

Extended Fuzzy Binary Soft Sets and Their Applications in Multi Parameter Decision Making Problems

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Abstract. Multi-criteria decision-making (MCDM) problems involve evaluating and selecting alternatives based on multiple criteria. This article aims to solve MCDM problems by extending the definition of fuzzy binary soft sets to two parameter sets, which are called extended fuzzy binary soft sets. Operations such as "AND" and "MaxMin" are defined and illustrated with examples. Additionally, an algorithm is presented to solve MCDM problems using extended fuzzy binary soft sets. Finally, an application of the proposed algorithm for decision-making is discussed.

Key words and Phrases: fuzzy sets, fuzzy soft sets, extended fuzzy soft sets, fuzzy binary soft sets, extended fuzzy binary soft sets

1. INTRODUCTION

Multi-criteria decision-making (MCDM) problems arise in a wide range of applications, including business, engineering, public policy and social sciences, where decision-makers need to consider various factors to arrive at an optimal decision. To handle such situations, Zadeh [1] introduced the concept of fuzzy set theory. Atanassov [2] generalized the fuzzy set theory to intuitionistic fuzzy set theory to handle more uncertainty precisely. Under these environments, researchers discussed several types of approaches to solving decision-making problems [3], [4], [5], [6], [7], [8], [9].

Since there is an absence of parametrization in these tools, Maldtsov [10] introduced the concept of soft theory. Soft sets emerged as a powerful tool for modelling uncertainty in a better way. Maji et al. [11] discussed MCDM problems using soft sets. Fuzzy sets were combined with soft sets to deal with more

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uncertainty and called fuzzy soft sets by Roy and Maji [12]. Many interesting applications of fuzzy soft set theory were discussed by many researchers [13], [14], [15], [16], [17], [18], [19].

Soft sets can deal with only one universal set. But as data becomes vaguer, it requires more powerful tool to deal with it. As a result, Acikgoz and Tas [20] defined binary soft sets and studied their properties. Fuzzy sets were combined with binary soft sets called the fuzzy binary soft sets by Metlida and Subhashini [21] and studied their properties. The application of fuzzy binary soft sets in MCDM problems was discussed by Patil et al. [22].

Some situations require more parameters to decide on uncertain things. To deal with such situations Anil and Patil [13] defined extended fuzzy soft sets and discussed their application in MCDM problems which deal with two parameter sets but only one universal set. Some decision-making problems involve two independent sets. The opinion of the decision makers is very crucial in making decision and there can be error while combining all the decision maker's opinion. Hence, it is very necessary to account the set of decision makers and there is a need to relate both sets and rank them in a pair. Also, in some cases, instead of decision makers there will be another parameter set which associates with the data. To address these kind of situations, this article defines with fuzzy binary soft sets with two parameter sets and is called extended fuzzy binary soft sets to solve MCDM problems. "AND" and "MaxMin" operations on extended fuzzy binary soft sets are defined and illustrated with some examples. An algorithm to solve MCDM problems is presented and illustrated with application in deciding the college-course combination.

This article is arranged into 6 sections. In section 2, basic concepts are discussed. Section 3 deals with extended fuzzy binary soft sets and their operations with examples. Section 4 gives an application of extended fuzzy binary soft sets in MCDM problems. The result and discussion are presented in section 5. The conclusion is given in section 6.

2. PRELIMINARIES

Definition 2.1. [1] Let X be a Universal set and A be a function defined by

$$A : X \longrightarrow [0, 1] \text{ or } \mu_X : X \longrightarrow [0, 1].$$

Then, the set $A = (x, A(x))/x \in X$ is called a fuzzy subset of X .

Definition 2.2. [10] A pair (F, E) is called a soft set over a universal set U if F is a mapping of E , a set of parameters into the set of all subsets of set U ,

$$F : E \longrightarrow P(U).$$

Definition 2.3. [12] Let $\tilde{P}(U)$ be the set of fuzzy subsets of U , a pair (\tilde{F}, A) is called a fuzzy soft set over U , where $\tilde{P}(U)$ is a mapping given by,

$$\tilde{F} : A \rightarrow \tilde{P}(U).$$

A fuzzy soft set (FSS) is a mapping from parameters to $\tilde{P}(U)$.

Definition 2.4. [16] Resultant matrix is a square matrix (c_{ij}) in which object names of universal set label rows and columns, and the entries are $c_{ij} = \sum_{k=1}^m \alpha_{ik} - \alpha_{jk}$ where α_{ik} is the membership value of the i^{th} object and k^{th} parameter.

Definition 2.5. [13] Suppose X is an initial universe and E and K are primary and secondary set of parameters. Let I^X denote family of all fuzzy sets over X and E^X denote family of all fuzzy soft sets over X with respect to the parameter set E . For any $A \subseteq E$, a pair (F^*, A) denoted by F_A^* is called extended fuzzy soft set over X , where F^* is a mapping given by

$$F^* : A \rightarrow E^X$$

defined by $F_A^*(k) = F_{EA}(k)$ for any $k \in A$.

Definition 2.6. [13] The cartesian “AND” product of two extended fuzzy soft sets F_A^* and F_B^* over a common universe X denoted by $H_C^* = F_A^* \wedge F_B^*$, is defined as $H_C^* : A \times B \rightarrow E^X$ and $H_C^*(a, b) = F_{EA}(a) \wedge F_{EB}(b)$, where $(a, b) \in A \times B$.

Definition 2.7. [20] Let U_1 and U_2 be two universal sets. E be a set of parameters, $A \subseteq E$. Let F be a function defined by

$$F : A \rightarrow P(U_1) \times P(U_2).$$

Then, the set (F, A) is called Binary Soft Set over U_1 and U_2 .

Definition 2.8. [21] Let U_1 and U_2 be two universal sets, E be the set of parameters, and $A \subseteq E$. Let F be a function defined by

$$F : A \rightarrow \tilde{P}(U_1) \times \tilde{P}(U_2)$$

where $\tilde{P}(U_1)$ and $\tilde{P}(U_2)$ are a set of all fuzzy sets of U_1 and U_2 , respectively. Then (F, A) is called fuzzy binary soft set over U_1 and U_2 .

Definition 2.9. [22] Let (F, A) be fuzzy binary soft set over U_1 and U_2 . Let $M_1(F, A)$ and $M_2(F, A)$ be expanded matrices of (F, A) then the “AND” operator of $M_1(F, A)$ and $M_2(F, A)$ with respect to the parameter e is denoted by $P_e^*(F, A)$ and defined by

$$(P_e^*(F, A))(x, y) = (M_1(F, A))(x) \wedge (M_2(F, A))(y).$$

Definition 2.10. [22] Let $M_1(F, A)$ and $M_2(F, A)$ be expanded matrices of fuzzy binary soft set (F, A) over U_1 and U_2 . An extended resultant matrix is a resultant matrix in which rows and columns are labeled with order pair elements of U_1 and U_2 .

3. EXTENDED FUZZY BINARY SOFT SETS

In this section, the definition of fuzzy binary soft sets is extended to two parameter sets and is called extended fuzzy binary soft sets.

Definition 3.1. Let U_1 and U_2 be two initial universal sets, E and P be two parameter sets. Let I^* be set of all fuzzy binary soft sets over U_1 and U_2 with respect to the parameter set E . For any $A \subseteq P$, (F^*, A) is denoted by F_A^* and is called extended fuzzy binary soft set over U_1, U_2 , where F^* is a mapping given by $F^* : A \rightarrow I^*$ and defined by $F_A^*(p) = F_{EA}(p)$.

Example 3.2. Let $U_1 = \{u_1, u_2\}$ and $U_2 = \{v_1, v_2\}$ be two initial universal sets. $E = \{e_1, e_2, e_3\}$ and $P = \{p_1, p_2, p_3, p_4\}$ be two parameter sets. Let $A = \{p_1, p_3\} \subseteq P$ and $B = \{p_2, p_4\} \subseteq P$. Then, (F^*, A) and (F^*, B) are extended fuzzy binary soft sets given by,

$$(F^*, A) = \{F_A^*(p_1), F_A^*(p_3)\}$$

$$(F^*, B) = \{F_B^*(p_2), F_B^*(p_4)\}.$$

$$F_{EA}(p_1) = \left\{ \left(e_1, \left(\left\{ \frac{u_1}{0.2}, \frac{u_2}{0.3} \right\}, \left\{ \frac{v_1}{0.6}, \frac{v_2}{0.4} \right\} \right) \right), \left(e_2, \left(\left\{ \frac{u_1}{0.6}, \frac{u_2}{0.4} \right\}, \left\{ \frac{v_1}{0.7}, \frac{v_2}{0.8} \right\} \right) \right), \right. \\ \left. \left(e_3, \left(\left\{ \frac{u_1}{0.5}, \frac{u_2}{0.4} \right\}, \left\{ \frac{v_1}{0.8}, \frac{v_2}{0.7} \right\} \right) \right) \right\}$$

$$F_{EA}(p_3) = \left\{ \left(e_1, \left(\left\{ \frac{u_1}{0.7}, \frac{u_2}{0.4} \right\}, \left\{ \frac{v_1}{0.8}, \frac{v_2}{0.3} \right\} \right) \right), \left(e_2, \left(\left\{ \frac{u_1}{0.4}, \frac{u_2}{0.5} \right\}, \left\{ \frac{v_1}{0.6}, \frac{v_2}{0.5} \right\} \right) \right), \right. \\ \left. \left(e_3, \left(\left\{ \frac{u_1}{0.7}, \frac{u_2}{0.6} \right\}, \left\{ \frac{v_1}{0.5}, \frac{v_2}{0.4} \right\} \right) \right) \right\}$$

$$F_{EB}(p_2) = \left\{ \left(e_1, \left(\left\{ \frac{u_1}{0.2}, \frac{u_2}{0.4} \right\}, \left\{ \frac{v_1}{0.3}, \frac{v_2}{0.7} \right\} \right) \right), \left(e_2, \left(\left\{ \frac{u_1}{0.4}, \frac{u_2}{0.2} \right\}, \left\{ \frac{v_1}{0.6}, \frac{v_2}{0.7} \right\} \right) \right), \right. \\ \left. \left(e_3, \left(\left\{ \frac{u_1}{0.2}, \frac{u_2}{0.4} \right\}, \left\{ \frac{v_1}{0.4}, \frac{v_2}{0.5} \right\} \right) \right) \right\}$$

$$F_{EB}(p_4) = \left\{ \left(e_1, \left(\left\{ \frac{u_1}{0.4}, \frac{u_2}{0.5} \right\}, \left\{ \frac{v_1}{0.6}, \frac{v_2}{0.5} \right\} \right) \right), \left(e_2, \left(\left\{ \frac{u_1}{0.4}, \frac{u_2}{0.8} \right\}, \left\{ \frac{v_1}{0.7}, \frac{v_2}{0.8} \right\} \right) \right), \right. \\ \left. \left(e_3, \left(\left\{ \frac{u_1}{0.6}, \frac{u_2}{0.4} \right\}, \left\{ \frac{v_1}{0.8}, \frac{v_2}{0.7} \right\} \right) \right) \right\}$$

The tabular representation of the example 3.2 is given in Table 1.

Definition 3.3. Let U_1 and U_2 be two initial universal sets. Let E and P be two parameters sets and $A, B \subseteq P$. Let (F^*, A) and (G^*) be two extended fuzzy binary soft sets (EFBSSs) over common universe U_1 and U_2 . Then, (G^*, B) is said to be extended fuzzy binary soft subset if

- (i) $B \subseteq A$
- (ii) $G_B^*(p) \subseteq F_A^*(p) \ \forall p \in B$ that is $G_{EB}(p) \subseteq F_{EA}(p) \ \forall p \in B$.

Example 3.4. Let $U_1 = \{u_1, u_2\}$, $U_2 = \{v_1, v_2\}$ be two initial universal sets. Let $E = \{e_1, e_2, e_3\}$ and $P = \{p_1, p_2, p_3, p_4\}$ be two parameter sets. Let $A = \{p_1, p_2\} \subseteq P$ and $B = \{p_1\} \subseteq P$. Let (F^*, A) and (G^*, B) be two EFBSSs over U_1, U_2 defined by

$$(F^*, A) = \{F_{EA}(p_i) / p_i \in A\}$$

$$(G^*, B) = \{G_{EB}(p_i) / p_i \in B\}$$

TABLE 1. Tabular representation of extended fuzzy binary soft set

$F_{EA}(p_1)$	u_1	u_2	v_1	v_2
e_1	0.2	0.3	0.6	0.4
e_2	0.6	0.4	0.7	0.8
e_3	0.5	0.4	0.8	0.7
$F_{EA}(p_3)$				
e_1	0.7	0.4	0.8	0.3
e_2	0.4	0.5	0.6	0.5
e_3	0.7	0.6	0.6	0.4
$F_{EB}(p_2)$				
e_1	0.2	0.4	0.3	0.7
e_2	0.4	0.2	0.6	0.7
e_3	0.2	0.4	0.4	0.5
$F_{EB}(p_4)$				
e_1	0.4	0.5	0.6	0.5
e_2	0.4	0.8	0.7	0.8
e_3	0.6	0.4	0.8	0.7

where

$$F_{EA}(p_1) = \left\{ \left(e_1, \left(\left\{ \frac{u_1}{0.9}, \frac{u_2}{0.8} \right\}, \left\{ \frac{v_1}{0.8}, \frac{v_2}{0.7} \right\} \right) \right), \left(e_2, \left(\left\{ \frac{u_1}{0.8}, \frac{u_2}{0.9} \right\}, \left\{ \frac{v_1}{0.9}, \frac{v_2}{0.7} \right\} \right) \right), \right. \\ \left. \left(e_3, \left(\left\{ \frac{u_1}{0.6}, \frac{u_2}{0.7} \right\}, \left\{ \frac{v_1}{0.8}, \frac{v_2}{0.7} \right\} \right) \right) \right\}$$

$$F_{EA}(p_2) = \left\{ \left(e_1, \left(\left\{ \frac{u_1}{0.3}, \frac{u_2}{0.4} \right\}, \left\{ \frac{v_1}{0.6}, \frac{v_2}{0.8} \right\} \right) \right), \left(e_2, \left(\left\{ \frac{u_1}{0.6}, \frac{u_2}{0.5} \right\}, \left\{ \frac{v_1}{0.7}, \frac{v_2}{0.8} \right\} \right) \right), \right. \\ \left. \left(e_3, \left(\left\{ \frac{u_1}{0.8}, \frac{u_2}{0.6} \right\}, \left\{ \frac{v_1}{0.8}, \frac{v_2}{0.5} \right\} \right) \right) \right\}$$

$$G_{EB}(p_1) = \left\{ \left(e_1, \left(\left\{ \frac{u_1}{0.8}, \frac{u_2}{0.7} \right\}, \left\{ \frac{v_1}{0.6}, \frac{v_2}{0.5} \right\} \right) \right), \left(e_2, \left(\left\{ \frac{u_1}{0.5}, \frac{u_2}{0.5} \right\}, \left\{ \frac{v_1}{0.8}, \frac{v_2}{0.3} \right\} \right) \right), \right. \\ \left. \left(e_3, \left(\left\{ \frac{u_1}{0.5}, \frac{u_2}{0.4} \right\}, \left\{ \frac{v_1}{0.6}, \frac{v_2}{0.5} \right\} \right) \right) \right\}$$

Proposition 3.5. Let U_1 and U_2 be two initial universal sets. Let E and P be two parameter sets and $A \subseteq P$. Let (F, E) be FBSS and (F^*, A) be EFBSS over common universe U_1, U_2 . Then $\{(F, E)\} \subseteq (F^*, A)$ if and only if $(F, E) \subseteq F_{EA}(p)$ for some $p \in P$.

Theorem 3.6. Let U_1, U_2 be two universal sets, E and P be two parameter sets and $A \subseteq P$. Let (F, E) be FBSS and (F^*, A) be EFBSS over common universe U_1, U_2 . Then $\{(F, E)\} = (F^*, A)$ if and only if $\{F_{EA}(p) = (F^*(p))\}$ and $(F, E) \in (F^*, A)$.

Proof. Suppose $\{(F, E)\} = (F^*, A)$. This implies $(F, E) \in (F^*, A)$ and A contain only one element precisely p .

Since $(F, E) \in (F^*, A)$ and A contain only one element, $\{F_{E_A}(p)\} = (F, E)$.

Hence, $\{F_{E_A}(p)\} = (F^*, A)$.

Conversely, If $\{F_{E_A}(p)\} = (F^*, A)$ then A contains only one element, precisely p .

Since $(F, E) \in (F^*, A)$ and A contain only one element $(F, E) = F_{E_A}(p)$.

Hence $\{F_{E_A}(p)\} = \{(F, E)\} = (F^*, A)$. \square

Definition 3.7. Union of two EFBSSs (F^*, A) and (G^*, B) over common universe U_1, U_2 is EFBSS $(H^*, C) = (F^*, A) \tilde{\cup} (G^*, B)$, $C = A \cup B$ and $\forall p \in C$

$$H^*(p) = H_{E_C}(p) = \begin{cases} F_{E_A}(p) & \text{if } p \in A - B \\ G_{E_A}(p) & \text{if } p \in B - A \\ F_{E_A}(p) \tilde{\cup} G_{E_B}(p) & \text{if } p \in A \cap B. \end{cases}$$

Example 3.8. Consider the example 3.4. Then $(H^*, C) = (F^*, A) \tilde{\cup} (G^*, B)$, where $C = \{p_1, p_2\}$,

$$H_{E_C}(p_1) = \left\{ \left(e_1, \left(\left\{ \frac{u_1}{0.9}, \frac{u_2}{0.8} \right\}, \left\{ \frac{v_1}{0.8}, \frac{v_2}{0.7} \right\} \right) \right), \left(e_2, \left(\left\{ \frac{u_1}{0.8}, \frac{u_2}{0.9} \right\}, \left\{ \frac{v_1}{0.9}, \frac{v_2}{0.7} \right\} \right) \right), \left(e_3, \left(\left\{ \frac{u_1}{0.6}, \frac{u_2}{0.7} \right\}, \left\{ \frac{v_1}{0.8}, \frac{v_2}{0.7} \right\} \right) \right) \right\}$$

$$H_{E_C}(p_2) = \left\{ \left(e_1, \left(\left\{ \frac{u_1}{0.3}, \frac{u_2}{0.4} \right\}, \left\{ \frac{v_1}{0.6}, \frac{v_2}{0.8} \right\} \right) \right), \left(e_2, \left(\left\{ \frac{u_1}{0.6}, \frac{u_2}{0.5} \right\}, \left\{ \frac{v_1}{0.7}, \frac{v_2}{0.8} \right\} \right) \right), \left(e_3, \left(\left\{ \frac{u_1}{0.8}, \frac{u_2}{0.6} \right\}, \left\{ \frac{v_1}{0.8}, \frac{v_2}{0.5} \right\} \right) \right) \right\}.$$

Definition 3.9. Intersection of two EFBSSs (F^*, A) and (G^*, B) over common universe U_1, U_2 is EFBSS $(K^*, D) = (F^*, A) \tilde{\cap} (G^*, B)$, $D = A \cap B$ and $\forall p \in D$

$$K^*(p) = K_{E_D}(p) = F_{E_A}(p) \tilde{\cap} G_{E_B}(p).$$

Example 3.10. Consider the example 3.4. Then $(K^*, D) = (F^*, A) \tilde{\cap} (G^*, B)$, where $D = \{p_1\}$,

$$H_{E_D}(p_1) = \left\{ \left(e_1, \left(\left\{ \frac{u_1}{0.8}, \frac{u_2}{0.7} \right\}, \left\{ \frac{v_1}{0.6}, \frac{v_2}{0.5} \right\} \right) \right), \left(e_2, \left(\left\{ \frac{u_1}{0.5}, \frac{u_2}{0.5} \right\}, \left\{ \frac{v_1}{0.8}, \frac{v_2}{0.3} \right\} \right) \right), \left(e_3, \left(\left\{ \frac{u_1}{0.5}, \frac{u_2}{0.4} \right\}, \left\{ \frac{v_1}{0.6}, \frac{v_2}{0.5} \right\} \right) \right) \right\}.$$

Proposition 3.11. Let $(F^*, A), (G^*, B)$ and (H^*, C) be three EFBSSs over common universe U_1, U_2 . Then

- (i) $(F^*, A) \tilde{\cup} (F^*, A) = (F^*, A)$
- (ii) $(F^*, A) \tilde{\cup} (G^*, B) = (G^*, B) \tilde{\cup} (F^*, A)$
- (iii) $(F^*, A) \tilde{\cup} [(G^*, B) \tilde{\cup} (H^*, C)] = (F^*, A) \tilde{\cup} [(G^*, B) \tilde{\cup} (H^*, C)]$
- (iv) $(F^*, A) \tilde{\cup} (G^*, B) = (F^*, A)$ if and only if $(G^*, B) \subseteq (F^*, A)$
- (v) $(F^*, A) \tilde{\cap} (F^*, A) = (F^*, A)$
- (vi) $(F^*, A) \tilde{\cap} (G^*, B) = (G^*, B) \tilde{\cap} (F^*, A)$

(vii) $(F^*, A) \tilde{\cap} [(G^*, B) \tilde{\cap} (H^*, C)] = (F^*, A) \tilde{\cap} [(G^*, B) \tilde{\cap} (H^*, C)]$
(viii) $(F^*, A) \tilde{\cap} (G^*, B) = (F^*, A)$ if and only if $(F^*, A) \subseteq (G^*, B)$.

Definition 3.12. The cartesian “AND” product of two extended fuzzy binary soft sets F_A^* and F_B^* over common universal sets U_1, U_2 denoted by $H_C^* = F_A^* \wedge F_B^*$, defined as $H_C^* : A \times B \rightarrow I^*$ and $H_C^*(a, b) = F_{EA}(a) \wedge F_{EB}(b)$, where $(a, b) \in A \times B$.

Example 3.13. Consider the example 3.2. The AND product on F_A^* and F_B^* is given by,

$$\begin{aligned} H_C^* &= F_A^* \wedge F_B^* \\ &= \{(p_1, p_2) (F_{EA}(p_1) \wedge F_{EB}(p_2))\}, \{(p_1, p_4) (F_{EA}(p_1) \wedge F_{EB}(p_4))\} \\ &\quad \{(p_3, p_2) (F_{EA}(p_3) \wedge F_{EB}(p_2))\}, \{(p_3, p_4) (F_{EA}(p_3) \wedge F_{EB}(p_4))\}. \end{aligned}$$

The extended fuzzy binary soft set H_C^* is given in Table 2.

TABLE 2. Extended fuzzy binary soft set H_C^*

(p_1, p_2)	u_1	u_2	v_1	v_2	(p_1, p_4)	u_1	u_2	v_1	v_2
(e_1, e_1)	0.2	0.3	0.3	0.4		0.2	0.3	0.3	0.4
(e_1, e_2)	0.2	0.2	0.6	0.4		0.2	0.2	0.6	0.4
(e_1, e_3)	0.2	0.3	0.4	0.4		0.2	0.3	0.6	0.4
(e_2, e_1)	0.2	0.4	0.3	0.7		0.4	0.4	0.6	0.5
(e_2, e_2)	0.4	0.2	0.6	0.7		0.4	0.4	0.7	0.8
(e_2, e_3)	0.2	0.4	0.4	0.5		0.6	0.4	0.7	0.7
(e_3, e_1)	0.2	0.4	0.3	0.7		0.4	0.4	0.6	0.5
(e_3, e_2)	0.4	0.2	0.6	0.7		0.4	0.4	0.7	0.7
(e_3, e_3)	0.2	0.4	0.4	0.5		0.5	0.4	0.8	0.7
(p_3, p_2)	u_1	u_2	v_1	v_2	(p_3, p_4)	u_1	u_2	v_1	v_2
(e_1, e_1)	0.2	0.4	0.3	0.3		0.4	0.4	0.6	0.3
(e_1, e_2)	0.4	0.2	0.6	0.3		0.4	0.4	0.7	0.3
(e_1, e_3)	0.2	0.4	0.4	0.3		0.6	0.4	0.8	0.3
(e_2, e_1)	0.2	0.4	0.3	0.5		0.4	0.5	0.6	0.5
(e_2, e_2)	0.4	0.2	0.6	0.5		0.4	0.5	0.6	0.5
(e_2, e_3)	0.2	0.4	0.4	0.5		0.4	0.4	0.6	0.5
(e_3, e_1)	0.2	0.4	0.3	0.4		0.4	0.5	0.5	0.4
(e_3, e_2)	0.4	0.2	0.5	0.4		0.4	0.6	0.5	0.4
(e_3, e_3)	0.2	0.4	0.4	0.4		0.6	0.4	0.5	0.4

Definition 3.14. The cartesian “OR” product of two extended fuzzy binary soft sets F_A^* and F_B^* over common universal sets U_1, U_2 denoted by $K_D^* = F_A^* \vee F_B^*$, defined as $K_D^* : A \times B \rightarrow I^*$ and $K_D^*(a, b) = F_{EA}(a) \vee F_{EB}(b)$, where $(a, b) \in A \times B$.

Example 3.15. Consider the example 3.2. The AND product on F_A^* and F_B^* is given by,

$$\begin{aligned} K_D^* &= F_A^* \vee F_B^* \\ &= \{\{(p_1, p_2) (F_{EA}(p_1) \vee F_{EB}(p_2))\}, \{(p_1, p_4) (F_{EA}(p_1) \vee F_{EB}(p_4))\} \\ &\quad \{(p_3, p_2) (F_{EA}(p_3) \vee F_{EB}(p_2))\}, \{(p_3, p_4) (F_{EA}(p_3) \vee F_{EB}(p_4))\}\}. \end{aligned}$$

The extended fuzzy binary soft set H_C^* is given in Table 3.

TABLE 3. Extended fuzzy binary soft set K_D^*

(p_1, p_2)	u_1	u_2	v_1	v_2	(p_1, p_4)	u_1	u_2	v_1	v_2
(e_1, e_1)	0.2	0.4	0.6	0.7		0.4	0.5	0.6	0.5
(e_1, e_2)	0.4	0.3	0.6	0.7		0.4	0.8	0.7	0.8
(e_1, e_3)	0.2	0.4	0.6	0.5		0.6	0.4	0.8	0.7
(e_2, e_1)	0.6	0.4	0.7	0.8		0.6	0.5	0.7	0.8
(e_2, e_2)	0.6	0.4	0.7	0.8		0.6	0.8	0.7	0.8
(e_2, e_3)	0.6	0.4	0.7	0.8		0.6	0.4	0.7	0.8
(e_3, e_1)	0.5	0.4	0.8	0.7		0.5	0.5	0.8	0.7
(e_3, e_2)	0.5	0.4	0.8	0.7		0.5	0.8	0.8	0.8
(e_3, e_3)	0.5	0.4	0.8	0.7		0.6	0.4	0.8	0.7
(p_3, p_2)	u_1	u_2	v_1	v_2	(p_3, p_4)	u_1	u_2	v_1	v_2
(e_1, e_1)	0.7	0.4	0.8	0.7		0.7	0.5	0.8	0.5
(e_1, e_2)	0.7	0.4	0.8	0.7		0.7	0.8	0.8	0.8
(e_1, e_3)	0.7	0.4	0.8	0.5		0.7	0.4	0.8	0.7
(e_2, e_1)	0.4	0.5	0.6	0.7		0.4	0.5	0.6	0.5
(e_2, e_2)	0.4	0.5	0.6	0.7		0.4	0.8	0.7	0.8
(e_2, e_3)	0.4	0.5	0.6	0.5		0.6	0.5	0.8	0.7
(e_3, e_1)	0.7	0.6	0.5	0.7		0.7	0.6	0.6	0.5
(e_3, e_2)	0.7	0.6	0.6	0.7		0.7	0.8	0.7	0.8
(e_3, e_3)	0.7	0.6	0.5	0.5		0.7	0.4	0.8	0.7

Definition 3.16. The MaxMin operator on “AND” products of two extended fuzzy binary soft sets F_A^* and F_B^* is given by

$$Max_a Min_b [H_C^*(a, b)] = \bigvee_{a \in A} \left\{ \bigwedge_{b \in B} (F_A^*(a) \wedge F_B^*(b)) \right\}$$

$$Max_b Min_a [H_C^*(a, b)] = \bigvee_{b \in B} \left\{ \bigwedge_{a \in A} (F_A^*(a) \wedge F_B^*(b)) \right\}.$$

Example 3.17. Consider the example 3.13. The MaxMin operator is given by,

$$\begin{aligned} \text{Max}_a \text{Min}_b [H_C^*(a, b)] &= \bigvee_{a \in A} \left\{ \bigwedge_{b \in B} (F_A^*(a) \wedge F_B^*(b)) \right\} \\ F_C &= \bigvee \{ \wedge \{ F_{EA}(p_1) \wedge F_{EB}(p_2), F_{EA}(p_1) \wedge F_{EB}(p_4) \} \\ &\quad \wedge \{ F_{EA}(p_3) \wedge F_{EB}(p_2), F_{EA}(p_3) \wedge F_{EB}(p_4) \} \}. \end{aligned}$$

The extended fuzzy binary soft set F_C is given in Table 4.

$$\begin{aligned} \text{Max}_b \text{Min}_a [H_C^*(a, b)] &= \bigvee_{b \in B} \left\{ \bigwedge_{a \in A} (F_A^*(a) \wedge F_B^*(b)) \right\} \\ F_D &= \bigvee \{ \wedge \{ F_{EA}(p_1) \wedge F_{EB}(p_2), F_{EA}(p_3) \wedge F_{EB}(p_2) \} \\ &\quad \wedge \{ F_{EA}(p_1) \wedge F_{EB}(p_4), F_{EA}(p_3) \wedge F_{EB}(p_4) \} \} \} \end{aligned}$$

The extended fuzzy binary soft set F_D is given in Table 5.

TABLE 4. Extended fuzzy binary soft set F_C

F_C	u_1	u_2	v_1	v_2
(e_1, e_1)	0.2	0.4	0.3	0.4
(e_1, e_2)	0.4	0.2	0.6	0.4
(e_1, e_3)	0.2	0.2	0.4	0.4
(e_2, e_1)	0.2	0.4	0.3	0.5
(e_2, e_2)	0.4	0.2	0.6	0.7
(e_2, e_3)	0.2	0.4	0.4	0.5
(e_3, e_1)	0.2	0.4	0.3	0.5
(e_3, e_2)	0.4	0.2	0.6	0.7
(e_3, e_3)	0.2	0.4	0.4	0.5

TABLE 5. Extended fuzzy binary soft set F_D

F_D	u_1	u_2	v_{-1}	v_{-2}
(e_1, e_1)	0.2	0.3	0.3	0.3
(e_1, e_2)	0.2	0.2	0.6	0.3
(e_1, e_3)	0.2	0.3	0.6	0.3
(e_2, e_1)	0.4	0.4	0.6	0.5
(e_2, e_2)	0.4	0.4	0.6	0.5
(e_2, e_3)	0.4	0.4	0.6	0.5
(e_3, e_1)	0.4	0.4	0.5	0.4
(e_3, e_2)	0.4	0.4	0.5	0.4
(e_3, e_3)	0.5	0.4	0.5	0.4

4. APPLICATION IN DECISION MAKING PROBLEMS

Suppose Mr. V wants to choose the best course in the best college. For this, he has a choice of four colleges and four courses. Let $U_1 = \{u_1, u_2, u_3, u_4\}$ be the set of colleges and $U_2 = \{v_1, v_2, v_3, v_4\}$ be the set of courses to be selected with respect to some parameters $E = \{e_1, e_2, e_3, e_4\}$. For this, he has hired two pairs of experts say (p_1, p_3) and (p_2, p_4) . Consider the parameter set P as experts, $P = \{p_1, p_2, p_3, p_4\}$. The experts assigned the scores between 0 to 100 given in Table 6. The data is converted on the scale of 0 to 1. This section provides an algorithm to solve MCDM problem and illustrated with example.

TABLE 6. The scores assigned by experts

p_1	u_1	u_2	u_3	u_4	v_1	v_2	v_3	v_4
e_1	94	94	87	86	66	74	58	72
e_2	37	50	31	38	40	100	96	96
e_3	73	89	57	56	49	95	54	63
e_4	96	78	73	41	36	85	56	43
p_2	u_1	u_2	u_3	u_4	v_1	v_2	v_3	v_4
e_1	95	57	32	73	78	43	91	93
e_2	70	60	91	74	90	48	38	63
e_3	69	76	34	93	60	70	72	96
e_4	66	100	86	56	56	67	51	94
p_3	u_1	u_2	u_3	u_4	v_1	v_2	v_3	v_4
e_1	77	80	37	44	88	61	72	60
e_2	65	71	85	59	74	53	42	99
e_3	91	48	63	92	47	83	87	52
e_4	39	56	76	68	95	66	81	90
p_4	u_1	u_2	u_3	u_4	v_1	v_2	v_3	v_4
e_1	38	67	57	82	45	93	58	73
e_2	64	86	41	96	46	89	62	50
e_3	84	75	55	43	94	40	69	98
e_4	51	79	97	78	84	69	49	70

4.1. Algorithm to solve MCDM problems:

Since, the problems involves two universal sets and more than one parameter set, there is a need of systematic method to combine all the alternative regarding to each parameter to make a decision. In this subsection, an algorithm to solve the MCDM problem is defined and its implication is discussed.

Step 1: Input extended fuzzy binary soft sets F_A^* and F_B^* .
 Step 2: Apply “AND” operator between F_A^* and F_B^* to obtain H_C^* .
 Step 3: Apply “AND” operator on expanded matrices of extended fuzzy binary soft set H_C^* to obtain P^* .
 Step 4: Apply MaxMin operator on P^* to get two extended fuzzy binary soft sets F_C and F_D .
 Step 5: Apply MaxMin operator between F_C and F_D to get four fuzzy sets.
 Step 6: Find the extended resultant matrix.
 Step 7: Find row sum of all the rows.
 Step 8: Highest row sum is given rank 1.

Implementation of algorithm

Step 1: Based on the scores of the experts (Table 6), let F_A^* and F_B^* be extended fuzzy binary soft sets given by

$$F_A^* = \{(p_1, F_{EA}(p_1)), (p_3, F_{EA}(p_3))\}$$

$$F_B^* = \{(p_2, F_{EB}(p_2)), (p_4, F_{EB}(p_4))\}$$

which are shown in Table 7.

Applying step 2 to step 6, the extended resultant matrix is obtained which is given in the table 8.

Step 7: Calculate the row sum (Table 9).

Step 8: rank them accordingly.

Table 7: Extended fuzzy binary soft sets F_A^* and F_B^*

$F_{EA}(p_1)$	u_1	u_2	u_3	u_4	v_1	v_2	v_3	v_4
e_1	0.94	0.94	0.87	0.86	0.66	0.74	0.58	0.72
e_2	0.37	0.50	0.31	0.38	0.40	1	0.96	0.96
e_3	0.73	0.89	0.57	0.56	0.49	0.95	0.54	0.63
e_4	0.96	0.78	0.73	0.41	0.36	0.85	0.56	0.43
$F_{EA}(p_3)$	u_1	u_2	u_3	u_4	v_1	v_2	v_3	v_4
e_1	0.77	0.80	0.37	0.44	0.88	0.61	0.72	0.60
e_2	0.65	0.71	0.85	0.59	0.74	0.53	0.42	0.99
e_3	0.91	0.48	0.63	0.92	0.47	0.83	0.87	0.52
e_4	0.39	0.56	0.76	0.68	0.95	0.66	0.81	0.90
$F_{EA}(p_2)$	u_1	u_2	u_3	u_4	v_1	v_2	v_3	v_4
e_1	0.95	0.57	0.32	0.73	0.78	0.43	0.91	0.93
e_2	0.70	0.60	0.91	0.74	0.90	0.48	0.38	0.63
e_3	0.69	0.76	0.34	0.93	0.60	0.70	0.72	0.96
e_4	0.66	1	0.86	0.56	0.56	0.67	0.51	0.94
$F_{EA}(p_4)$	u_1	u_2	u_3	u_4	v_1	v_2	v_3	v_4

Table 7 – continued

	u_1	u_2	u_3	u_4	v_1	v_2	v_3	v_4
e_1	0.38	0.67	0.57	0.82	0.45	0.93	0.58	0.73
e_2	0.64	0.86	0.41	0.96	0.46	0.89	0.62	0.50
e_3	0.84	0.75	0.55	0.43	0.94	0.40	0.69	0.98
e_4	0.51	0.79	0.97	0.78	0.84	0.69	0.49	0.70

TABLE 8. Extended resultant matrix

	$(u_1, v_1)(u_1, v_2)(u_1, v_3)(u_1, v_4)(u_2, v_1)(u_2, v_2)(u_2, v_3)(u_2, v_4)(u_3, v_1)(u_3, v_2)(u_3, v_3)(u_3, v_4)(u_4, v_1)(u_4, v_2)(u_4, v_3)(u_4, v_4)$
$(u_1, v_1)0$	-0.02 0.1 -0.09 -0.22 -0.09 -0.14 -0.44 0.19 0.15 0.25 0.06 -0.08 -0.1 -0.04 -0.17
$(u_1, v_2)0.02$	0 0.12 -0.07 -0.2 -0.07 -0.12 -0.42 0.21 0.17 0.27 0.08 -0.06 -0.08 -0.02 -0.15
$(u_1, v_3)-0.1$	-0.12 0 -0.19 -0.32 -0.19 -0.24 -0.54 0.09 0.05 0.15 -0.04 -0.18 -0.2 -0.14 -0.27
$(u_1, v_4)0.09$	0.07 0.19 0 -0.13 0 -0.05 -0.35 0.28 0.24 0.34 0.15 0.01 -0.01 0.05 -0.08
$(u_2, v_1)0.22$	0.2 0.32 0.13 0 0.13 0.08 -0.22 0.41 0.37 0.47 0.28 0.14 0.12 0.18 0.05
$(u_2, v_2)0.09$	0.07 0.19 0 -0.13 0 -0.05 -0.35 0.28 0.24 0.34 0.15 0.01 -0.01 0.05 -0.08
$(u_2, v_3)0.14$	0.12 0.24 0.05 -0.08 0.05 0 -0.3 0.33 0.29 0.39 0.2 0.06 0.04 0.1 -0.03
$(u_2, v_4)0.44$	0.42 0.54 0.35 0.22 0.35 0.3 0 0.63 0.59 0.69 0.5 0.36 0.34 0.4 0.27
$(u_3, v_1)-0.19$	-0.21 -0.09 -0.28 -0.41 -0.28 -0.33 -0.63 0 0 -0.04 0.06 -0.13 -0.27 -0.29 -0.23 -0.36
$(u_3, v_2)-0.15$	-0.17 -0.05 -0.24 -0.37 -0.24 -0.29 -0.59 0.04 0 0.1 -0.09 -0.23 -0.25 -0.19 -0.32
$(u_3, v_3)-0.25$	-0.27 -0.15 -0.34 -0.47 -0.34 -0.39 -0.69 -0.06 -0.1 0 -0.19 -0.33 -0.35 -0.29 -0.42
$(u_3, v_4)-0.06$	-0.08 0.04 -0.15 -0.28 -0.15 -0.2 -0.5 0.13 0.09 0.19 0 -0.14 -0.16 -0.1 -0.23
$(u_4, v_1)0.08$	0.06 0.18 -0.01 -0.14 -0.01 -0.06 -0.36 0.27 0.23 0.33 0.14 0 -0.02 0.04 -0.09
$(u_4, v_2)0.1$	0.08 0.2 0.01 -0.12 0.01 -0.04 -0.34 0.29 0.25 0.35 0.16 0.02 0 0.06 -0.07
$(u_4, v_3)0.04$	0.02 0.14 -0.05 -0.18 -0.05 -0.1 -0.4 0.23 0.19 0.29 0.1 -0.04 -0.06 0 -0.13
$(u_4, v_4)0.17$	0.15 0.27 0.08 -0.05 0.08 0.03 -0.27 0.36 0.32 0.42 0.23 0.09 0.07 0.13 0

TABLE 9. Row sum of extended resultant matrix

College-Course Pair	Row sum	College-Course Pair	Row sum
(u_1, v_1)	-0.64	(u_3, v_1)	-3.68
(u_1, v_2)	-0.32	(u_3, v_2)	-3.04
(u_1, v_3)	-2.24	(u_3, v_3)	-4.64
(u_1, v_4)	0.8	(u_3, v_4)	-1.6
(u_2, v_1)	2.88	(u_4, v_1)	0.64
(u_2, v_2)	0.8	(u_4, v_2)	0.96
(u_2, v_3)	1.6	(u_4, v_3)	0
(u_2, v_4)	6.4	(u_4, v_4)	2.08

5. RESULT AND DISCUSSION

The performance of the algorithm is illustrated with an example of ranking college-course combination based on the reports by experts (considered as set of second parameters) considering four parameters. The ranking strategy is based on the value of row sum as given in Table 9. The pair with highest row sum is ranked 1. For the example under discussion, the pair (u_2, v_4) got highest score with value 6.4, indicating that college u_2 has the best performance in course v_4 . Also, u_2 has highest score across multiple courses v_1, v_2, v_3 and v_4 . This shows, for any course, the college u_2 is the best choice. Further, v_4 is the best course in all the colleges. Hence, college u_2 is the most preferred college in overall with strong performance across multiple courses, where as u_3 appears to be least preferred. Similarly, v_4 is the most preferred course and v_3 is the least preferred course. The differences in the scores between the college-course combination suggests that students may have different preferences and priorities when choosing a college and course. Some may prioritize specific course, while others may focus on the overall performance of the college. This article provide the solution for all the situations.

6. CONCLUSION

This article aims to solve MCDM problem involving multiparameter sets by extending the definition of fuzzy binary soft sets for two parameter sets. “AND” operator and MaxMin operators are defined on extended fuzzy binary soft sets. An algorithm to solve decision making problems was presented. The algorithm is illustrated with an example, providing the solution to choosing the college-course combination. This MCDM approach provides a quantitative way to evaluate and compare the performance of college-course combinations. This approach can be

extended for more than two parameter sets and can be improved by considering additional factors, such as cost, availability and some personal preference while making final decision.

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