J. Indones. Math. Soc.
Vol. 30, No. 02 (2024), pp. 236–255.

CONTROLLING THE BORROWER POPULATION OF P2P LENDING MODELS

Audri Utami Gunadi[1]* Dewi Handayani [2]

Department of Mathematics, Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung, Indonesia 40132
[1]audriutamigunadi@students.itb.ac.id, [2]dhandayanimtk@itb.ac.id

Abstract. P2P lending, commonly called online lending, is a service provider institution that provides borrowing and lending services in rupiah currency through an electronic system. The growth of P2P lending has increased rapidly since the pandemic of Covid-19 and led to an increase in the number of borrowers. Meanwhile, crime has also increased as many people can't repay their loans. The chain of P2P lending must be controlled to suppress the growth of the population of people with online loans. This study constructs two P2P lending models by modifying the Kermack-McKendrick Epidemic Model. The population is divided into three subpopulations: potential individuals, borrowers, and payers. Optimal control is used to suppress the population growth of borrower individuals through socialization with potential individuals or people with work potential and providing payment assistance for borrowers. This study constructs several optimal control scenarios for the two P2P lending models. From the comparison of optimal control scenarios, the optimal control recommendations that can suppress the population growth of borrower is to provide socialization to people with work potential and payment assistance for the borrower population.

Key words and Phrases: P2P lending model, borrower, optimal control, socialization, payment assistance.

1. INTRODUCTION

P2P lending, commonly called online lending, is a service provider institution that provides borrowing and lending services in rupiah currency through an electronic system [6]. The reasons why people have loans in P2P lending are that the administrative requirements in P2P lending are relatively easier than formal financial service loans [11]. Furthermore, the decline in employment due to the COVID-19 pandemic has caused an increase in the number of people who are not working so they apply in P2P lending to fulfill their needs [1].

2020 Mathematics Subject Classification: 34A34, 37N35, 49J15
Received: 16-09-2023, accepted: 23-08-2024.

There are two types of P2P lending: formal/legal and informal/illegal [6]. Formal P2P lending is online lending that is registered and supervised by Financial Service Authority of Indonesia and BI Checking [6]. Informal P2P lending should be avoided because there is no supervision, which causes a lot of public distress due to high interest rates, impolite and threatening debt collectors even though the deadline has not yet arrived, the spread and misuse of personal data, and many crimes such as fraud, money laundering, etc. [4, 11]. While informal P2P lending should be avoided, having a formal P2P lending is also not recommended as borrowing is not a good habit and will have a negative impact if it continues. The security of formal P2P lending also cannot be guaranteed because cybercrime is on the rise [11].

Over time, more and more people have loans in P2P lending. There are more than 14.17 million accounts of formal P2P lending customer in Indonesia as of October 2022 [5]. Therefore, controls are needed to suppress the growth of the borrower population. Optimal control is used to determine an effective strategy to minimize the number of borrowers [8, 9]. Optimal control is carried out by providing socialization as a way of prevention and providing payment assistance as a way of recovery.

2. THE MATHEMATICAL MODEL OF P2P LENDING

The P2P lending model adapts SIR Model by Kermack-McKendrick. The compartments used in the model are: potential individuals (S), borrowers (L), and payers (P). This first model will be called as simple P2P lending model. An extension of the model will be made so that borrowers are divided into formal/legal P2P lending borrowers (L_F) and informal/illegal P2P lending borrowers (L_{IF}). This extension model will be called formal and informal P2P lending model.

In this study, it is assumed that a person can engage in P2P lending because influenced by recommendations of the borrower in P2P lending (α) or influenced by advertisements of P2P lending businesses (β). Simple P2P lending model involves both borrower influence (α) and P2P lending advertisement influence (β), while formal and informal P2P lending model only involves the borrower influence (α).

People who have the potential to borrow from P2P lending are people who do not have enough income to cover their living costs. This community will be called "people with work potential".

There are assumptions used in constructing both models:

(1) the average addition of people with work potential per day is considered constant,
(2) no deaths in the population of potential individuals because the tenor of online loans is assumed to be short,
(3) no deaths in the population of borrowers because if the borrower dies, the loan will become the responsibility of his/her closest relatives,
(4) individuals who have fully repaid their loans can return to being potential individuals.

a. Simple P2P Lending Model

TABLE 1. Variable Description for Simple P2P Lending Model

Symbol	Description	Unit
$S(t)$	Potential individual at time t	person
$L(t)$	Borrowers at time t	person
$P(t)$	Payers at time t	person

TABLE 2. Parameter Description for Simple P2P Lending Model

Symbol	Description	Value
A	Average of people with work potential	80 $(person.day^{-1})$
α	Rate of potential individual becoming a borrower due to the influence of a borrower	0.001 $(person^{-1}.day^{-1})$
β	Rate of potential individual engaging in P2P lending due to influence of P2P lending advertisement	0.001 (day^{-1})
γ	Rate of individual repayment P2P lending	0.0005 (day^{-1})
μ	Rate of the payers returning to being potential individuals	0.003 (day^{-1})
δ	Rate of individual mortality	$1/365(day^{-1})$

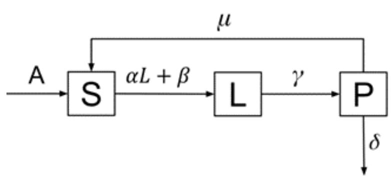


FIGURE 1. Compartment diagram of the simple P2P lending model

Based on the diagram in Figure 1, the following model is obtained:

$$
\begin{aligned}
\frac{dS}{dt} &= A - \alpha SL - \beta S + \mu P \\
\frac{dL}{dt} &= \alpha SL + \beta S - \gamma L \\
\frac{dP}{dt} &= \gamma L - P(\mu + \delta) \\
S(0) > 0,\ & L(0) \geq 0,\ P(0) \geq 0.
\end{aligned}
\tag{1}
$$

TABLE 3. Variable Description for Formal and Informal P2P Lending Model

Symbol	Description	Unit
$S(t)$	Potential individual at time t	person
$L_F(t)$	Borrowers in formal P2P lending at time t	person
$L_{IF}(t)$	Borrowers in informal P2P lending at time t	person
$P(t)$	Payers at time t	person

TABLE 4. Parameter Description for Formal and Informal P2P Lending Model

Symbol	Description	Value
A	Average of people with work potential	80 $(person.day^{-1})$
α	Rate of potential individual becoming a borrower due to the influence of a borrower	0.001 $(person^{-1}.day^{-1})$
γ_F	Rate of individual repayment formal P2P lending	0.0005 (day^{-1})
γ_{IF}	Rate of individual repayment informal P2P lending	0.0001 (day^{-1})
μ	Rate of the payers returning to being potential individuals	0.003 (day^{-1})
δ	Rate of individual mortality	1/365(day^{-1})
p	Proportion of an individual engaging in formal P2P lending	0.6

b. Formal and Informal P2P Lending Model

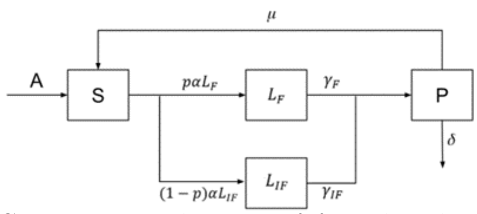


FIGURE 2. Compartment diagram of formal and informal P2P lending model

Based on the diagram in Figure 2, the following model is obtained:

$$\begin{aligned}
\frac{dS}{dt} &= A - p\alpha S L_F - (1-p)\alpha S L_{IF} + \mu P \\
\frac{dL_F}{dt} &= p\alpha S L_F - \gamma_F L_F \\
\frac{dL_{IF}}{dt} &= (1-p)\alpha S L_{IF} - \gamma_{IF} L_{IF} \\
\frac{dP}{dt} &= \gamma_F L_F + \gamma_{IF} L_{IF} - \mu P - \delta P \\
S(0) > 0,\ & L_F(0) \geq 0,\ L_{IF}(0) \geq 0,\ P(0) \geq 0.
\end{aligned} \tag{2}$$

3. THE EXISTENCE AND STABILITY OF EQUILIBRIUM

3.1. **Equilibrium Point.**

Equilibrium points of P2P lending models are divided into borrower-free equilibrium point and endemic equilibrium point. The borrower-free equilibrium point is a condition where there are no borrowers in the population. The endemic equilibrium point is a condition where there is at least one borrower who will affect the community so that borrowers will increase in the population [2].

3.1.1. *Equilibrium Point for Simple P2P Lending Model.*

Based on the system of differential equations (1), only endemic equilibrium point is obtained for simple P2P lending model. The equilibrium point when $L > 0$ is:

$$E_1^* = (S^*, L^*, P^*) = \left(\frac{A\gamma(\mu+\delta)}{A\alpha(\mu+\delta) + \beta\delta\gamma}, \frac{A(\mu+\delta)}{\delta\gamma}, \frac{A}{\delta} \right). \tag{3}$$

3.1.2. *Equilibrium Point for Formal and Informal P2P Lending Model.*

Based on the system of differential equations (2), only borrower-free equilibrium point is obtained for formal and informal P2P lending model.

a. The equilibrium point when $L_F = 0$ is:

$$E_{2_0} = (S^0, L_F^0, L_{IF}^0, P^0) = \left(\frac{\gamma_{IF}}{\alpha(1-p)}, 0, \frac{A(\mu+\delta)}{\delta\gamma_{IF}}, \frac{A}{\delta} \right). \tag{4}$$

b. The equilibrium point when $L_{IF} = 0$ is:

$$E_{3_0} = (S^0, L_F^0, L_{IF}^0, P^0) = \left(\frac{\gamma_F}{\alpha p}, \frac{A(\mu+\delta)}{\delta\gamma_F}, 0, \frac{A}{\delta} \right). \tag{5}$$

3.2. **Stability Analysis of Equilibrium Point.**

Stability analysis of equilibrium point is useful to determine whether the population will be free of borrowers or will continue to exist. The method used to test the stability of the equilibrium point in this paper is the eigenvalue criteria [2].

3.2.1. *Stability of Equilibrium Point for Simple P2P Lending Model.*

Jacobi matrix of the system (1) at equilibrium point (3) is given by:

$$J(E_1^*) = \begin{pmatrix} -\alpha\left(\frac{A(\mu+\delta)}{\delta\gamma}\right) - \beta & -\alpha\left(\frac{A\gamma(\mu+\delta)}{A\alpha(\mu+\delta)+\beta\delta\gamma}\right) & \mu \\ \alpha\left(\frac{A(\mu+\delta)}{\delta\gamma}\right) + \beta & \alpha\left(\frac{A\gamma(\mu+\delta)}{A\alpha(\mu+\delta)+\beta\delta\gamma}\right) - \gamma & 0 \\ 0 & \gamma & -\mu-\delta \end{pmatrix}. \tag{6}$$

The characteristic equation of $J(E_1^*)$ is:

$$a_0\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0 \tag{7}$$

with

$$\begin{aligned} a_0 &= 1, \\ a_1 &= \frac{\beta\delta^2\gamma^3 + (A\alpha(\mu+\delta)+\beta\delta\gamma)^2 + (\mu+\delta)\delta\gamma(A\alpha(\mu+\delta)+\beta\delta\gamma)}{\delta\gamma(A\alpha(\mu+\delta)+\beta\delta\gamma)}, \\ a_2 &= \frac{\beta\delta^2\gamma^3(\mu+\delta) + (A\alpha(\mu+\delta)+\beta\delta\gamma)^2(\mu+\delta+\gamma)}{\delta\gamma(A\alpha(\mu+\delta)+\beta\delta\gamma)}, \\ a_3 &= A\alpha(\mu+\delta)+\beta\delta\gamma. \end{aligned}$$

The basic reproduction number is obtained as follows:

$$R_{0_1} = \frac{\delta^2\gamma^2c^3}{\left(c^2 + A\alpha d^2\left(\beta\delta^2\gamma^2(d+\gamma)\right)\right)\left(c^2(d+\gamma)+\beta\delta^2\gamma^3d\right)} \tag{8}$$

with $c = A\alpha(\mu+\delta)+\beta\delta\gamma$ and $d = \mu+\delta$.

The Routh-Hurwitz criteria will be applied to determine the value of λ. Since a_0, a_1, $a_3 > 0$, according to The Routh criteria, all eigenvalues of the characteristic equation (7) have negative real part if $a_1a_2 > a_0a_3$. Then, $a_1a_2 > a_0a_3$ if $R_{0_1} > 1$. Thus, E_1^* is locally asymptotically stable when $R_{0_1} > 1$.

3.2.2. *Stability of Equilibrium Point for Formal and Informal P2P Lending Model.*

By linearizing system (2) using Jacobi matrix as in simple P2P lending model, the characteristics equation for formal and informal P2P lending model are obtained as follows:

a. The characteristic equation of $J(E_2^0)$ is:

$$(\lambda p - \gamma_F + p\gamma_F + p\gamma_{IF})(b_0\lambda^3 + b_1\lambda^2 + b_2\lambda + b_3) = 0 \tag{9}$$

with

$$\begin{aligned} b_0 &= \delta\gamma_F, \\ b_1 &= (\mu+\delta)(\delta\gamma_F + A\alpha p), \\ b_2 &= (A\alpha p)(\mu+\delta)(\mu+\delta+\gamma_F), \\ b_3 &= (A\alpha p)(\delta\gamma_F)(\mu+\delta). \end{aligned}$$

The basic reproduction number is obtained as follows:

$$R_{0_2} = \frac{(p(A-\alpha)+\delta\gamma_F)(\mu+\delta)(\mu+\delta+\gamma_F)}{(\delta\gamma_F)^2}. \tag{10}$$

For the equilibrium point E_2^0, all eigenvalues of the characteristic equation (9) have negative real part if $b_1b_2 > b_0b_3$. Then, $b_1b_2 > b_0b_3$ if $R_{0_2} > 1$. Thus, E_2^0 is locally asymptotically stable when $R_{0_2} > 1$ and $\gamma_F < \frac{p}{(1-p)}\gamma_{IF}$.

b. The characteristic equation of $J(E_3^0)$ is:

$$((p-1)\lambda + (p-1)\gamma_F + p\gamma_{IF})(c_0\lambda^3 + c_1\lambda^2 + c_2\lambda + c_3) = 0 \tag{11}$$

with

$$\begin{aligned}
c_0 &= \delta\gamma_{IF},\\
c_1 &= (\mu+\delta)(\delta\gamma_{IF} + (1-p)(A\alpha)),\\
c_2 &= (1-p)(A\alpha)(\mu+\delta)(\mu+\delta+\gamma_{IF}),\\
c_3 &= (1-p)(A\alpha)(\delta\gamma_{IF})(\mu+\delta),
\end{aligned}$$

The basic reproduction number is obtained as follows:

$$R_{0_3} = \frac{((1-p)(A\alpha)+\delta+\gamma_I F)(\mu+\delta)(\mu+\delta+\gamma_{IF})}{(\delta\gamma_{IF})^2}. \tag{12}$$

For the equilibrium point E_3^0, all eigenvalues of the characteristic equation (10) have negative real part if $c_1c_2 > c_0c_3$. Then, $c_1c_2 > c_0c_3$ if $R_{0_3} > 1$. Thus, E_3^0 is locally asymptotically stable when $R_{0_3} > 1$ and $\gamma_{IF} < \frac{1-p}{p}\gamma_F$.

4. OPTIMAL CONTROL FOR P2P LENDING MODELS

The controls that will be applied to the models are socialization and payment assistance. Socialization is carried out by educating the public about the negative impacts of P2P lending. Payment assistance is provided by the government to borrowers in formal P2P lending with the condition that the loan is used to fulfill their urgent life needs. Note that u_1 is a control variable that represents the percentage of people who avoid P2P lending after obtaining socialization, so the percentage of people who engage in P2P lending is $(1-u_1)$. Then, u_2 is a control variable that represents the percentage of borrowers who received payment assistance.

TABLE 5. The optimal control scenarios for both P2P lending models

Model	Scenario	Socialization	Payment Assistance
Simple P2P Lending Model	1	To potential individual	-
	2	To potential individual	✓
	3	To people with work potential	✓
Formal and Informal P2P Lending Model	1	To potential individual	-
	2	To people with work potential	-

4.1. **Optimal Control for The Simple P2P Lending Model.**

The optimal control problem to be solved from the system in the simple P2P lending model is to minimize the following cost function:

$$min\ J_1(t_0) = \int_0^T (\omega L^2 + \sum_{i=1}^{n} r_i u_i^2) dt \tag{13}$$

with $0 \leq t \leq T$, $0 \leq u_i(t) \leq 1$, n is the number of controls, and ω, r_i are the weights that balance the cost.

Using Pontryagin's Minimum Principle, the Hamiltonian function is defined as follows:

$$H = (\omega L^2 + \sum_{i=1}^{n} r_i u_i^2) + \frac{dS}{dt}\lambda_1 + \frac{dL}{dt}\lambda_2 + \frac{dP}{dt}\lambda_3 \tag{14}$$

with $\boldsymbol{\lambda}^{\boldsymbol{T}} = (\lambda_1\ \lambda_2\ \lambda_3)$ is the Lagrange multiplier.

From the Hamiltonian function, we obtain the state equation $\left(\dot{\boldsymbol{x}}(t) = \frac{\partial H}{\partial \boldsymbol{\lambda}}\right)$ with $\boldsymbol{x}(0) = \boldsymbol{x_0}$, costate equation $\left(\dot{\boldsymbol{\lambda}}(t) = -\frac{\partial H}{\partial \boldsymbol{x}}\right)$ with $\boldsymbol{\lambda}(T) = \boldsymbol{0}$, and stationary condition $\left(\frac{\partial H}{\partial u_i} = 0\right)$.

4.1.1. *The First Scenario of The Simple P2P Lending Model.*

In the first scenario, the control variable used to suppress the growth of the borrower population is providing socialization to potential individuals (S). The rate of potential individuals who avoid P2P lending after obtaining socialization is u_1. So, the rate of potential individuals involved in P2P lending is $(1 - u_1)$.

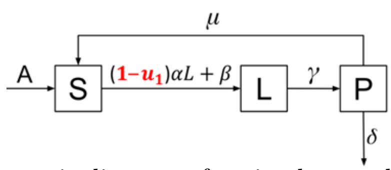


Figure 3. Systematic diagram of optimal control for the first scenario of the simple P2P lending model

Based on the diagram in Figure 3, the following model is obtained:

$$\begin{aligned}
\frac{dS}{dt} &= A - (1 - u_1)\alpha SL - \beta S + \mu P, \\
\frac{dL}{dt} &= (1 - u_1)\alpha SL + \beta S - \gamma L, \\
\frac{dP}{dt} &= \gamma L - P(\mu + \delta), \\
S(0) > 0,\ & L(0) \geq 0,\ P(0) \geq 0.
\end{aligned}$$

a. State equation

$$\begin{aligned}
\dot{S}(t) &= \frac{\partial H}{\partial \lambda_1} = A - (1-u_1)\alpha SL - \beta S + \mu P,\\
\dot{L}(t) &= \frac{\partial H}{\partial \lambda_2} = (1-u_1)\alpha SL + \beta S - \gamma L,\\
\dot{P}(t) &= \frac{\partial H}{\partial \lambda_3} = \gamma L - P(\mu+\delta),
\end{aligned} \tag{15}$$

with known initial value $[S(0), L(0), P(0)] = [S_0, L_0, P_0]$.

b. Costate equation

$$\begin{aligned}
\dot{\lambda_1}(t) &= -\frac{\partial H}{\partial S} = (\alpha L - u_1\alpha L + \beta)(\lambda_1 - \lambda_2),\\
\dot{\lambda_2}(t) &= -\frac{\partial H}{\partial L} = (\alpha S - u_1\alpha S)(\lambda_1 - \lambda_2) + \gamma(\lambda_2 - \lambda_3) - 2\omega L,\\
\dot{\lambda_3}(t) &= -\frac{\partial H}{\partial P} = \lambda_3(\mu+\delta) - \mu\lambda_1,
\end{aligned} \tag{16}$$

with known final value $\lambda_1(T) = \lambda_2(T) = \lambda_3(T) = 0$.

c. Stationary condition

$$u_1 = \frac{\alpha SL(\lambda_2 - \lambda_1)}{2r_1}. \tag{17}$$

Based on the Pontryagin's Minimum Principle, the optimal control u_1^* for the first scenario of the simple P2P lending model is obtained as follows:

$$u_1^*(t) = min\left\{1; max\left\{0, \frac{\alpha SL(\lambda_2 - \lambda_1)}{2r_1}\right\}\right\}. \tag{18}$$

4.1.2. *The Second Scenario of The Simple P2P Lending Model.*

In the second scenario, two control variables are applied to suppress the growth of the borrower population, that are providing socialization to potential individuals (S) and providing payment assistance to borrowers (L). The rate of potential individuals who avoid P2P lending after obtaining socialization is u_1. So, the rate of potential individuals involved in P2P lending is $(1-u_1)$ and the rate of borrowers who providing payment assistance is u_2.

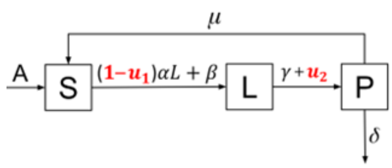


Figure 4. Systematic diagram of optimal control for the second scenario of the simple P2P lending model

Based on the diagram in Figure 4, the following model is obtained:

$$\begin{aligned}\frac{dS}{dt} &= A - (1-u_1)\alpha SL - \beta S + \mu P,\\ \frac{dL}{dt} &= (1-u_1)\alpha SL + \beta S - L(\gamma + u_2),\\ \frac{dP}{dt} &= L(\gamma+u_2) - P(\mu+\delta),\\ S(0) > 0,\ & L(0) \geq 0,\ P(0) \geq 0.\end{aligned} \tag{19}$$

a. State equation

$$\begin{aligned}\dot{S}(t) &= \frac{\partial H}{\partial \lambda_1} = A - (1-u_1)\alpha SL - \beta S + \mu P,\\ \dot{L}(t) &= \frac{\partial H}{\partial \lambda_2} = (1-u_1)\alpha SL + \beta S - L(\gamma+u_2),\\ \dot{P}(t) &= \frac{\partial H}{\partial \lambda_3} = L(\gamma+u_2) - P(\mu+\delta),\end{aligned} \tag{20}$$

with known initial value $[S(0), L(0), P(0)] = [S_0, L_0, P_0]$.

b. Costate equation

$$\begin{aligned}\dot{\lambda_1}(t) &= -\frac{\partial H}{\partial S} = (\alpha L - u_1\alpha L + \beta)(\lambda_1 - \lambda_2),\\ \dot{\lambda_2}(t) &= -\frac{\partial H}{\partial L} = (\alpha S - u_1\alpha S)(\lambda_1-\lambda_2) + (\gamma+u_2)(\lambda_2-\lambda_3) - 2\omega L,\\ \dot{\lambda_3}(t) &= -\frac{\partial H}{\partial P} = \lambda_3(\mu+\delta) - \mu\lambda_1,\end{aligned} \tag{21}$$

with known final value $\lambda_1(T) = \lambda_2(T) = \lambda_3(T) = 0$.

c. Stationary condition

$$u_1 = \frac{\alpha SL(\lambda_2-\lambda_1)}{2r_1},\ u_2 = \frac{L(\lambda_2-\lambda_3)}{2r_2}. \tag{22}$$

Based on the Pontryagin's Minimum Principle, the optimal control u_1^* and u_2^* for the second scenario of the simple P2P lending model is obtained as follows:

$$\begin{aligned}u_1^*(t) &= min\left\{1; max\left\{0, \frac{\alpha SL\,(\lambda_2-\lambda_1)}{2r_1}\right\}\right\},\\ u_2^*(t) &= min\left\{1; max\left\{0, \frac{L\,(\lambda_2-\lambda_3)}{2r_2}\right\}\right\}.\end{aligned} \tag{23}$$

4.1.3. *The Third Scenario of The Simple P2P Lending Model.*

This scenario is similar to the second scenario, but the socialization is given to people with work potential (A). The rate of people with work potential who avoid P2P lending after obtaining socialization is u_1. So, the rate of people with work potential involved in P2P lending is $(1-u_1)$ and the rate of borrowers who providing payment assistance is u_2.

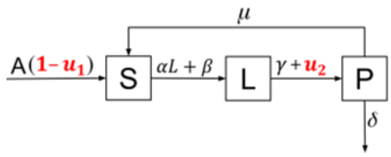


FIGURE 5. Systematic diagram of optimal control for the third scenario of the simple P2P lending model

Based on the diagram in Figure 5, the following model is obtained:

$$\begin{aligned}
\frac{dS}{dt} &= A(1-u_1) - \alpha SL - \beta S + \mu P, \\
\frac{dL}{dt} &= \alpha SL + \beta S - L(\gamma + u_2), \\
\frac{dP}{dt} &= L(\gamma + u_2) - P(\mu + \delta), \\
S(0) > 0, L(0) &\geq 0,\ P(0) \geq 0.
\end{aligned} \tag{24}$$

a. State equation

$$\begin{aligned}
\dot{S}(t) &= \frac{\partial H}{\partial \lambda_1} = A(1-u_1) - \alpha SL - \beta S + \mu P, \\
\dot{L}(t) &= \frac{\partial H}{\partial \lambda_2} = \alpha SL + \beta S - L(\gamma + u_2), \\
\dot{P}(t) &= \frac{\partial H}{\partial \lambda_3} = L(\gamma + u_2) - P(\mu + \delta),
\end{aligned} \tag{25}$$

with known initial value $[S(0), L(0), P(0)] = [S_0, L_0, P_0]$.

b. Costate equation

$$\begin{aligned}
\dot{\lambda_1}(t) &= -\frac{\partial H}{\partial S} = (\alpha L + \beta)(\lambda_1 - \lambda_2), \\
\dot{\lambda_2}(t) &= -\frac{\partial H}{\partial L} = \alpha S(\lambda_1 - \lambda_2) + (\gamma + u_2)(\lambda_2 - \lambda_3) - 2\omega L, \\
\dot{\lambda_3}(t) &= -\frac{\partial H}{\partial P} = \lambda_3(\mu + \delta) - \mu \lambda_1,
\end{aligned} \tag{26}$$

with known final value $\lambda_1(T) = \lambda_2(T) = \lambda_3(T) = 0$.

c. Stationary condition

$$u_1 = \frac{\lambda_1}{2r_1},\ u_2 = \frac{L(\lambda_2 - \lambda_3)}{2r_2}. \tag{27}$$

Based on the Pontryagin's Minimum Principle, the optimal control u_1^* and u_2^* for the third scenario of the simple P2P lending model is obtained as follows:

$$u_1^*(t) = min\left\{1; max\left\{0, \frac{\lambda_1}{2r_1}\right\}\right\},$$
$$u_2^*(t) = min\left\{1; max\left\{0, \frac{L\left(\lambda_2 - \lambda_3\right)}{2r_2}\right\}\right\}. \tag{28}$$

4.2. **Optimal Control for The Formal and Informal P2P Lending Model.**

The optimal control problem to be solved from the system in the formal and informal P2P lending model is to minimize the following cost function:

$$min\ J_2(t_0) = \int_0^T \omega L_{IF}^2 + r_1 u_1^2 dt \tag{29}$$

with $0 \leq t \leq T$, $0 \leq u_1(t) \leq 1$, and ω, r_1 are the weights that balance the cost.

Using Pontryagin's Minimum Principle, the Hamiltonian function is defined as follows:

$$H = \omega L_{IF}^2 + r_1 u_1^2 + \frac{dS}{dt}\lambda_1 + \frac{dL_F}{dt}\lambda_2 + \frac{dL_{IF}}{dt}\lambda_3 + \frac{dP}{dt}\lambda_4 \tag{30}$$

with $\boldsymbol{\lambda^T} = (\lambda_1\ \lambda_2\ \lambda_3\ \lambda_4)$ is the Lagrange multiplier.

From the Hamiltonian function, we obtain the state equation $\left(\dot{\boldsymbol{x}}(t) = \frac{\partial H}{\partial \boldsymbol{\lambda}}\right)$ with $\boldsymbol{x}(0) = \boldsymbol{x_0}$, costate equation $\left(\dot{\boldsymbol{\lambda}}(t) = -\frac{\partial H}{\partial \boldsymbol{x}}\right)$ with $\boldsymbol{\lambda}(T) = \boldsymbol{0}$, and stationary condition $\left(\frac{\partial H}{\partial u_i} = 0\right)$.

4.2.1. *The First Scenario of The Formal and Informal P2P Lending Model.*

In the first scenario, the control variable used to suppress the growth of informal borrower population (L_{IF}) is providing socialization to potential individuals (S). The rate of potential individuals who choose formal P2P lending after obtaining socialization is u_1. So, the rate of potential individuals involved in informal P2P lending is $(1 - u_1)$.

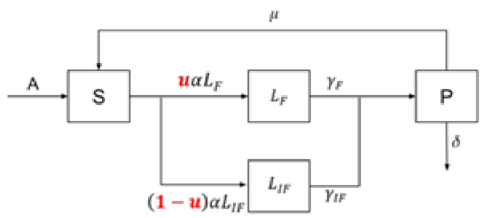


FIGURE 6. Systematic diagram of optimal control for the first scenario of the formal and informal P2P lending model

Based on the diagram in Figure 6, the following model is obtained:

$$\begin{aligned}\frac{dS}{dt} &= A - \alpha S(u_1 L_F + (1-u_1)L_{IF}) + \mu P,\\ \frac{dL_F}{dt} &= u_1 \alpha S L_F - \gamma_F L_F,\\ \frac{dL_{IF}}{dt} &= (1-u_1)\alpha S L_{IF} - \gamma_{IF} L_{IF},\\ \frac{dP}{dt} &= \gamma_F L_F + \gamma_{IF} L_{IF} - P(\mu+\delta),\\ S(0) > 0,\ &L_F(0) \geq 0,\ L_{IF}(0) \geq 0,\ P(0) \geq 0.\end{aligned} \tag{31}$$

a. State equation

$$\begin{aligned}\dot{S}(t) &= \frac{\partial H}{\partial \lambda_1} = A - \alpha S(u_1 L_F + (1-u_1)L_{IF}) + \mu P,\\ \dot{L}_F(t) &= \frac{\partial H}{\partial \lambda_2} = u_1 \alpha S L_F - \gamma_F L_F,\\ \dot{L}_{IF}(t) &= \frac{\partial H}{\partial \lambda_3} = (1-u_1)\alpha S L_{IF} - \gamma_{IF} L_{IF},\\ \dot{P}(t) &= \frac{\partial H}{\partial \lambda_4} = \gamma_F L_F + \gamma_{IF} L_{IF} - P(\mu+\delta),\end{aligned} \tag{32}$$

with known initial value $[S(0), L_F(0), L_{IF}(0), P(0)] = [S_0, L_{F_0}, L_{F_0}, P_0]$.

b. Costate equation

$$\begin{aligned}\dot{\lambda}_1(t) &= -\frac{\partial H}{\partial S} = \alpha(L_F u_1(\lambda_1 - \lambda_2) + L_{IF}(1-u_1)(\lambda_1 - \lambda_3)),\\ \dot{\lambda}_2(t) &= -\frac{\partial H}{\partial L_F} = \alpha S u_1(\lambda_1 - \lambda_2) + \gamma_F(\lambda_2 - \lambda_4),\\ \dot{\lambda}_3(t) &= -\frac{\partial H}{\partial L_{IF}} = \alpha S(1-u_1)(\lambda_1 - \lambda_3) + \gamma_{IF}(\lambda_3 - \lambda_4) - 2\omega L_{IF},\\ \dot{\lambda}_4(t) &= -\frac{\partial H}{\partial P} = \lambda_4(\mu+\delta) - \mu\lambda_1,\end{aligned} \tag{33}$$

with known final value $\lambda_1(T) = \lambda_2(T) = \lambda_3(T) = \lambda_4(T) = 0$.

c. Stationary condition

$$u_1 = \frac{\alpha S(L_F(\lambda_1 - \lambda_2) - L_{IF}(\lambda_1 - \lambda_3))}{2r_1}. \tag{34}$$

Based on the Pontryagin's Minimum Principle, the optimal control u_1^* for the first scenario of the formal and informal P2P lending model is obtained as follows:

$$u_1^*(t) = min\left\{1; max\left\{0, \frac{\alpha S(L_F(\lambda_1 - \lambda_2) - L_{IF}(\lambda_1 - \lambda_3))}{2r_1}\right\}\right\}. \tag{35}$$

4.2.2. *The Second Scenario of The Formal and Informal P2P Lending Model.*

In the second scenario, the control variable used to suppress the growth of informal borrower population (L_{IF}) is providing socialization to people with work potential (A). The rate of people with work potential who avoid P2P lending is u_1. So, the rate of people with work potential who become potential individual is $(1 - u_1)$.

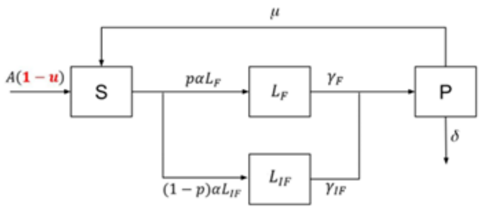


FIGURE 7. Systematic diagram of optimal control for the second scenario of the formal and informal P2P lending model

Based on the diagram in Figure 7, the following model is obtained:

$$\begin{aligned}
\frac{dS}{dt} &= A(1-u_1) - \alpha S(pL_F + (1-p)L_{IF}) + \mu P,\\
\frac{dL_F}{dt} &= p\alpha S L_F - \gamma_F L_F,\\
\frac{dL_{IF}}{dt} &= (1-p)\alpha S L_{IF} - \gamma_{IF} L_{IF},\\
\frac{dP}{dt} &= \gamma_F L_F + \gamma_{IF} L_{IF} - P(\mu+\delta),\\
S(0) > 0,\ & L_F(0) \geq 0,\ L_{IF}(0) \geq 0,\ P(0) \geq 0.
\end{aligned} \tag{36}$$

a. State equation

$$\begin{aligned}
\dot{S}(t) &= \frac{\partial H}{\partial \lambda_1} = A(1-u_1) - \alpha S(pL_F + (1-p)L_{IF}) + \mu P,\\
\dot{L}_F(t) &= \frac{\partial H}{\partial \lambda_2} = p\alpha S L_F - \gamma_F L_F,\\
\dot{L}_{IF}(t) &= \frac{\partial H}{\partial \lambda_3} = (1-p)\alpha S L_{IF} - \gamma_{IF} L_{IF},\\
\dot{P}(t) &= \frac{\partial H}{\partial \lambda_4} = \gamma_F L_F + \gamma_{IF} L_{IF} - P(\mu+\delta),
\end{aligned} \tag{37}$$

with known initial value $[S(0), L_F(0), L_{IF}(0), P(0)] = [S_0, L_{F_0}, L_{F_0}, P_0]$.

b. Costate equation

$$
\begin{aligned}
\dot{\lambda}_1(t) &= -\frac{\partial H}{\partial S} = \alpha(L_F p(\lambda_1 - \lambda_2) + L_{IF}(1-p)(\lambda_1 - \lambda_3)), \\
\dot{\lambda}_2(t) &= -\frac{\partial H}{\partial L_F} = \alpha S p(\lambda_1 - \lambda_2) + \gamma_F(\lambda_2 - \lambda_4), \\
\dot{\lambda}_3(t) &= -\frac{\partial H}{\partial L_{IF}} = \alpha S(1-p)(\lambda_1 - \lambda_3) + \gamma_{IF}(\lambda_3 - \lambda_4) - 2\omega L_{IF}, \\
\dot{\lambda}_4(t) &= -\frac{\partial H}{\partial P} = \lambda_4(\mu + \delta) - \mu\lambda_1,
\end{aligned} \tag{38}
$$

with known final value $\lambda_1(T) = \lambda_2(T) = \lambda_3(T) = \lambda_4(T) = 0$.

c. Stationary condition

$$u_1 = \frac{A\lambda_1}{2r_1}. \tag{39}$$

Based on the Pontryagin's Minimum Principle, the optimal control u_1^* for the second scenario of the formal and informal P2P lending model is obtained as follows:

$$u_1^*(t) = min\left\{1; max\left\{0, \frac{A\lambda_1}{2r_1}\right\}\right\}. \tag{40}$$

5. NUMERICAL SIMULATION RESULTS

Numerical simulations were run over a period of 150 days for both models. The Backward Forward Sweep method and 4th Order Runge-Kutta method were used in this numerical simulation. The numerical simulation was resolved with the MATLAB program.

5.1. **Numerical Simulation Result for The Simple P2P Lending Model.**

The initial values for the state in the simple P2P lending model are $[S(0), L(0), P(0)] = [150, 50, 10]$ with the weight values in the cost function is $[\omega, r_1, r_2] = [0.0001, 0.3, 0.01]$.

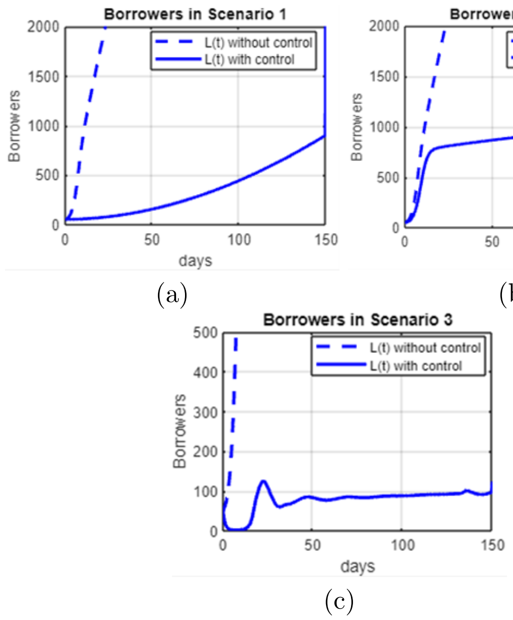


FIGURE 8. Numerical simulation result for the simple P2P lending model in (a) the first scenario, (b) the second scenario, and (c) the third scenario.

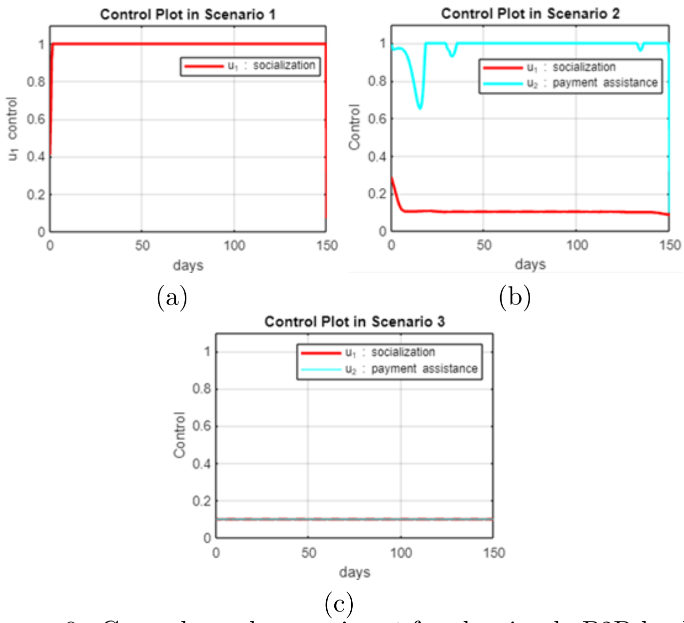


FIGURE 9. Control graph over time t for the simple P2P lending model in (a) the first scenario, (b) the second scenario, and (c) the third scenario.

Table 6. Result of the control scenarios in the simple P2P lending model

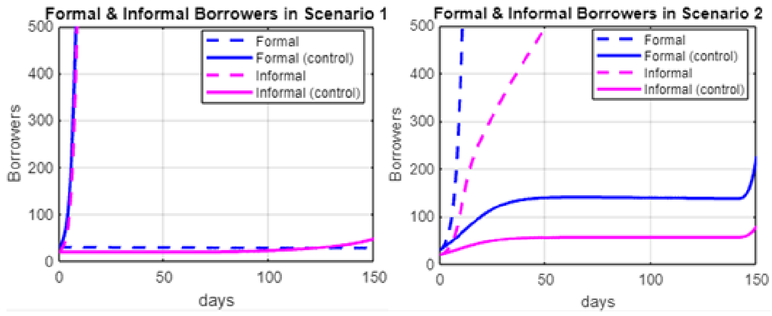
Scenario	Control Description	Borrowers	Cost Function Value
-	Without Control	11763	-
1	Socialization to potential individual	899	1.739×10^4
2	Socialization to potential individual and payment assistance for borrowers	1053	1.199×10^4
3	Socialization to people with work potential and payment assistance for borrowers	97	1.037×10^2

Based on Table 6, at the end observation the given control can reduce the number of borrowers. The first scenario can reduce the number of borrowers by 92.36%. The second scenario can reduce the number of borrowers by 91.05%. The third scenario can reduce the number of borrowers by 99.18%. Based on the value of the cost function, the lowest cost function value of the simple P2P lending model is in the third scenario.

Based on Figure 9(a), socialization to potential individuals are given at maximum all the time, meaning that socialization is highly emphasized in this scenario so that all potential individuals understand the dangers of P2P lending so that they are not influenced by borrowers. Only marketing influence can make potential individuals become borrowers. On Figure 9(b), socialization provided decreased at the initial time and then remained constant after day 10, while payment assistance provided fluctuated at the initial time and then remained constant at the maximum value after day 40, meaning that in Scenario 2, the control in the form of payment assistance is more emphasized to reduce the number of borrowers. On Figure 9(c), socialization and payment assistance are given constantly throughout time t, but the effect given can significantly reduce the number of borrowers, so this control is quite effective in reducing the number of borrowers.

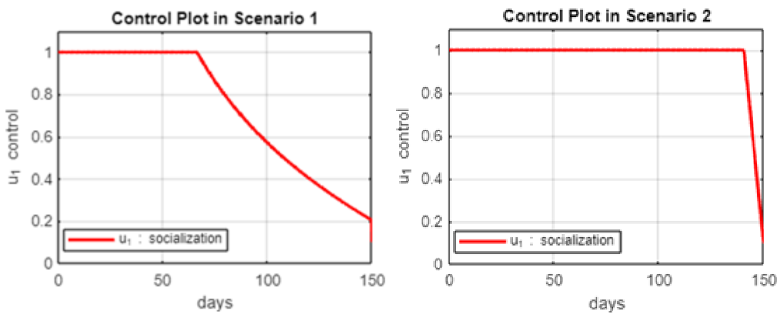
5.2. **Numerical Simulation Result for The Formal and Informal P2P Lending Model.**

The initial values for the state in the second P2P lending model are $[S(0), L_F(0), L_{IF}(0), P(0)] = [150, 30, 20, 10]$ with the weight values in the cost function is $[\omega, r_1] = [0.0004, 0.3]$.



(a) (b)

FIGURE 10. Numerical simulation result for the formal and informal P2P lending model in (a) the first scenario, and (b) the second scenario.



(a) (b)

FIGURE 11. Control graph over time t for the formal and informal P2P lending model in (a) the first scenario, and (b) the second scenario.

TABLE 7. Result of the control scenarios in the simple P2P lending model

Scenario	Control Description	Borrowers	Cost Function Value
-	Without control	$L_F(T) = 10760$ $L_{IF}(T) = 1045$	-
1	Socialization to potential individual	$L_F(T) = 11706$ $L_{IF}(T) = 47$	1.410×10^4
2	Socialization to people with work potential	$L_F(T) = 218$ $L_{IF}(T) = 78$	2.179×10^4

Based on Table 7, at the end observation the given control can reduce the number of borrowers in informal P2P lending. The first scenario can reduce the number of borrowers in informal P2P lending by 95.50%. The second scenario can reduce the number of borrowers in informal P2P lending by 92.54%. Although the first scenario provides a lower cost function value, the number of borrowers in formal P2P lending increases, while in the second scenario, the number of borrowers in formal P2P lending can be reduced by 97.97%.

Based on Figure 11, socialization was initially given at maximum for both scenarios, but in Scenario 1 there was a decrease after day 70 while in Scenario 2 there was a very rapid decrease after day 140. This means that socialization in Scenario 2 is more emphasized, while in Scenario 1, potential individuals are more quickly educated by the socialization so that there is a faster decline in the control given.

6. CONCLUDING REMARKS

Based on the result, it can be concluded that the simple P2P lending model can be modified by adding socialization to people with work potential and payment assistance for borrowers to minimize the number of borrowers as in Scenario 3 of the first model. The formal and informal P2P lending model can be modified by adding socialization to people with work potential to minimize the number of borrowers, both formal and informal P2P lending as in Scenario 2 of the second model.

REFERENCES

[1] Abdullah, A., *Analysis of Online Loan Knowledge in Muslim Communities in Surakarta*, (Journal of Indonesian Sharia Economics, 2021), 9(2), pp. 110-111.

[2] Boyce, W. E., and DiPrima, R. C., *Elementary Differential Equations and Boundary Value Problem 9th Edition*, (John Wiley and Sons, 2009), pp. 513.

[3] Brauer, F., Castillo-Chavez, C., and Feng, Z., *Mathematical Models in Epidemiology*, (Springer-verlag, New York, 2019), 69, pp. 26–35.

[4] Budiyanti, E., *Resolving Illegal Financial Technology Businesses*, (Journal of Brief Info, 2019), 11(4), pp. 20.

[5] CNBC Indonesia (2022, December 17), *Data on Indonesian who cannot Pay Online Loans*. Retrieved April 21, 2023, from https://www.cnbcindonesia.com/tech/20221217130201-37-397750/ini-bukti-makin-banyak-warga-ri-gak-bayar-pinjol-cek-datanya.

[6] Financial Services Authority: *Financial Services Authority Regulation Number: 77/POJK.01/2016 concerning Information Technology-Based Money Lending and Borrowing Services*, (Financial Services Authority, 2016), pp. 1–29.

[7] Hale, J. K. and Kocak, H., *Dynamic and Bifurcation*, (Springer-verlag, New York, 1991).

[8] Lewis L., Frank, Draguna L. Vrabie, and Vassilis L.Syrmos, *Optimal Control*, (John Wiley and Sons, 2012), pp. 112–115, 143–145.

[9] Madubueze, C. E., Dachollom, S., and Onwubuya, I. O., *Controlling the Spread of COVID-19: Optimal Control Analysis*, (Computational and Mathematical Methods in Medicine, 2020), pp. 5–10.

[10] Santoso, W., Trinugroho, I., and Risfandy, T., *What Determine Loan Rate and Default Status in Financial Technology Online Direct Lending? Evidence from Indonesia*, (Emerging Markets Finance and Trade, 2020), 56(2), pp. 351–369.

[11] Wahyuni, R. A. E., and Turisno, B. E.: *Illegal Financial Technology Practices in the Form of Online Loans in Review of Business Ethics*, (Journal of Indonesian Legal Development, 2019), 1(3), pp. 380–381.

[12] Wiggins, S., *Introduction to Applied Nonlinear Dynamical System and Chaos*, (Springer-verlag, New York, 1990).

[13] Wonglimpiyarat, J., *Fintech Banking Industry: A Systemic Approach*, (Foresight, 2017), 19(6), pp. 590–603.