Premiums of Deposit Insurance with Maximum Limit Under the Black-Scholes Model

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Abstract. Deposit insurance is an important mechanism in protecting bank customers from the risk of bankruptcy and providing a sense of security for their savings. Under deposit insurance, customers will still receive a refund of their funds up to a certain limit determined by the deposit insurance agency. This research aims to construct a formula to calculate the premium of deposit insurance with a given upper claim limit. To the best of the authors' knowledge, this article is the first study that gives a formula for deposit insurance premiums with a coverage limit. First, we present a theorem that shows the equivalence between the claim of deposit insurance with coverage limit with the payoff of two put options. Secondly, under the assumption that the asset follows Geometric Brownian Motion, we determine the fair price of the premium of deposit insurance. The main research findings indicate that the sum of two Black-Scholes options with different strike prices can be used to determine the premium value of deposit insurance while considering the applied coverage limits. Finally, we simulate some sensitivity analysis to gain a deeper understanding on the impact of several important variables on the magnitude of premium. Based on sensitivity analysis, it is found that the premium value is inversely proportional to interest rates and directly proportional to asset price volatility.

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1. INTRODUCTION

Banks play a crucial role in the economy, gathering funds from the public and channeling them as loans to borrowers. Despite this strategic role, banks also deal with various risks that can threaten their stability. Two primary risks are liquidity risk, where banks are unable to meet large withdrawal requests, and credit risk, where borrowers fail to repay loans. Alongside operational and market risks, these can lead to bank bankruptcy, causing significant financial losses for customers.

As a protection mechanism for customers, the concept of deposit insurance emerged. When a bank goes bankrupt, deposit insurance ensures that customers still receive their funds up to a certain limit set by the deposit insurance agency. In the United States, deposit insurance was first introduced in 1933 by the Federal Deposit Insurance Corporation (FDIC), as a response to the banking crisis during the Great Depression [1]. According to Kadir [2], insurance is a legal agreement where the insurer promises compensation for losses under agreed conditions. Within the deposit insurance scheme, customers feel secure knowing their funds are safeguarded even in unfavorable situations like a bank's bankruptcy.

The deposit insurance scheme involves banks or financial institutions paying premiums to the deposit insurance agency. Through these premiums, banks become participants in the deposit insurance program, gaining financial protection for their customers. The determination of these premiums is crucial, and the Black-Scholes method is used to assess the cost of deposit insurance premiums [3]. Merton's research [4] connected European put options with deposit insurance, later expanded upon by Marcus and Shaked [1] by considering dividends. This research evolved further through the work of Ronn and Verma [5] and Zhang and Shi [6], who integrated systematic risk into the Merton model. Chiang and Tsai [7] proposed a model by incorporating the interest rate spread (defined as the difference between the lending rate and the borrowing rate) and the risk of a bank's early bankruptcy in the derivation of a closed-form pricing formula for calculating deposit insurance premiums. Wu et al. [8] constructs an explicit deposit insurance scheme and derives a closed form pricing formula for fair premiums that comprises three components: early closure, capital forbearance, and a grace period. Next, Chang et al. [9] explored Ronn and Verma's model by considering depositor preference laws and contingent capital bonds (CoCos) which gave the result that the issuance of CoCos can reduce the insolvency risks of insured banks, whereas depositor preference laws can lower the deposit insurer's resolution costs in liquidation. Lee et al. [10] introduces new types of exchange options, i.e., early exchange options and barrier exchange options which allow multi-step boundaries to deal with the insurance guaranty funds' payoff structure.

A crucial aspect of the deposit insurance scheme is the presence of limits that regulate the amount of protection provided by the insurance agency. These limits influence the size of the insurance premiums that banks or financial institutions must pay. The limit of the insurance contract, indeed will reduce the premium of the deposit insurance. This research extends the Merton formula with an additional maximum claim limit. To the best of authors' knowledge, there is no literature that takes into account the variable of maximum claim limit in the premium calculation of deposit insurance.

In Section 2, we explain the definition of deposit insurance and some background knowledge on option pricing which help to determine the pricing formula. The main result of the pricing formula to calculate the premium of deposit insurance with maximum claim is presented in Subsection 2.5. Section 3 shows some simulation results and sensitivity analysis of some variables to the premium price based on the proposed model. Finally, Section 4 concludes the research.

2. Construction of the Premium of Deposit Insurance with Maximum Limit

2.1. Deposit Insurance.

Insurance is a reciprocal agreement between the insurer and the insured, binding them to compensate for agreed-upon losses at the time of agreement closure, in the event of an uncertain occurrence, with the insured party paying a premium. According to Subekti [11], insurance is an agreement in which the guarantor promises the insured party to receive a premium payment as compensation for losses suffered by the insured party, due to the effects of an event that has not yet occurred. Generally, insurance can be described as a form of assurance or compensation for losses provided by the insurer to the insured party if there is a risk or uncertainty of future losses.

Deposit insurance is a form of protection offered to bank customers for their stored funds. Banks play a pivotal role in collecting and distributing funds within society, contributing to economic stability and public trust. Recognizing the need to minimize banking risks, many countries provide tangible assurance to customers. This assurance aims to maintain stable banking operations, ensure sufficient lending services, sustain economic stability, and prevent excessive risks.

Direct protection for customers comes in the form of deposit insurance agencies, like the Federal Deposit Insurance Corporation (FDIC) in the United States, which offers insurance coverage up to a certain limit, currently USD250,000 per depositor per bank, in case of bank failure. FDIC instills confidence in customers that their funds are secure, mitigating the impact of bank failures and upholding trust in the banking sector.

Indonesia's Deposit Insurance Corporation (LPS) provides protection for bank customers' deposits within its member banks, with coverage up to a set maximum limit, currently at IDR 2 billion per depositor per bank. LPS compensates depositors in the event of bank failure, ensuring each deposit is safeguarded within the stipulated maximum. The purpose of LPS and deposit insurance in general is to build confidence in the banking system and provide security for customers. Deposit insurance assures customers that their savings are protected, even in the face of bank failures. LPS's vital role in financial stability and trust-building in the banking sector becomes evident, especially with the establishment of a maximum coverage limit that provides extensive protection for the majority of depositors, minimizing the financial impact of bank failures.

2.2. Options.

Options are contracts that grant the buyer the right to buy or sell shares at a predetermined price and time. They are divided into American options and European options, each with two types of contracts: call options and put options. Merton [4] calculates the premium of deposit insurance following the fair price of European Put Options.

In European put options, the following notations are used: S_T for the stock price at time T, K for the strike price, and T for the maturity date. The intrinsic value (payoff) of a European put option is the maximum of $K - S_T$ and 0. If $S_T < K$, the option will be executed and is considered to be in the money with a profit of $K - S_T$. However, if $S_T \ge K$, the option will not be executed and is considered to be out of the money. In the first case, the owner of the European put option will gain a profit, while in the second case, they will not experience any gain or loss. On the other hand, the option writer potentially faces significant losses. To address this imbalance, the option buyer is required to pay a premium to the option writer when the option agreement is made. Understanding European put options serves as a crucial foundation for determining deposit insurance.

The following is the formula used to calculate a European put option:

$$P(S,t) = Ke^{-r(T-t)}N(x_2) - S_t N(x_1)$$
(1)

with

$$\begin{split} x_1 &\equiv \frac{\left\{ \log\left(\frac{K}{S_t}\right) - \left(r + \frac{\sigma^2}{2}\right)(T-t) \right\}}{\sigma\sqrt{T-t}} \\ x_2 &\equiv x_1 + \sigma\sqrt{T-t}. \end{split}$$

 ${\cal N}$ represents the cumulative distribution function of the standard normal distribution

2.3. Deposit Insurance and Put Options.

The interconnection between put options and deposit insurance lies in their shared goal of financial risk protection [12]. Despite operating in different contexts, both aim to safeguard value or assets from financial losses or declines. Put options are financial instruments granting holders the right to sell assets, such as stocks, at a specific price (strike price) within a designated time frame. The purpose of put options is to guard against future asset value decreases.

There are three potential conditions regarding the execution of put options. Firstly, if the stock price in the market is higher than the strike price, the put owner won't exercise his right. In this scenario, he could sell it on the open market at a higher price. Secondly, if the stock price in the stock market is equal to the strike price, the put owner can decide whether or not to exercise his right. Thirdly, if the stock price was lower than the strike price, then the put owner would exercise his right, and the value of the put option would be the difference between the strike price and the stock price.

On the other hand, deposit insurance is a protective mechanism offered by deposit guarantee institutions or financial organizations. Customers pay premiums to protect their deposits from potential bank or financial institution bankruptcies. The deposit guarantee institution provides reimbursement within the defined protection limit in case of bankruptcy. In this context, the strike price of put options corresponds to the obligations the bank owes to customers, while the stock price relates to the asset value. When the asset value V_T falls below the bank's obligation B to customers at maturity T, the deposit guarantee institution steps in to protect the bank's ability to fulfill its obligations.

The same principle is applied to determining payouts in European put options and deposit insurance. The payout value of deposit insurance is the maximum between $B - V_T$ and 0. If $V_T < B$, the deposit insurance is considered in the money, and the insurer pays the difference between B and V_T . However, if $V_T \ge B$, the deposit insurance is out of the money and remains inactive. In the first case, the bank receives protection from the insurer, whereas in the second case, the bank does not receive protection. The insurer faces the risk of significant losses. To counter this imbalance, the bank pays premiums to the insurer.

In the context of deposit insurance, banks pay premiums to deposit guarantee institutions in exchange for the provided protection. These premiums also serve as reserves to handle customer claims in case of bank bankruptcy. The premiums reflect the risk assessment conducted by deposit guarantee institutions. By paying premiums, banks ensure protection for their customers, providing them with a sense of security and certainty about the safety of their deposits.

2.4. The analytical solution of deposit insurance pricing model.

Using put option concepts, Merton [4] shows the analytical solutions for deposit insurance as follows:

Let $H(V_t, t)$ be the value of the deposit insurance at time t with asset value V_t

$$H(V_T, T) = (B - V_T)^+ = \max\{0, B - V_T\}$$

$$H(V_t, t) = Be^{-r(T-t)}N(x_2) - V_tN(x_1)$$
(2)

with

$$x_1 = \frac{\ln \frac{B}{V_t} - \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$
$$x_2 = x_1 + \sigma\sqrt{T-t}.$$

 $N(\cdot)$ represents the cumulative distribution function of the standard normal distribution.

Annotation:

 $H(V_T, T)$: deposit Insurance values B: the amount of the bank's liability V_T : asset value at maturity date T: maturity date r: interest rate σ^2 : variance.

2.5. Determination of Deposit Insurance Premiums with Limits.

In case there is a limit or restriction on the coverage amount, then the payoff or total claim amount of deposit insurance can be set as the following Theorem.

Theorem 2.1. Let B represent the liability that the bank must pay to the customer, V_T denote the asset value at maturity, Suppose L represents the acceptable coverage limit, and by subtracting B - L from B_0 , we obtain an equation: $B - B_0 = L$. This represents the insurance equation with the limit on the claim amount. Then the value of the insurance equation with the limit on the number of claims is

$$F(T) = \begin{cases} 0 & if \quad V_T \ge B\\ B - V_T & if \quad B_0 \le V_T < B\\ B - B_0 & if \quad V_T < B_0 \end{cases}$$
(3)

Equivalent to the following equation

$$G(T) = G_1(T) - G_2(T)$$
(4)

where

$$G_1(T) = \begin{cases} B - V_T & \text{if } B > V_T \\ 0 & \text{if } B \le V_T \end{cases}$$
(5)

$$G_2(T) = \begin{cases} 0 & \text{if } V_T \ge B_0 \\ B_0 - V_T & \text{if } V_T < B_0 \end{cases}.$$
 (6)

PROOF. To prove the theorem, it will be demonstrated that under any condition, the theorem holds as follows: G(T) = F(T)

- If $V_T \geq B$,
- $G_1(T) = 0$ and $G_2(T) = 0$ therefore G(T) = 0 = F(T).
- If $B_0 \leq V_T < B$,
- $G_1(T) = B V_T$ and $G_2(T) = 0$ therefore $G(T) = B V_T = F(T)$. • If $V_T < B_0$, since $B_0 < B$, then
- $G_1(T) = B V_T$ and $G_2(T) = B_0 V_T$ therefore $G(T) = B B_0 = F(T)$.

So, the analytical solution for determining deposit insurance premiums with a protection limit is

$$G(T) = G_1(T) - G_2(T)$$
(7)

$$G(V_t, t) = Be^{-r(T-t)}N(x_2) - V_tN(x_1) - B_0e^{-r(T-t)}N(x_4) + V_tN(x_3)$$
(8)

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where

$$x_{1} = \frac{\ln \frac{B}{V_{t}} - \left(r + \frac{1}{2}\sigma^{2}\right)(T - t)}{\sigma\sqrt{T - t}}$$

$$x_{2} = x_{1} + \sigma\sqrt{T - t}$$

$$x_{3} = \frac{\ln \frac{B_{0}}{V_{t}} - \left(r + \frac{1}{2}\sigma^{2}\right)(T - t)}{\sigma\sqrt{T - t}}$$

$$x_{4} = x_{3} + \sigma\sqrt{T - t}.$$

 $N(\cdot)$ represents the cumulative distribution function of the standard normal distribution.

3. SIMULATION AND MODEL ANALYSIS

This model simulation was conducted using MATLAB software. There are several inputs used in the simulation including the value of the liability B = 2000, $B_0 = 1000$ (for models with limits), the interest rate r = 0.0575, volatility sigma = 0.3, and t = 0 to T = 1 (years).



FIGURE 1. Deposit Insurance Premium

The simulation results in Figure 1 indicate a relationship between the asset value and the premium amount in deposit insurance. As the asset value increases, the premium tends to approach zero, while at lower asset values, the premium becomes higher. In this context, there is an inverse relationship between the premium to be paid by the bank and the asset value it holds. The simulation also demonstrates that the larger the asset value, the smaller the premium amount the

bank needs to pay. Conversely, as the asset value decreases, the required premium payment increases.

In determining the insurance premium for deposit insurance with a specified coverage limit, there exists a particular value that constrains the premium amount to a specific threshold as shown in Figure 2. Referencing the provided illustration depicting the computation of deposit insurance premiums under a prescribed coverage limit, the simulation also demonstrates that as the asset value increases, the required premium decreases for the bank. Conversely, as the asset value diminishes, the premium payment escalates, yet remains capped at a certain level.



B= 2000, r=0.0575, sigma=0.3, T=1, B0=1000

FIGURE 2. Deposit Insurance Premium With Limit

3.1. Sensitivity to Interest Rate.

Subsequently, we will delve into the sensitivity of interest rates concerning the magnitude of the premium.

Figure 3 shows the tendency for the premium value to rise as the interest rate declines. For instance, at a very low interest rate such as r = 0.005, the premium becomes noticeable for relatively substantial asset values. However, significant changes in the interest rate, such as r = 0.5, do not exert a substantial influence on the premium value, except for lower asset values where a premium increase becomes apparent. Thus, it can be concluded that the interest rate does impact the premium value in deposit insurance. As the interest rate decreases, premiums tend to increase. This phenomenon arises because higher interest rates result in substantial returns on invested funds, leading to larger protection amounts and consequently smaller premiums required.

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3.1.1. Sensitivity Deposit Insurance without Limit to Interest Rate.

The analysis results in Figure 3 to Figure 6 show the sensitivity deposit insurance without limit to interest rate with the different sigma. As the value of the asset increases, the premium value will decrease. For an interest rate of 0.5, the premium value will reach 0 when the asset is valued at 1588.6 Meanwhile, for an interest rate of 0.0575, the premium value will reach 0 when the asset is valued at 1990. In the case of extremely low interest rates, namely 0.005 and 0.0005, the premium value will also be 0 at the same asset value of 2040.1. We can conclude that with different sigma values, the pattern of the correspondence between premium and interest rate remains the same. Therefore, we can infer that interest rates are sensitive to premiums.



FIGURE 3. Premium Value for Sigma=0.003 with Zoom in Version











FIGURE 6. Premium Value for Sigma=0.9

3.1.2. Sensitivity Deposit Insurance with Limit to Interest Rate.

However, due to the presence of coverage limits, there exists a point where very small asset values result in a constant premium amount, in accordance with the established premium cap by the deposit insurance institution. In other words, despite fluctuations in the interest rate, the premium remains constrained within the bounds set by the deposit insurance institution. Thus, sensitivity analysis regarding interest rates in deposit insurance with coverage limitations reveals a correlation between interest rates and premium values. Changes in interest rates tend to influence premium values, especially for low asset values. However, for high asset values, fluctuations in interest rates do not significantly impact premium values. The analysis results in Figure 7 to Figure 10 show the sensitivity deposit insurance with limit to interest rate with the different sigma.



FIGURE 7. Premium Value for Sigma=0.003 With Zoom in Version











FIGURE 10. Premium Value for Sigma=0.9

In contrast to the interest rate, varying sigma from 0.003 to 0.03, 0.3, and 0.9, with an initial asset value of 0, yields the same premium value of 1995.1. However, as the asset value increases, the premium value progressively decreases. The divergence in premium values becomes evident when the asset value reaches 1000. With a high sigma value of 0.9, as shown in Figure 10, the premium tends to be larger than with other sigma values. In Figure 10, the premium value starts to decrease when the asset value is at 484.95 with premium values for interest rates 0.5, 0.0575, 0.005, and 0.0005 being 738.0085, 945.9025, 973.1341, and 975.4959, respectively. At this elevated sigma value, with the largest asset value of 5000, the premium approaches 0. Conversely, at a very low volatility value of 0.003, the premium has already reached 0 when the asset value is 1990.

3.2. Sensitivity to volatility.

Subsequently, we will delve into the sensitivity of volatility concerning the magnitude of the premium

3.2.1. Sensitivity Deposit Insurance without Limit to Volatility.

From Figure 11 to Figure 14, it can be concluded that volatility has a positive relationship with the magnitude of premiums, the higher asset's volatility will result the higher premium price. The analysis results in Figure 11 to Figure 14 show the sensitivity deposit insurance without limit to volatility with the different interest rate. Lower volatility corresponds to lower premium payments. Conversely, in the case of high volatility like $\sigma = 0.9$, the premium will increase. For instance, in the scenario of low volatility such as $\sigma = 0.03$, it is evident that as the asset price rises, the premium value decreases. In fact, for significantly high asset prices, the premium paid can approach zero. However, under high volatility conditions like $\sigma = 0.9$, the premium is very high compared to other volatility conditions. This is due to the presence of significant risk factors, which leads to a higher probability of loss. Therefore, it can be concluded that volatility significantly affects the premium value in deposit insurance. The higher the volatility, the more likely the premium will increase.

3.2.2. Sensitivity Deposit insurance with limit to volatility.

The sensitivity analysis of asset price volatility in deposit insurance with imposed coverage limits draws the same conclusion as the influence of asset price volatility on deposit insurance without coverage limits. The distinguishing factor lies in the existence of a point where very small asset values result in a constant premium amount, in accordance with the premium cap set by the deposit insurance institution. For example, in Figure 15 to Figure 17, the premium is constant at 1000 as the asset value gets smaller, even though the volatility is varied. But for Figure 18, the premium value decreases, due to the very high interest rate. But it will still give a constant value at 800 when the asset value is getting smaller even though the volatility is varied. Consequently, volatility gives a positive relationship with the magnitude of premiums. Lower volatility corresponds to lower premium payments. Conversely, in the case of high volatility like $\sigma = 0.9$, the premium will



FIGURE 11. Premium Value for Interest Rate=0.0005 with Zoom in Version



FIGURE 12. Premium Value for Interest Rate=0.005

increase. The analysis results in Figure 15 to Figure 18 show the sensitivity deposit insurance with limit to volatility with the different interest rate.







FIGURE 14. Premium Value for Interest Rate=0.5



FIGURE 15. Premium Value for interest rate=0.0005 With Zoom in Version



FIGURE 16. Premium Value for interest rate=0.005



FIGURE 17. Premium Value for interest rate=0.0575



FIGURE 18. Premium Value for interest rate=0.5

4. CONCLUDING REMARKS

Based on our research, we can conclude that the analytical solution for calculating the deposit insurance premium with coverage limits involves adding two European put options with different strike prices, as explained in Theorem 2.1. In terms of the sensitivity analysis, we found that changes in interest rates and volatility have an impact on deposit insurance premiums, specifically:

- (1) interest rates have an inverse relationship with premium value; when interest rates are low, premiums tend to increase, whereas when interest rates are high, premiums tend to decrease. This is because the higher interest rates generate greater returns on invested funds, so the amount of protection provided is greater and consequently the premium required is smaller.
- (2) volatility has a positive relationship with the magnitude of premiums; when asset price volatility increases, deposit insurance premiums also tend to increase. This is because the greater the risk factor, the higher the likelihood of bankruptcy.

REFERENCES

- A. J. Marcus and I. Shaked, "The valuation of fdic deposit insurance using option-pricing estimates," *Journal of Money, Credit and Banking*, vol. 16, no. 4, pp. 446–460, 1984. https: //doi.org/10.2307/1992183.
- [2] M. A. Kadir, Hukum Asuransi. Aditya Bakti, 2006.
- F. Black and M. Scholes, "The pricing of options and corporate liabilities," Journal of Political Economy, vol. 81, pp. 637-654, 1973. https://www.jstor.org/stable/1831029.
- [4] R. C. Merton, "An analytic derivation of the cost of deposit insurance and loan guarantees an application of modern option pricing theory," *Journal of Banking & Finance*, vol. 1, pp. 3–11, 1977. https://doi.org/10.1016/0378-4266(77)90015-2.
- [5] E. I. Ronn and A. K. Verma, "Pricing risk-adjusted deposit insurance: An option-based model," *The Journal of Finance*, vol. 41, pp. 871–895, 1986. https://doi.org/10.1111/j. 1540-6261.1986.tb04554.x.
- [6] Y. Zhang and B. Shi, "Systematic risk and deposit insurance pricing: based on market model and option pricing theory," *China Finance Review International*, vol. 7, pp. 390–406, 2017. https://doi.org/10.1108/CFRI-12-2016-0133.
- [7] S.-L. Chiang and M.-S. Tsai, "The valuation of deposit insurance allowing for the interest rate spread and early-bankruptcy risk," *The Quarterly Review of Economics and Finance*, vol. 76, pp. 345–356, 2020. https://doi.org/10.1016/j.qref.2019.09.008.
- [8] Y.-C. Wu, T.-F. Chen, and S.-K. Lin, "Risk management of deposit insurance corporations with risk-based premiums and credit default swaps," *Quantitative Finance*, pp. 1–16, 2020. https://doi.org/10.1080/14697688.2020.1726437.
- C.-C. Chang, S.-L. Chung, R.-J. Ho, and Y.-J. Hsiao, "Revisiting the valuation of deposit insurance," *Journal of Futures Markets*, vol. 42, pp. 77–103, 2022. https://doi.org/10. 1002/fut.22284.
- [10] H. Lee, S. Song, and G. Lee, "Insurance guaranty premiums and exchange options," Mathematical Finance and Economics, vol. 17, pp. 49–77, 2023. https://doi.org/10.1007/s11579-022-00326-4.
- [11] R. Subekti, Hukum Perjanjian. Jakarta: PT Intermasa, 2010.
- [12] L. Allen and A. Saunders, "Forbearance and valuation of deposit insurance as a callable put," *Journal of Banking & Finance*, vol. 17, pp. 629–643, 1993. https://doi.org/10.1016/ 0378-4266(93)90004-W.