similar to those in Rothschild & Stiglitz [4] which incorporates the expected claims from high-risk policyholders into the loading factor and yet this is not the case. However, Rothschild & Stiglitz's [4] results is based on deriving optimal premiums whereas our result comes from modeling optimal capacity, which might explain the difference between the results.

Proposition 2.2. *The optimal level of high-risk participants is inversely related to the difference in expected claims payable between high-risk and low-risk participants.*

Proof. Based on (3) and again making use of the fact that $\beta'(c) = -\beta(c)$, we obtain:

$$\beta^*(c)n^* = -\beta'(c^*)n^* = \frac{1}{q_H X_H - q_L X_L} \tag{5}$$

which represents the optimal amount of high-risk participants. □

Proposition 2.3. *The optimal level of low-risk participants is positively related to the difference in expected claims payable between high-risk and low-risk participants and has a positive relationship with optimal capacity.*

Proof. By definition, the optimal number of low-risk participants is the optimal total number of participants n^* less the optimal amount of high-risk participants which is represented by equation (5). The optimal level of low-risk participants is therefore:

$$n^*(1 - \beta^*(c)) = n^* - \frac{1}{q_H X_H - q_L X_L} \tag{6}$$

□

Note that this implies the optimal mix of high-risk and low-risk participants is indirectly affected by the factors affecting n^*, namely the implicit loading factor (i.e. difference between the contributions collected) as well as the expected value of claims to be made by low-risk participants.

We consider these propositions are results which are novel but sensible with a somewhat interesting interpretation. The direct interpretation is that the optimal number of high-risk participants (and therefore how important it is to screen for them) depends on the difference in value between the expected claims they make and the ones made by low-risk participants. If both types of participants are expected to make claims that are high but not too different in value, the model implies that the operator might as well not screen at all which makes sense. It is slightly different for low-risk participants as the optimal amount of low-risk participants is influenced by the optimal number of participants and therefore indirectly affected by the implicit loading factor. The overall implication here therefore is that if the difference in expected claims value between low-risk and high-risk participants is not significant, resources would be better spent extracting more contributions from participants.

Proposition 2.4. *The optimal level of screening costs is (i) positively affected by the difference in expected claims payable between high-risk and low-risk participants (ii) positively affected by the optimal capacity of the takāful organization (iii) affected indirectly by the factors affecting the optimal capacity of the takāful organization.*

PROOF. Beginning with equation (5), simply divide both sides by n and take the inverse of $\beta(c)$, which results in:

$$c^* = \log\left[(q_H X_H - q_L X_L)n^*\right]$$

□

As the interpretation of the expression for optimal screening costs is mostly similar to what has been discussed concerning the optimal number of high-risk and low-risk participants, we shall not elaborate further.

3. COMMUNITY *TAKĀFUL*

In this section, we discuss the implications of our characterization of a community *takāful* organization's optimization problem. Note that we have not used the terms "manager" or "operator" as decision making for this type of organization might not be centralized. In fact, we assume that decision-making occurs at the individual level. Other assumptions we make regarding community *takāful* are as follows:

- There are n participants in the community but there is no discrimination between high and low-risk participants; we assume that each member of the community naively believes that everyone has a probability q of incurring some loss X
- All participants care about their wealth W_i and do not like spending it
- All participants contribute the same amount p to the community takāful pool of funds
- All participants have what Clavien & Chapuisat [11] refer to as *degree altruism* such that their utility function has the form

$$U_i = u_{i,s}(\cdot) + \phi u_{i,o}(\cdot)$$

  this means that each participant i's total utility U_i is dependent on their own utility $u_{i,s}$ (or utility of *self*) and the utility of *others* $u_{i,o}$ with ϕ being a parameter representing how much a given participant cares about the utility of others
- All participants are risk-averse with respect to their wealth and the community's pool of funds (because it is suboptimal for those who are risk-neutral and risk-loving to buy insurance), which is represented by having utility functions of the form $u(x) = x^\alpha$ where $0 < \alpha < 1$ and lower levels of α represent higher levels of risk aversion
- Each participant maximizes their expected utility by making a claim C_i which will be paid out if there are enough funds in the community takāful

pool of funds; note that this claim does not have to be equal to the actual loss incurred
- Each participant assumes that each claim made by other participants is equal to X

Each participant therefore faces the following maximization problem:

$$\max_{C_i} \quad E[U_i] = E[u_{i,s} + \phi u_{i,o}]$$

where each participant's personal net wealth is assumed to be affected only by the following factors: (i) their contribution to the community takāful pool p (ii) whether or not they incur a loss X (iii) how much compensation they receive C_i when they incur a loss (iv) the probability of incurring a loss q. A participant's expected utility concerning their personal wealth is therefore expressed as:

$$E[u_{i,s}] = q(W_i - p - X + C_i)^\alpha + (1-q)(W_i - p)^\alpha \tag{7}$$

As for the community takāful pool of funds, it is assumed to be affected only by inflows and outflows of funds. However, the aggregate amount of losses incurred by the other $n-1$ participants is a binomial random variable which can complicate the economic aspect of the analysis. For parsimony, we assume that there are only two participants such that the possible outcomes are (i) either both participants make claims (ii) only one makes a claim, or (iii) neither participant makes a claim. A participant's expected utility concerning the community pool of funds is therefore:

$$E[u_{i,o}] = q^2(2p-X-C_i)^\alpha + q(1-q)(2p-C_i)^\alpha + q(1-q)(2p-X)^\alpha + (1-q)^2(2p)^\alpha \tag{8}$$

The first order condition of a given participant's expected utility maximization problem is therefore:

$$\frac{\partial E[U_i]}{\partial C_i} = (W_i - p - X + C_i)^{\alpha-1} - \phi q(2p-X-C_i)^{\alpha-1} - \phi(1-q)(2p-C_i)^{\alpha-1} = 0 \tag{9}$$

which can be rewritten as:

$$(W_i - p - X + C_i)^{\alpha-1} = \phi q(2p - X - C_i)^{\alpha-1} + \phi(1-q)(2p - C_i)^{\alpha-1} \tag{10}$$

This rearrangement of the first-order condition is common in the economics field, in this case the left side represents the marginal benefit of increasing C_i whereas the right side represents the marginal cost of decreasing it. A simple interpretation is that while demanding more compensation for loss increases one's personal wealth, it also detracts from the participant's other concern which is ensuring that there are enough funds in the pool for the community. From this perspective, it can be seen that the "altruism parameter" ϕ helps to psychologically penalize the participant from demanding too much compensation from the community pool of funds. Assuming that the other participant incurs a loss with certainty $q = 1$ allows for a closed-form solution and an easier analysis without loss of generality:

$$(W_i - p - X + C_i)^{\alpha-1} = \phi(2p - X - C_i)^{\alpha-1} \tag{11}$$

which can be rearranged to obtain the following closed-form expression for a given participant's optimal claim:

$$C_i^* = \frac{1}{1+\phi^{\frac{1}{\alpha-1}}}\left([1+2\phi^{\frac{1}{\alpha-1}}]p + [1-\phi^{\frac{1}{\alpha-1}}]X - W_i\right) \tag{12}$$

Proposition 3.1. *The optimal level of claims C_i^* made by a given community takāful participant i is (i) negatively affected by the amount of wealth W_i they have (ii) positively affected by the contributions p paid by each participant (iii) positively or negatively affected by the loss incurred X depending on the level of altruism ϕ (iv) positively or negatively affected by the level of altruism ϕ, depending ultimately on the amount of funds in the community pool after the claims of other participants have been paid out (v) indirectly affected by risk-aversion α in that increases in risk-aversion weaken the effect of altruism*

Proof. Points (i)-(iii) in Proposition 3.1 follow from equation (12); the variables p and W_i have unambiguous effects on C_i^* whereas the effect of X on C_i^* depends on whether or not $\phi^{\frac{1}{\alpha-1}} > 1$. For points (iv) and (v) it helps to rearrange equation (12) into the following form:

$$C_i^* = \frac{1}{1/\phi^{\frac{1}{\alpha-1}}+1}(2p - X) + \frac{1}{1+\phi^{\frac{1}{\alpha-1}}}(p + X - W_i) \tag{13}$$

which should help accentuate the fact that at higher levels of altruism ϕ, optimal compensation depends only on how many funds are left in the community pool after all other claims have been paid out. Equation (13) also makes it easier to see that as α decreases (i.e. risk-aversion increases), the exponent of all altruism parameters ϕ decreases and therefore the effect of the altruism parameters on C_i^* also decreases. □

Proposition 3.1 highlights some key implications of the different operational basis of community-based takāful organizations compared with corporate takāful organizations. One of the more important implications is that the more decentralized nature of community-based takāful can be a double-edged sword. One one hand, Proposition 3.1 shows that a sufficiently high level of altruism prevents the moral hazard of participants making higher claims then necessary. In fact equation (13) allows for negative claims if altruism is sufficiently high and if losses are sufficiently high relative to contributions. On the other hand, a significant amount of the model depends on the beliefs of each participant regarding the condition and behavior of other participants which can make managing such an organization very complex and difficult.

4. CONCLUDING REMARKS

We have constructed and discussed mathematical representations of two forms of *takāful* organizations based on some assertions as to what they should be like. The "corporate" form is profit maximizing and risk-neutral whereas the "community" form is built upon a community of participants with varying degrees of risk-aversion and concern for the good of the community. Key insights we have demonstrated include that there are theoretically significant differences between both *takāful* organizational formss; while both are ultimately driven by market conditions, the community form is also driven by personal perceptions and concern for society. Further questions to be investigated include conditions in which one form is preferred to the other, whether there are social concern aspects of corporate *takāful*, and how different social dynamics can affect the decision-making associated with both organizational forms.

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