SUKUK VERSUS BONDS: MATHEMATICALLY LITERATURE STUDY

Wahyuning Murniati[1a], Novriana Sumarti[2b], Dila Puspita[2c]

[1]Doctoral Program, Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung, Ganesha 10 Bandung, Indonesia

[2]Industrial and Financial Mathematics Research Group, Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung, Ganesha 10 Bandung, Indonesia.

[a]30122001@mahasiswa.itb.ac.id, [b]novriana@itb.ac.id, [c]dpuspita@itb.ac.id

Abstract. Based on IIFM Sukuk Report 2022, global sukuk issuance has shown a single digit increase of around 7.72%, the highest value of yearly sukuk issuances to date. This fact shows that sukuk is increasingly popular and will continue to grow in the future, encourages more research related to this subject. Sukuk, as one of the innovative investment instruments in Islamic finance, is often equated to conventional bonds. However, the existing literature primarily concentrates on its structures and mechanisms, largely overlooking the pricing aspects of both sukuk and bonds. Therefore, this study will discuss pricing in mathematical terms for the two investment instruments discussed. More particularly, this paper covers the Indonesian Sukuk system's operation. Numerical simulations have been done, using the characteristics of several varieties of Indonesian Sukuk and a variety of defined inputs. These mathematics simulation used to comprehend the technical aspects of sukuk and understand the principles of Islamic finance. The results of this study indicate that while some studies claim that sukuk's structure resembles bonds and that the pricing is determined accordingly, the comparison of both instruments prices provided no results consistent with the claims. Further study is required to examine the sukuk pricing mechanism, particularly in determining the value of profit sharing on sukuk.

Key words and Phrases: Sukuk, Bond, Pricing Mechanism, Mathematics Finance

1. INTRODUCTION

The International Islamic Financial Market (IIFM) is a prominent global organization within the Islamic Financial Services Industry (IFSI). Its primary

objective is to establish and promote standardized financial documentation that adheres to Shariah principles, as well as to ensure the validation of Islamic financial products and the adherence to related standards. Based to the Sukuk Report 2022 published by the International Islamic Financial Market (IIFM) [1], the global sukuk market is seeing a notable surge in popularity and is expected to sustain its growth trajectory in the foreseeable future. The global sukuk issuance in 2020 to 2021 reached a significant milestone, with a single-digit growth rate of around 7.72% or USD 174.641 billion to USD 188.121 billion. This represents the biggest annual value of sukuk issuance recorded thus far. Similar scenarios also place in Indonesia. Based to Laporan Perkembangan Keuangan Syariah Indonesia 2022 by Otoritas Jasa Keuangan (OJK) [2], as of December 2022, the number of outstanding corporate sukuks has increased to 221 series, indicating a growth of 16.93% compared to the previous year, where there were 189 series. Value-wise, it rose by 22.24% during the course of the previous year to IDR 42.50 trillion. Similarly, the total value of State Sukuk has witnessed a significant rise, reaching Rp. 2,223.01 trillion, subsequent to the enactment of UU Nomor 19 Tahun 2008, which pertains to Surat Berharga Syariah Negara (SBSN). The rise in the issuance of SBSN highlights the growing significance of SBSN in funding infrastructure projects, addressing state budget shortfalls, and expanding the Islamic financial system.

The increasing prevalence of sukuk issuance has led to a broadening scope of study within the field of this financial instrument. The existent literature predominantly centers on the empirical, legal, regulatory, supervisory, and other facets pertaining to research. Furthermore, a majority of the research employs statistical approach as the basis for distinguishing between the two investment instrument, sukuk and bonds. Sukuk were initially developed under the premise that they have similarities to conventional bonds, but incorporate Islamic principles. However, the existing literature presents varying findings about the differentiation between bonds and sukuks. According to the findings of Ayturk et al. [3], it can be observed that the primary market spreads of bonds and sukuk are influenced by a common set of attributes. This suggests that market participants perceive these two financial products in a similar manner. Hossain et al.[4] also observe no statistically significant disparities in the returns of sukuk and bonds. According to Miller et al. [5] and Wilson [6], it is argued that sukuk products frequently adhere to Western securitization standards and do not embody substantial financial innovation. In contrast, the study conducted by Balli et al. [7] reveals that bonds exhibit a significantly higher level of sensitivity compared to sukuk when exposed to fluctuations in global markets. Additionally, the research highlights substantial differences in the methodologies employed to calculate the yields of sukuk and bonds. Furthermore, the study conducted by Naifar and Hammoudeh [8] finds a lack of correlation between uncertainty in traditional bond markets and the returns observed in the Middle Eastern sukuk that were examined. Moreover, Asmuni and Tan [9] provides evidence supporting the existence of substantial variations in yields between sukuk and bonds issued by the government in the Malaysian market. The researchers

ascribe these variations to the influence of liquidity factors. Fathurahman and Fitriati [10] made a comparison between the yields of sukuk and conventional bonds listed on the Indonesia Stock Exchange. The findings of the study revealed that sukuk demonstrated superior returns in comparison to conventional bonds. Cakir and Raei [11] present an alternative viewpoint by asserting that sukuk possess distinctive risk-mitigation advantages when incorporated into a fixed income securities portfolio, hence distinguishing them from conventional bonds. All variations in the findings of these studies can be explained by Almaskati [12], the existing literature presents contrasting findings about the distinction between sukuk and bonds and the disagreement in the literature might be caused largely by the methodologies employed in many these study.

Generally the present focus on empirical studies, sukuk and bonds, there is a notable lack of attention given to the pricing mechanism. Whereas, pricing mechanism is a crucial factor for investors to consider before to making a decision regarding the utilisation of a financial instruments. Due of the perceived similarity between sukuk and bonds, the pricing mechanism of the two investment products will be studied in mathematical terms in this study. More particularly, this paper analyzes the system of the Indonesian sukuk. This study has provided us with the data necessary to motivate our future research, particularly concerning pricing mechanisms.

2. THE DEVELOPMENT OF SUKUK IN INDONESIA

The Islamic financial market in Indonesia is observed to exhibit a higher growth rate compared to the conventional financial industry. The enactment of the SBSN Law presents a favorable circumstance for the advancement of Islamic financial institutions in the creation of various Islamic financial products, such as sukuk [13]. This expansion of sukuk in Indonesia is characterized by notable swiftness; yet, it is imperative that the employed method aligns with the contractual principles rooted in Islamic Law. Dewan Syariah Nasional of Indonesia, in Fatwa No. 137/DSN-MUI/IX/2020, has outlined several provisions regarding sukuk issuance. These provisions emphasize the necessity of adhering to Sharia principles in the contractual framework of sukuk; *mudharabah, ijarah, wakalah,musharakah, murabahah, salam, istishna'* and others [14]. Additionally, OJK releases the frameworks for corporate sukuk in Indonesia. According to the Himpunan Skema Sukuk Korporasi 2019 by Otoritas Jasa Keuangan, Indonesia exclusively employs the *mudharabah, wakalah,* and *ijarah* akad in its practical implementation [15].
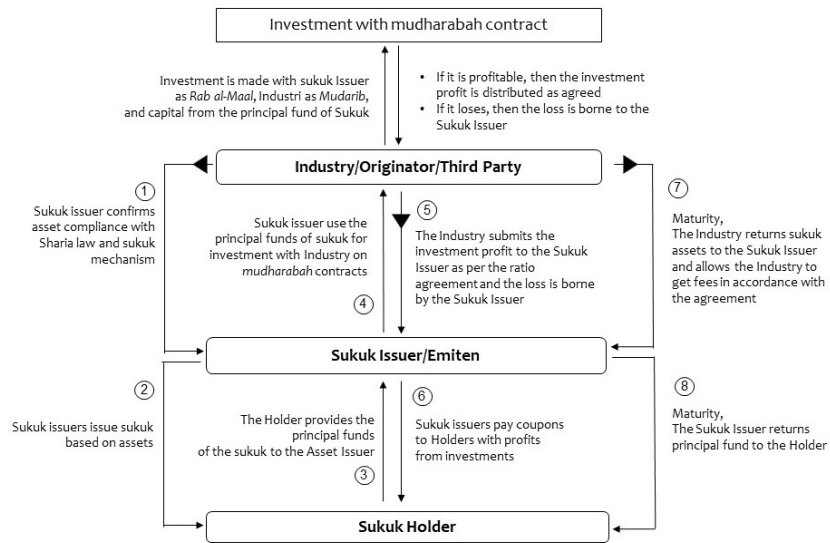
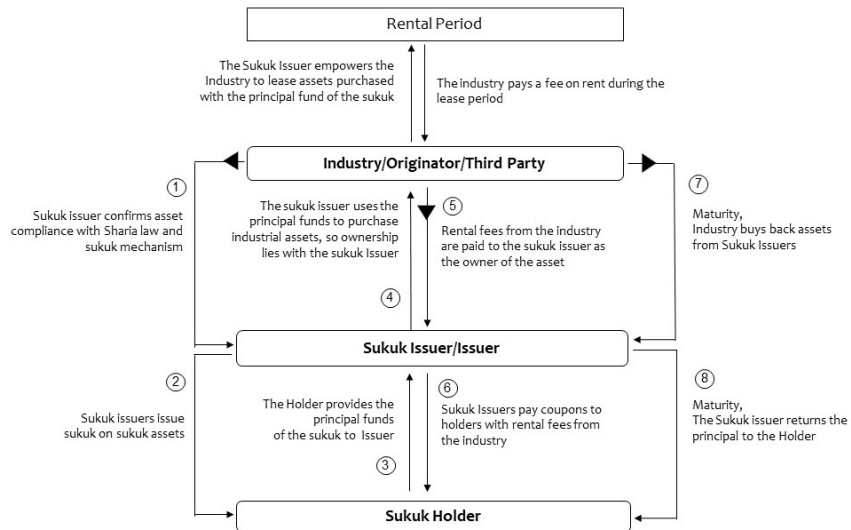FIGURE 1. *Mudharabah* Sukuk Cash Flow Framework

FIGURE 2. *Ijarah* Sukuk Cash Flow Framework

The term *mudharabah* is commonly interpreted as a type of partnership struc-
ture that is based on equity. In this arrangement, one partner, known as the *rab*

al-maal, contributes capital, while the other partner, known as the *mudarib*, gives management expertise [16]. Based on the structure sukuk discussed in Sukuk Guide Book (2009) and some applicable regulation, we obtained the cash flow of sukuk *mudharabah* on Figure 1. The next sukuk contract in Indonesia, is *ijarah*. The concept of *ijarah* is widely recognized as the act of transferring the right to use and enjoy an asset to another individual in return for a rental payment. The Himpunan Skema Sukuk Korporasi by OJK, state that the total of 232 sukuk issuances consist of 28 prospectuses utilize *mudharabah* contracts, 47 prospectuses employ *ijarah* contracts, and 2 prospectuses rely on *wakalah* contracts [15]. We compose a comprehensive analysis of the *ijarah* cash flow scheme on Figure 2. The establishment of sukuk *wakalah* represents a notable development in contemporary sukuk structures. The aforementioned structure is derived from the concept of a *wakalah*, which can be defined as a contractual arrangement wherein one party grants permission to another party to act on its behalf. *Wakalah* contract can be comprehended as a type of agency arrangement. The following is a demonstrative representation of a sukuk *wakala* framework on Figure 3.
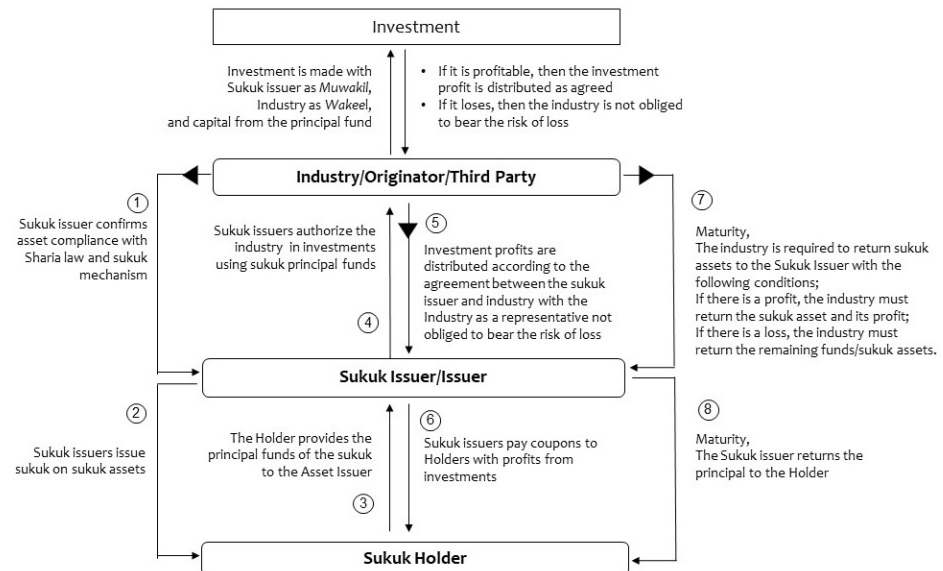

Figure 3. *Wakalah* Sukuk Cash Flow Framework

3. THE PRICING OF BONDS

As previously elucidated, a majority of studies employ a statistical methodology to differentiate between sukuk and bonds. It has been posited that sukuk possess a structure similar to conventional bonds. So this section will delve into

the process of ascertaining bond prices within the context of the literature being examined in this study.

Bonds are among the financial instruments that are carefully evaluated by a discerning trader. A bond is a security that accrues interest and guarantees the payment of a specified quantity of money at a future date [17]. In a similar vein, Vaaler, Harper and Daniel [18] provides a definition of a bond as a financial instrument that is issued by either a governmental body or a corporate organization, with the purpose of guaranteeing specific payments to be made at a designated future period. Bonds are generally redeemed at the end of a fixed period of time. The fixed duration of the bond is referred to as its term. The termination of a bond's contractual period is referred to as its maturity date. Therefore, the pricing of a bond necessitates the equality between its price and the combined present value of its redemption value and the future coupon payments. There exist four distinct types of formulas that can be used to determine the price of a bond P [17, 18], on Table 1, with following assumptions are;

(1) The bond issuer is responsible for making all payments on the designated payment dates. Disregard any market fluctuates that may impact the price of bonds.
(2) The bond has a fixed maturity date.
(3) The price of a bond is typically sought shortly following a coupon payment date

TABLE 1. Bond Price Formulas

Basic
$$P = Fra_{\overline{n}|} + Cv^n \qquad (1)$$
Premium/discount
$$P = Fra_{\overline{n}|} + C(1 - ia_{\overline{n}|}) = C + (Fr - Ci)a_{\overline{n}|} \qquad (2)$$
Base
$$P = Gia_{\overline{n}|} + Cv^n = G\left(1 - v^n\right) + Cv^n = G + (C - G)v^n \qquad (3)$$
Makeham
$$P = Cg\left(\frac{1 - v^n}{i}\right) + Cv^n = \frac{g}{i}(C - Cv^n) + Cv^n = \frac{g}{i}(C - K) + K \qquad (4)$$

Source: Kellison (2009) [17]

The following symbols are utilized in the bond price formula;

Based on definition, the first one, referred to as the basic formula, is the most direct and straightforward. The basic formula represents the sum of two elements, the present value of coupon payments throughout the term and the value of redemption at either the issue date or maturity. In usage, it is observed that bonds have the potential to be sold either at a premium or at a discount.A bond

F = par or face value, the amount is required to be paid at the specified date of maturity for n number of coupon payment period

r = coupon rate

Fr = amount of coupon

C = redemption value of bond

K = present value of redemption at the yield rate

i = yield rate of a bond

n = number of coupon payment period

G = the base amount of bond, defined $Gi = Fr$, invested amount at the yield rate i for producing periodic interest payment equal to the coupons

g = the modified coupon rate of a bond, is defined as being compatible with the formula $Fr = Cg$

is considered to be trading at a premium when its price exceeds its redemption value and contrasting financial instrument is the discount bond, characterized by a market price that is lower than its redemption value, then we have,

$$\text{premium } = P - C = (Fr - Ci)\, a_{\overline{n}|i} = C(g - i) a_{\overline{n}|i}$$
$$\text{discount } = C - P = (Ci - Fr)\, a_{\overline{n}|i} = C(i - g) a_{\overline{n}|i} \tag{5}$$

The base formula is built by define G as invested amount at interest rate i, and the last formula is the Makeham formula, proves to be advantageous in cases where the present value K of redemption is known, but the number of coupon n has not been provided [17]. In addition, this formula is also applicable for computing serial bonds. The pricing of an issue of serial bond can most efficiently be performed using Makeham's formula [17]. Assume that the serial bonds are redeemable at z different redemption dates, thus we have,

$$P_1 = \tfrac{g}{i}(C_1 - K_1) + K_1$$
$$P_2 = \tfrac{g}{i}(C_2 - K_2) + K_2$$
$$\vdots$$
$$P_z = \tfrac{g}{i}(C_z - K_z) + K_z$$

and summing, we obtain

$$P' = \frac{g}{i}(C' - K') + K' \tag{6}$$

where,

$$P' = \sum_{x=1}^{z} P_x \quad C' = \sum_{x=1}^{z} C_x \quad K' = \sum_{x=1}^{z} K_x$$

Thus, the price of an entire issue of series bonds is denoted by P'.

3.1. Determination of Yield Rates.

Based on the aforementioned formulas, it is evident that the yield i holds significant importance as an input in the computation of bond prices. Kellison [17] clarify that the process of ascertaining the yield to maturity on a bond bears

resemblance to the process of determining an unknown rate of interest for an annuity. One effective method for ascertaining the yield to maturity of a bond with an unknown value is to using a financial calculator. The approach initiates by employing a formula for premium bond price and define the variable $d = \frac{P-C}{C}$, denoting the ratio of the discrepancy between price and redemption. Using the formula for premium pricing, we have;

$$(g - i)\, a_{\overline{n}|} = \frac{P - C}{C} = d$$

then,

$$i = g - \frac{d}{a_{\overline{n}|}}$$

The solution to the aforementioned equation is derived through the utilisation of a series expansion, namely the binomial series. This approach allows for the determination of the yield to maturity, irrespective of any rank greater than i. Bond Salesman's Methods applied with replacing $\frac{(n+1)}{2n}$ with $\frac{1}{2}$.

$$\frac{1}{a_{\overline{n}|}} = \frac{i}{1 - (1+i)^{(-n)}}$$
$$= \frac{1}{n}\left[1 + \frac{n+1}{2}i + \frac{n^2-1}{12}i^2 + \ldots\right]$$

This allows for the establishment of a correlation for i formula is,

$$i = \frac{g - \frac{d}{n}}{1 + \frac{n+1}{2n}d} \approx \frac{g - \frac{d}{n}}{1 + \frac{1}{2}d} \tag{7}$$

4. DESIGN OF SUKUK PRICING

The widespread introduction of sukuk, frequently referred to as Islamic bonds, was initiated in February 1988, subsequent to the fourth session of the Council of the Islamic Fiqh Academy of the Organization of the Islamic Conference (OIC) in Jeddah, Saudi Arabia. The issuing of sukuk, as an initial purpose, serves as a reaction to the demand from issuers and investors in Muslim countries that desire an alternative financial and investment model that aligns with the principles of Islamic Sharia. In May 2003, the Accounting and Audit Organization for Islamic Financial Institutions released a publication titled "Standards for Sukuk Investment." This document provided a definition of Sukuk as certificates that hold an equivalent value to those indicating undivided share ownership in tangible assets, facilities, and services [19]. This factor contributes to the expanding field of study concerning sukuk. Patrinieri et al. [20] conducted an analysis of the existing literature on sukuk from the years 1950 to 2018. This paper presents an analysis of the co-authorship network and identifies three distinct research streams. This paper examines three key aspects related to sukuk: (1) a summary of sukuk and its growth; (2) the relationship between sukuk and finance theories; and (3) the impact of sukuk on stock market behavior. This study is in line with Zulkhibri [19], who also stated

that the current inventory of literature on sukuk primarily consists of qualitative research rather than quantitative studies.

4.1. **Sukuk Pricing Mechanism.**

Razak, Saiti and Dinc [21] stated the pricing mechanism of sukuk is a significant topic of discussion; however, it has been largely overlooked in existing academic research. This study claims that the general concept of pricing in sukuk bears resemblance to that of bonds. Razak, Saiti and Dinc [21] analyze six types of sukuk in Malaysia; Cagamas Mudharabah Bond(CMB), Bank Negara Monetary Notes (BNMN-i) Sukuk, The Government Investment Issues (GII), Ijarah Sukuk, Murabaha and Wakalah Sukuk. The price formula that is examined in this study is presented in Table 2.

Cagamas Mudharabah Bond (CMB) have two steps in pricing this kind of sukuk [22]. The starting point involves establishing the book price B on a perface value basis (F), Bank Negara Malaysia set RM100, for the sukuk. Next type is **Bank Negara Monetary Notes (BNMN-i)** sukuk, is a transactional arrangement in which a commodity is initially sold to a buyer on a deferred payment basis (known as *Thaman mu'ajjal*), and subsequently repurchased by the original seller for immediate cash payment at a price lower than the originally agreed deferred price. Furthermore the government issued **Government Investment Issues** (GII) for the necessary funding. The government will initially engage in the sale of its Shariah-compliant assets, like as equities, to financial institutions in exchange for immediate cash payment. After the conclusion of the sale, financial institutions will proceed to sell the assets back to the government at a profit that will be deferred until a later time.

TABLE 2. Malaysian Sukuk Price Formulas

CMB (*mudharabah* sukuk)	
$$B = F\frac{\left(1 + \left(\frac{r \times E}{365}\right)\right)}{\left(1 + \frac{i \times T}{365}\right)} - \left(F \times \frac{r \times t}{365}\right)$$	(8)
$$P = \frac{G \times B}{F} + \left(G \times \frac{r \times t}{365}\right)$$	(9)
BNMN-i (*bay al-inah* sukuk)	
$$P = C\left(1 - \frac{i \times k}{365}\right)$$	(10)
GII (*bay ad-dayn* sukuk)	
$$P = \left(\frac{C}{(1 + \frac{i}{2})^{N-1+\frac{T}{E}}}\right) + \left(\sum_{M=1}^{N} \frac{\frac{Fr}{2}}{(1 + \frac{i}{2})^{N-1+\frac{T}{E}}}\right)$$	(11)
Note: the coupon is paid semi-annually	

Source: Razak dkk. (2019) [21]

where,

E = number of days in current coupon period
T = number of days from transaction date to next coupon payment day
t = number of days from last coupon payment date to the value date
k = number of days left to maturity
M = 1, the first payment coupon
N = number coupon payment from transaction date to maturity

Razak, Saiti and Dinc [21] also discuss the pricing of ijarah sukuk, called **Sukuk Bank Negara Malaysia Ijarah (SBNMI)**. In a typical sukuk ijarah structure, an investor is entitled to receive two distinct forms of cash flows in return for an initial capital investment. The pricing of these fundamental sukuk instruments would be determined by the present value of the cash flows that can be obtained through sukuk investment. Nevertheless, if the sale price is not pre-established, it becomes necessary to generate an approximation of the anticipated sale price. One alternative approach entails the assignment of many probable sale prices, along by the allocation of probability to each respective price point. The projected sale price is determined by calculating the weighted average of this value. If each potential selling price is referred to the l_{th} scenario, the following equation represents [22];

$$\text{Expected sale price} = \sum \left(Prob_l \times SP_l\right) \tag{12}$$

where,

$$Prob_l \; = \; \text{probability of the scenario } l$$
$$SP_l \quad = \; \text{sale price of the asset under the scenario } l$$

Other sukuk discussed by Razak, Saiti and Dinc [21] is *murabaha* and *wakalah* sukuk. *Murabaha* sukuk is refers to the issuance of securities in which the obligor and issuer enter into an agreement for the ostensible to sell an asset and subsequently repurchase it at a markup. As for the *murabaha* sukuk (cost plus profit sale-based Islamic bond), it is an Islamic trust contract where the original price and the markup need to be disclosed. The last type is *wakalah* sukuk (agency based Islamic bond), is among the famous sukuk in the global Islamic finance market. This type of sukuk has similarities with the *mudharabah* structure. The only difference is that there is no pre-agreed profit ratio in a *wakalah* structure. The *wakeel* will only be given performance or incentive fee from the excess of the agreed profit.

4.2. **Pricing Model of *Ijarah* Corporate Sukuk.**

Rosadi's research illustrates how *ijarah* contracts use the sukuk model for corporate type [23]. The structure model built based on the cash flow of *ijarah* sukuk [24] as follow;

(1) At time zero $y = 0$, the sukuk holders transfer the funds generated by *ijarah* sukuk to the Special Purpose Vehicle (SPV), which acts as an autonomous governing body mediating between the issuer and the sukuk holders.

(2) During the contractual agreement, sukuk holders are entitled to receive periodic payment from SPV that are equivalent to the leasing payment received from the borrower.

(3) In the maturity $y = Y$, upon the conclusion of the contractual agreement, (SPV) proceeds to relinquish ownership of the asset by selling it back to the borrower. In turn, the borrower compensates the SPV by paying the repurchase price P_r for the asset. SPV allocates the amount of dissolution distribution to the holders of sukuk.

Based on the aforementioned cash flows, it is evident that the cash flows of *ijarah* sukuk exhibit similarities to those of conventional instruments. Sukuk *ijarah* is a rather costly variant of the standard bond. The instrument is seen as costly because of the considerable transaction costs associated with it [23]. The transaction associated with sukuk *ijarah* can be delineated into the following components:

(1) Periodic payment. During the sukuk tenure, SPV disburses periodic payments to the holder, which are equivalent to the rental payments received from the borrower. The present value of the lease payments or the periodic payment amounts is,

$$\sum_{y=1}^{Y} \frac{Fr}{(1+i)^y} = \frac{Fr}{1+i} + \frac{Fr}{(1+i)^2} + \cdots + \frac{Fr}{(1+i)^Y}$$

(2) Repurchase Price P_r by asset value. If asset value follows Brownian Motion Geometry, then the present value formula is

$$K_r = \frac{P_0 e^{\mu Y}}{(1+i)^Y}$$

where P_0 is asset value on $y = 0$ with $P_r = P_0 e^{\mu Y}$. If value of fixed assets based on the agreement, the present value formula is,

$$K_r = \frac{P_r}{(1+i)^y}$$

(3) Transaction Cost consist of SPV fee is $Fee = P_0 \times a$ where a is proportion of the annual asset price, fee of Legal and Shari'ah Board is S, and fee of maintenance and insurance according to the present value formula

$$\sum_{y=1}^{Y} \frac{O}{(1+i)^y} = \frac{O}{1+i} + \frac{O}{(1+i)^2} + \cdots + \frac{O}{(1+i)^Y}$$

where O is the maintenance and insurance fee per coupoun payment during the period of sukuk.

The price of sukuk *ijarah* is the net present value of all cash flows for this instrument. The study acquire the price sukuk formula, especially for corporate type, by combining all elements of the transaction [24],

$$P = \sum_{y=1}^{Y} \frac{Fr}{(1+i)^y} + \frac{P_r}{(1+i)^Y} + P_0.a + S + \sum_{y=1}^{Y} \frac{O}{(1+i)^y}$$

$$= \sum_{y=1}^{Y} \frac{Fr + O}{(1+i)^y} + \frac{P_r}{(1+i)^Y} + P_0.a + S$$

The following is then obtained by applying the geometric series concept to the coupon present value and maintenance costs equation,

$$\sum_{y=1}^{Y} \frac{Fr + O}{(1+i)^y} = \frac{Fr + O}{(1+i)^1} + \frac{Fr + O}{(1+i)^2} + \cdots + \frac{Fr + O}{(1+i)^Y}$$

$$= Fr + O \left(\sum_{y=1}^{Y} \frac{1}{(1+i)^y} \right)$$

$$= Fr + O \left(\frac{1}{(1+i)^1} + \frac{1}{(1+i)^2} + \cdots + \frac{1}{(1+i)^Y} \right)$$

For $\frac{1}{1+i}$ as first term and ratio, then we have,

$$S_Y = \frac{a(1-ra^Y)}{1-ra}$$

$$= \frac{\frac{1}{1+i}\left(1-\left(\frac{1}{1+i}\right)^Y\right)}{1-\frac{1}{1+i}}$$

$$= \frac{\left(1-\frac{1}{(1+i)^Y}\right)}{i}$$

furthermore,

$$\sum_{y=1}^{Y} \frac{Fr+O}{(1+i)^y} = Fr+O\frac{\left(1-\frac{1}{(1+i)^Y}\right)}{i}$$

The price of corporate sukuk with an ijarah contract [24] is,

$$\begin{aligned}
P &= \frac{Fr+O}{i}\left(1-\frac{1}{(1+i)^Y}\right) + \frac{P_r}{(1+i)^Y} + P_0.a + S \\
&= \frac{Fr+O}{i} - \frac{Fr+O}{i}\left(\frac{1}{(1+i)^Y}\right) + \frac{P_r}{(1+i)^Y} + P_0.a + S \\
&= \frac{Fr+O}{i} + P_0.a + S + \frac{P_r}{(1+i)^Y} - \frac{Fr+O}{i}\left(\frac{1}{(1+i)^Y}\right) \\
&= \frac{Fr+O}{i} + P_0.a + S + \left(P_r - \frac{Fr+O}{i}\right)\frac{1}{(1+i)^Y}
\end{aligned} \tag{13}$$

where,

$$\begin{aligned}
i = \rho &= \frac{U}{V}(1-b+bq) \\
b &= f + s \\
q &= \frac{V}{R}
\end{aligned} \tag{14}$$

U = expected firm's accounting earnings
V = market price of firm
f = expected retention rate
s = expected stock financing rate
R = firm's replacement cost

Variable $i = \rho$ relates the cost of equity capital to Tobin's q [25]. This study provides a methodology that enables the measurement of the cost of capital without relying on a set and predetermined interest rate. As stated by Rosadi [23], it is proposed to replace the discount rate value with the value of expected yields. The calculation of this expected yield relies on various factors, such as the yield to

maturity, credit risk premium, inflation rate, liquidity premium, and other relevant considerations.

4.3. **Retail Sukuk Pricing.**

Sumarti has developed a framework for analysing the revenue and income generated by Sukuk Ritel of Indonesia in both the main and secondary markets [26]. The primary market is the initial venue where securities are originated and offered to the general public for the first time. In contrast, the secondary market pertains to the trading of these securities among investors, encompassing notable exchanges such as the New York Stock Exchange, the Nasdaq, and various other global exchanges [27]. Let p is percentage of income tax and m is the duration of having sukuk, then price for purchasing on the primary market,

$$P = F \times r \times m \times (1 - p) + F = F(1 + rm(1 - p)) \tag{15}$$

Then the sukuk is sold on the secondary market with Y is sukuk period, sukuk on par price in accordance with the funds invested $G(0) = F$ if $m = Y$. Hence for $m < Y$ with $m = b$ months h days, $G(m) > F$ for premium sukuk and $G(m) < F$ for discount sukuk. Let's say x_p is capital gain from the price, $G(m) = F + x_p$ and x_d is capital loss from the price, $G(m) = F + x_d, x_d < 0$. If $G(m) = F + x_p > F$, the price of sukuk is,

$$\begin{aligned}
P &= F \times r \times \frac{b}{12} \times (1 - p) + F \times r \times \frac{h}{365} \times (1 - p) + x_p(1 - p) + F \\
&= \left(Fr\frac{b}{12} + Fr\frac{h}{365} + x_p \right)(1 - p) + F
\end{aligned} \tag{16}$$

If $G(m) = F - x_d < F$, the price of sukuk is,

$$\begin{aligned}
P &= F \times r \times \frac{b}{12} \times (1 - p) + F \times r \times \frac{h}{365} \times (1 - p) + x_d + F \\
&= \left(Fr\frac{b}{12} + Fr\frac{h}{365} \right)(1 - p) + x_d + F
\end{aligned} \tag{17}$$

while the rate of return, $r_r = \frac{P - F}{F}$ per m years.

Purchase in the secondary market if remaining time $k < Y$ sukuk is bought in the secondary market, say $k = b$ months h days, sukuk price is,

$$\begin{aligned}
P &= F \times r \times \frac{b}{12} \times (1 - p) + F \times r \times \frac{h}{365} \times (1 - p) + F \\
&= \left(Fr\frac{b}{12} + Fr\frac{h}{365} \right)(1 - p) + F
\end{aligned} \tag{18}$$

while rate of return $r_r = \frac{P - G(k)}{(G(k))}$.

5. THE NUMERICAL SIMULATION

In this section, we employ various Indonesian Sukuk in the preceding price formula under predefined characteristics. Here we consider using the transaction date is October 12, 2021 with daily yield of Indonesia, 6.205% [28]. Suppose we have fund $F =$IDR. 30.000.000,00 for investment, we continue with calculating the price of each investment instrument here.

TABLE 3. Sustainable Sukuk *Mudharabah* I Sarana Multigriya Finansial Phase III 2021

	Characteristics of Sukuk
Code	SMSMFP01CN3
Issue dates	February 10, 2021
Maturity	February 20, 2022
Total fund	IDR 1,000,010,000,000.00
Coupon rate	4.75%
Payment coupon date	Quarterly pay
First coupon payment	May 10, 2021
Last coupon payment	Februari 20, 2022

Source: Sukuk Prospectus

The first sukuk is SMSMFP01CN3, which refers to the Sustainable Sukuk *Mudharabah* I Sarana Multigriya Finansial Phase III 2021. Based on Table 3, numerical stimulation of this sukuk calculated for CMB, BNMN-i and GII formula. This simulation specifically modifies the base fave value currency to IDR in order to correlate with the specific characteristic of the sukuk, particularly for the CMB formula. The simulation result given by Table 4, indicates that the price of BNMN-i is comparatively lower than that of other financial instruments and the face value itself. This discrepancy can be attributed to the implementation of a discounted basis mechanism, as indicated by the provided main value. This table also presents the results obtained by increasing the yield value. The findings suggest that alterations in yield have a discernible impact on the results of the formula. An augmentation in SMSMFP01CN3 yields leads to a corresponding decrease in prices for transactions executed on the same date, with identical coupon rates and nominal values. Here we assume that the price of this sukuk is impacted by the prevailing yield. However, further simulation is required to ascertain the extent of this impact.

TABLE 4. SMSMFP01CN3 on price formula

$i(\%)$	CMB	BNMN-i	GII
6.205	IDR 30,241,923.49	IDR 29,331,900.00	IDR 30,099,126.42
6.500	IDR 30,000,000.61	IDR 29,300,136.99	IDR 30,070,505.49

According to Razak, Saiti and Dinc study [21], the pricing of basic ijarah sukuk is determined by the combine present value of periodic payments received and the anticipated sale price of the underlying asset at the time of maturity. Rosadi's study [23] presents the design of the ijarah sukuk model, drawing upon the findings of Dinna's study [24]. Moreover, this study incorporates the characteristics of SIPPLN02ACN1, on Table 5, in Formula 13 with some determined cost. As a result, the calculated price is IDR 32,179,188.74 in addition of a 2.5% SPV fee, a IDR 500,000 legal board fee and an operating cost of IDR 50,000 per coupon payment period. Next, when the yield is augmented by 6.5%, the price of the sukuk is IDR 31,936,526.01 means that SIPPLN02ACN1 encounters an identical scenario to the aforementioned sort of sukuk in terms of the rise in yield value. However, besides to the yield value, the determination of the cost value is expected to have a substantial impact on the price value based on this *ijarah* sukuk formula.

TABLE 5. Sustainable Sukuk *Ijarah* PLN II Phase I 2017 Series A

	Characteristics of Sukuk
Code	SIPPLN02ACN1
Issue dates	July 11, 2017
Maturity	July 11, 2022
Total fund	IDR 186,000,000,000.00
Coupon rate	7.70%
Payment coupon date	Quarterly pay
First coupon payment	October 11, 2017
Last coupon payment	July 11, 2022

Source: Sukuk Prospectus

Next study is conducted by Sumarti [26], presents a pricing formula that is specifically associated with Indonesian retail sukuk both the primary and secondary market. Here we employ SR011 issued on March 28, 2019 with three years of period [29]. Table 6 describes the characteristics of SR011, utilised for incorporation into the resultant equation, which yield does not include.

Let the investor acquired SR011 on the primary market on March 28, 2019. When the investment was held until its maturity on March 10, 2022, spanning a period of 3 years or 36 months. Consequently, the investor received a total revenue is IDR 36,158,250.00 as a price of sukuk with a tax rate of 15%. However, when the investor were to sell the sukuk on the secondary market after purchasing it on the primary market, specifically on October 12, 2021, at 101.9% capital gain, the resulting price based on the equation 16 is IDR 35,705,181.59. Nevertheless, if the capital loss is 98% of the price then the value in Indonesian Rupiah is IDR 34,610,610.62. Next if investors acquire the sukuk on the secondary market on the specified transaction date at a price of 101.9%, the resulting value will amount to IDR 30,830,473.29. From this perspective, it can be observed that retail sukuk exhibit a comparatively elevated pricing in relation to corporate sukuk, although

the calculation of the formula does not use any yield on. Since that the coupon rate has a considerable effect on the price, we increase its value and observe a corresponding increase in the result.

Table 6. SR011 Indonesian Retail Sukuk

	Characteristics of Sukuk
Code	SR011
Issue dates	March 28, 2019
Maturity	March 10, 2022
Total fund	IDR 21,117,570,000,000.00
Coupon rate	8.05%
Payment coupon date	Monthly on 10th
First coupon payment	May 10, 2019
Last coupon payment	March 10, 2022

Source: Official Website of the Ministry of Finance, IDX and OJK

For the purpose of comparison, we compute bond prices based on the specific characteristics of the sukuk employed in this study. Table 7 presents the outcome of a numerical simulation of Formula 1, which is the basic formula for price bond calculating. The findings indicate that, according to the sukuk pricing formula examined in this study, bond prices generally exhibit a lower value compared to sukuk prices, given an equivalent yield. The prices of SMSMFP01CN3 and SIP-PLN02ACN1 are more expensive than the prices of bonds with the same instrument characteristics. Meanwhile, SR011 offers a more affordable pricing compared to the bond price for direct transactions in the secondary market. If the first transaction takes place in the primary market, whether resulting in capital gains or losses, sukuk prices generally remain higher than the prices of bonds.

Table 7. Bond Price on Basic Formula

$i(\%)$	Sukuk		
	SMSMFP01CN3	SIPPLN02ACN1	SR011
6.205	IDR 29,202,017.23	IDR 31,194,313.30	IDR 32,318,598.01
6.500	IDR 29,044,171.13	IDR 30,953,451.18	IDR 31,932,390.94

Even so, Rosadi and Muslim (2017) argue that sukuk is a more costly iteration of the conventional bond [23]. The consideration of various costs is necessary when determining the price of sukuk in order to enable the implementation of the sukuk mechanism in accordance with Islamic sharia principles. Nevertheless, we suggest doing a comprehensive analysis is necessary to ascertain the valuation of this sukuk. Although it is acknowledged that the fundamental principle of pricing

in sukuk is comparable to that of bonds [21], and numerous methods for calculating sukuk values are based on this principle, the determination of optimal returns holds significant importance within this framework. Further research is required to examine the implementation of profit sharing in the sukuk mechanism in order to develop a more efficient sukuk pricing model.

6. CONCLUDING REMARKS

Indonesia has witnessed substantial progress in the domain of sukuk, although the utilization of sukuk remains constrained to a just three contracts. The guidelines outlined in Fatwa of Dewan Syariah Nasional Indonesi No. 137/DSN-MUI/IX/2020 differ from the aforementioned statement. Nevertheless, the prevalence of qualitative research methods in the existing literature on sukuk has resulted in a limited exploration of the quantitative analysis of the pricing mechanism. Through a literature review, it emerged that the fundamental mechanism for determining sukuk prices is similar to that of bonds. Indeed, some research suggests that sukuk is a comparatively more costly type of bond. Numerical simulation findings provide divergent interpretations. Pricing comparisons between sukuk and bonds have not confirmed the claim. Besides from the price discrepancy between the two securities, not all bond prices are less expensive than sukuk. Hence, further study is required to further comprehend the pricing analysis of these two investment products. Considering the application of profit and loss sharing in mathematical models for determining some variables on sukuk prices, such as coupon rate, cost or yield, could be a valuable area for future research.

References

[1] I. I. F. Market, IIFM Sukuk Report: a Comprehensive Study of The Global Sukuk Market, **2022**.

[2] D. P. S. O. J. Keuangan, Leporan Perkembangan Keuangan Syariah Indonesia, **2022**.

[3] Y. Ayturk, M. Asutay, E. Aksak, *Research in International Business and Finance* **2017**, *40*, 141–149.

[4] M. S. Hossain, M. H. Uddin, S. H. Kabir, *Emerging Markets Finance and Trade* **2021**, *57*, 3792–3817.

[5] N. D. Miller, J. Challoner, A. Atta, *Int'l Fin. L. Rev.* **2007**, *26*, 24.

[6] R. Wilson, *Humanomics* **2008**, *24*, 170–181.

[7] F. Balli, H. Ghassan, E. H. Al Jeefri, *Journal of Economics and Finance* **2021**, *45*, 529–543.

[8] N. Naifar, S. Hammoudeh, *Pacific-Basin Finance Journal* **2016**, *39*, 57–69.

[9] N. H. Asmuni, K. S. Tan, *Journal of Emerging Market Finance* **2021**, *20*, 165–191.

[10] H. Fathurahman, R. Fitriati, *American Journal of Economics* **2013**, *3*, 159–163.
[11] S. Cakir, F. Raei, **2007**.
[12] N. Almaskati, *Borsa Istanbul Review* **2022**, *22*, 1033–1038.
[13] D. A. Fatah, *Al-'Adalah* **2011**, *10*, 35–46.
[14] Dewan Syariah Nasional, Fatwa Dewan Syariah Nasional - Majelis Ulama Indonesia No: 137/DSN-MUI/IX/2020 Tentang Sukuk, **2020**.
[15] Otoritas Jasa Keuangan, *Himpunan Skema Sukuk Korporasi Di Indonesia*, Otoritas Jasa Keuangan, **2019**.
[16] Dubai International Financial Centre, *Sukuk Guide Book*, Clifford Chance LLP, **2009**.
[17] S. G. Kellison, *The Theory of Interest Third Edition*, McGRAW-HILL INTERNATIONAL, **2009**.
[18] L. J. F. Vaaler, S. K. Harper, J. W. Daniel, *Mathematical interest theory*, Vol. 57, American Mathematical Soc., **2021**.
[19] M. Zulkhibri, *Borsa Istanbul Review* **2015**, *15*, 237–248.
[20] A. Paltrinieri, M. K. Hassan, S. Bahoo, A. Khan, *International Review of Economics & Finance* **2019**.
[21] S. S. Razak, B. Saiti, Y. Dinç, *Borsa istanbul review* **2019**, *19*, S21–S33.
[22] O. I. Bacha, A. Mirakhor, *Islamic capital markets: A comparative approach*, World Scientific, **2019**.
[23] D. Rosadi et al. in Journal of Physics: Conference Series, *Vol. 943*, IOP Publishing, **2017**, p. 012006.
[24] D. Rohmatunnisa, *Unpublished MA in Finance and Investment University of Nottingham Nottingham UK* **2008**.
[25] A. Mirakhor, *Islamic economic studies* **1996**, *4*.
[26] N. Sumarti, *Matematika Keuangan Syariah*, ITB Press Bandung, **2019**.
[27] W. N. Kamilah, N. Sumarti, K. A. Sidarto in AIP Conference Proceedings, *Vol. 2423*, AIP Publishing, **2021**.
[28] Imbal Hasil Obligasi Indonesia 10 Tahun, `%7Bhttps://id.investing.com/rates-bonds/indonesia-10-year-bond-yield%7D`, accessed: 20.09.2021.
[29] SBN Benchmark Series, `%7Bhttps://www.phei.co.id/en-us/Data/Fair-Prices-and-Yield%7D`, accessed: 27.06.2023.