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MODIFIED MULTIPLE DECREMENT TABLE AND ITS CREDIBILITY BASED ON FACTOR CHARACTERISTICS

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Abstract. The 2019 Indonesian Mortality Table IV (TMI IV) involved 52 life insurance companies in Indonesia during the study period from 2013 to 2017. From the data, there may be differences in the characteristics of company customers so that the use of TMI IV is not in accordance with these characteristics. In life insurance companies, there are types of coverage (causes), namely: NDPA, which means death due to illness or accident; PAD, which means reimbursement of medical expenses; and SRD, which is the cancellation of the policy so that the coverage ends. The Companies can construct a Life Table involving multiple causes called a Multiple Decrement (MD) Table. This table is modified into a Modified Multiple Decrement Table (MDT) by adding factors to the causes in the form of regions. The clustering of factors needs to be done to reduce the complexity of the calculation. Using the K-means method, the grouping of regions R1R9 is divided into the following: PAD causes (3 groups) and SRD (2 groups). MDT is obtained from the relationship between MD and the Associated Single Decrement (ASD). The Annual Exposure Method was used to calculate the probability of causes. Furthermore, extrapolation is performed on the probability of cause, for which there is no value, and graduation is performed on the less smooth probability of cause. Then, credibility theory is used to determine the credibility level of the industry. The industry-credible probability of cause has a value between the observed value and the industry value (TMI IV).

Key words and Phrases: associated single decrement, clustering, extrapolation and graduation, credibility theory, modified multiple decrement

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1. INTRODUCTION

In the preparation of TMI IV, 52 life insurance companies in Indonesia were involved in the study period from 2013 to 2017. The data provided by the 52 life insurance companies may represent the data of each life insurance company. However, it is possible that there are differences in the distribution of insurance agents in each life insurance company, which means that there are also differences in the characteristics of life insurance participants, so that the data is feared to be less representative of the data of all life insurance companies. Therefore, every life insurance company in Indonesia is expected to have a mortality table that describes the characteristics of the life insurance company participants by considering TMI IV.

In life insurance companies, there are several types of coverage that are commonly used, namely NDPA, which means death due to illness or accident; PAD, which means reimbursement of medical expenses; and SRD means surrender which participants who withdraw from the policy. These types of coverage are referred to as causes that cause the end of coverage, and the insurance company must pay a sum of money or, specifically, for withdrawal, the company must return the remaining premium paid by the participant in accordance with the agreed-upon agreement.

Based on the above conditions, life insurance companies can construct a modified mortality table with various causes, or the so-called Multiple Decrement Table. This model builds a distribution between two random variables applied to one life, namely the remaining time until a person leaves the system [1], [2] used the Kendall Model to construct the Multiple Increment-Decrement Table of the HIV population, [3] modified the Multiple Decrement Model based on the conditions of various diseases in Northern Ghana, [4] used Markov chains to calculate the worst condition in multiple decrement, and [5] built multiple decrement using an associated single decrement assuming constant and linear acceleration as well as [6] constructed the multiple decrement table using the fractional age assumption expansion method. As of [7] used markov model in a Multiple State Model for premium calculation when several premium-paid states are involved and [8] explored Hierarchical Markov Model in life insurance and social benefit schemes. Then, [9] applied a multiple decrement life table model for orphan daughters in Turkey and [10] contruct decrement rates and a numerical method under competing risks.

In this study, the Multiple Decrement Model (MDM) will be modified by including a factor for each cause. This factor can be a region or another factor. Each cause can have the same or different factors, depending on the selection of factors that have a significant influence on the cause. This model is referred to as the Modified Multiple Decrement Model (MMDM). The Associated Single Decrement Model is used directly to obtain the multiple decrement probability. The cause probability (associated single decrement) is calculated using the annual exposure method for each cause per factor. Here is the MMDM and how it differs from the MDM.

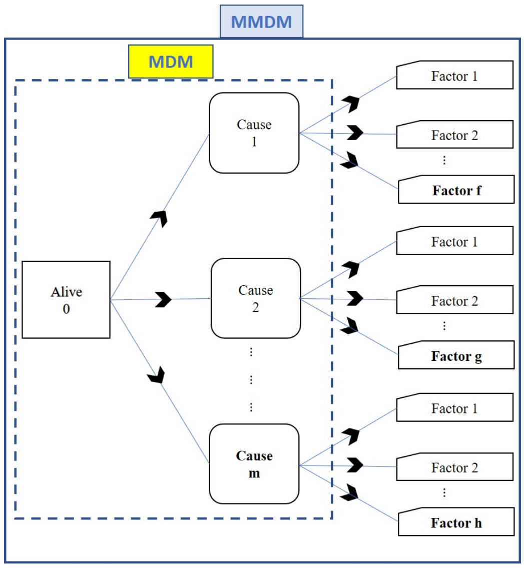

FIGURE 1. Modified multiple decrement model (non-dashed line) and multiple decrement model (dashes line)

The cause factors should be grouped under the same type of risk. This is done to streamline computation, thus saving research time and cost as well as allowing for clear risk segmentation. Since the company experience data is not as extensive as the industry data, there may be empty cause probability, so extrapolation and graduation are used to obtain smooth cause probability. Furthermore, credibility theory is used to obtain credible cause probability for the industry (TMI IV), especially for death probability.

2. MODIFIED MULTIPLE DECREMENT MODEL

In Figure 1, the Modified Multiple Decrement Model is an extension of the Multiple Decrement Model, namely by adding factors to each cause. In Chapter 2.1 and 2.2, the Multiple Decrement Model, the Associated Single Decrement Model and their relationship will be explained. Next, the Multiple Decrement Model with Factors (Modified Multiple Decrement Model) will be elaborated.

2.1. **Multiple Decrement Model (MD).** Based on [11] let J be a discrete r.v. representing the cause of an individual (x), a person's age at x, leaving the system and T be a continuous r.v. representing the time until (x) leaves the system. The values of the two r.v's are,

$$T \geq 0 \text{ and } J = 1, 2, \cdots, m.$$

The joint probability and marginal probability of T and J, respectively,

$$f_{T,J}(t,j) = {}_tp_x^\tau \mu_{x+t}^j$$
$$f_T(t) = {}_tp_x^\tau \mu_{x+t}^\tau$$
$$f_J(j) = {}_\infty q_x^{(j)}$$

as well as the total of distribution function, survival function, and hazard function, respectively,

$${}_tq_x^{(\tau)} = \sum_{j=1}^{m} {}_tq_x^{(j)}$$
$${}_tp_x^{(\tau)} = 1 - {}_tq_x^{(\tau)}$$
$$\mu_{x+t}^{(\tau)} = \sum_{j=1}^{m} \mu_{x+t}^{(j)}$$

2.2. **Associated Single Decrement Model (ASD).** There is a relationship between the MD Model and the ASD Model. The probability of (x) staying in the system due to all causes in $(x, x+t]$,

$${}_tp_x^{(\tau)} = \prod_{j=1}^{m} {}_tp_x^{'(j)} \tag{1}$$

Proof.

$${}_tp_x^{(\tau)} = \exp\left(-\int_0^t \mu_{x+s}^{(\tau)} ds\right) = \exp\left(-\int_0^t \sum_{j=1}^{m} \mu_{x+s}^{(j)} ds\right) = \prod_{j=1}^{m} \exp\left(-\int_0^t \mu_{x+s}^{(\tau)} ds\right)$$
$$= \prod_{j=1}^{m} {}_tp_x^{'(j)}.$$

It is known that ${}_tp_x^{'(j)} \in (0,1)$. Then, ${}_tp_x^{(\tau)} \leq {}_tp_x^{'(j)} \in (0,1) \forall j$ so, ${}_tqx^{(\tau)} \geq {}_tqx^{'(j)}$, consequence, $q_x^{(j)} \leq q_x^{'(j)} \leq q_x^{(\tau)}$.

Proof.

$$\int_0^1 {}_tp_x^{'(j)} \mu_{x+t}^{(j)} dt = -\int_0^1 \frac{d}{dt} {}_tp_x^{'(j)} 1 dt = -\left({}_tp_x^{'(j)}\right)_0^1 + \int_0^1 {}_tp_x^{'(j)} \frac{d}{dt} 1 = 1 - p_x^{'(j)} = q_x^{'(j)}.$$

Then

$${}_tp_x^{'(j)} \mu_{x+t}^{(j)} \geq {}_tpx^{(\tau)} \mu_{x+t}^{(j)}$$
$$q_x^{'(j)} = \int_0^1 {}_tp_x^{'(j)} \mu_{x+t}^{(j)} dt \geq \int_0^1 {}_tpx^{(\tau)} \mu_{x+t}^{(j)} dt = q_x^{(j)}.$$

Based on Equation (1), it is obtained relation between ${}_tq_x^{(\tau)}$ and ${}_tq_x^{'(j)}$ as following,

$$ {}_tq_x^{(\tau)} = \sum_{n=1}^{m} (-1)^{n-1} {}_tN_n \tag{2} $$

where ${}_tN_n$ is sum of combination n ${}_tq_x^{'(j)}$ from all its possibilities. As example with $m = 3$, ${}_tN_2 = {}_tq_x^{'(1)} {}_tq_x^{'(2)} + {}_tq_x^{'(1)} {}_tq_x^{'(3)} + {}_tq_x^{'(2)} {}_tq_x^{'(3)}$.

Proof.

$$ \begin{aligned} {}_tq_x^{(\tau)} &= 1 - \prod_{j=1}^{m} {}_tp_x^{'(j)} \\ &= 1 - {}_tp_x^{'(1)} {}_tp_x^{'(2)} \cdots {}_tp_x^{'(m)} \\ &= 1 - \left(1 - {}_tq_x^{'(1)}\right)\left(1 - {}_tq_x^{'(2)}\right) \cdots \left(1 - {}_tq_x^{'(m)}\right) \\ &= 1 - \left[1 + (-1)^1 {}_tN_1 + (-1)^2 {}_tN_2 + \cdots + (-1)^m {}_tN_m\right] \\ &= {}_tN_1 + (-1) {}_tN_2 + (-1)^2 {}_tN_3 + \cdots + (-1)^{m-1} {}_tN_m \end{aligned} $$

Thus,

$$ {}_tq_x^{(\tau)} = \sum_{n=1}^{m} (-1)^{n-1} {}_tN_n $$

Furthermore, from Equation (1) and (2) are obtained relation between ${}_tq_x^{(j)}$ and ${}_tq_x^{'(j)}$ which shows the direct relation between MD and ASD,

$$ {}_tq_x^{(j)} = {}_tq_x^{'(j)} \left[1 + \sum_{k=1}^{m-1} (-1)^k \left(\frac{1}{k+1}\right) {}_tC_k\right] \tag{3} $$

where ${}_tC_k$ is a sum of combination k ${}_tq_x^{'(j)}$ from all its possibilities, except ${}_tq_x^{'(j)}$ corresponding with ${}_tq_x^{(j)}$. As example, let $m = 3$ and $j = 2$,

$$ {}_tq_x^{(2)} = {}_tq_x^{'(2)} \left[1 + (-1)^1 \left(\frac{1}{2}\right) {}_tC_1 + (-1)^2 \left(\frac{1}{3}\right) {}_tC_2\right] $$

where

$$ \begin{aligned} {}_tC_1 &= {}_tq_x^{'(1)} + {}_tq_x^{'(3)} \\ {}_tC_2 &= {}_tq_x^{'(1)} {}_tq_x^{'(3)}. \end{aligned} $$

2.3. **Modified Multiple Decrement Model (MDT).** Previously defined r.v J and T, respectively denote causes and time until termination of system. Let $m = 3$, with $(j = 1) = D$, $(j = 2) = O$, and $(j = 3) = W$. Define D_k denote cause D of factor k, where k as much f; D_k where k as much g; W_k where k as much h. Based on Equation (1) is written,

$$ {}_tp_x^{(\tau)} = {}_tp_x^{'(D)}\,{}_tp_x^{'(O)}\,{}_tp_x^{'(W)} \left(\prod_{k=1}^{f} {}_tp_x^{(''D_k)}\right)\left(\prod_{k=1}^{g} {}_tp_x^{(''O_k)}\right)\left(\prod_{k=1}^{h} {}_tp_x^{(''W_k)}\right) $$

where ${}_tp_x^{(''D_k)}$ is probability of cause D for factor k, that is without involve other factor and ${}_tp_x^{('D_k)}$ involve other factor. Based on above equation is obtained,

$$ {}_tq_x^{(\tau)} = \sum_{n=1}^{f+g+h} (-1)^{n-1}\,{}_tN_n. $$

As example with $f = 2$, $g = 2$, $h = 2$, get

$$ \begin{aligned} {}_tN_2 &= {}_tq_x^{''(D_1)}{}_tq_x^{''(D_2)} + {}_tq_x^{''(D_1)}{}_tq_x^{''(O_1)} + {}_tq_x^{''(D_1)}{}_tq_x^{''(O_2)} + {}_tq_x^{''(D_1)}{}_tq_x^{''(W_1)} \\ &+ {}_tq_x^{''(D_1)}{}_tq_x^{''(W_2)} + {}_tq_x^{''(D_2)}{}_tq_x^{''(O_1)} + {}_tq_x^{''(D_2)}{}_tq_x^{''(O_2)} + {}_tq_x^{''(D_2)}{}_tq_x^{''(W_1)} \\ &+ {}_tq_x^{''(D_2)}{}_tq_x^{''(W_2)} + {}_tq_x^{''(O_1)}{}_tq_x^{''(W_1)} + {}_tq_x^{''(O_1)}{}_tq_x^{''(W_2)} + {}_tq_x^{''(O_2)}{}_tq_x^{''(W_1)} \\ &+ {}_tq_x^{''(O_2)}{}_tq_x^{''(W_2)} + {}_tq_x^{''(W_1)}{}_tq_x^{''(W_2)}. \end{aligned} $$

Furthermore, from Equation (3) is obtained,

$$ {}_tq_x^{(j)} = {}_tq_x^{''(j)} \left[1 + \sum_{k=1}^{f+g+h-1} (-1)^n \left(\frac{1}{n+1}\right) {}_tC_n\right], \tag{4} $$

with same value of f, g, and h, as well as $k = 2$, for $(j = 1)_k = D_k$, get

$$ {}_tq_x^{(D_2)} = {}_tq_x^{''(D_2)} \left[1 + \sum_{k=1}^{5} (-1)^n \left(\frac{1}{n+1}\right) {}_tC_n\right], $$

where ${}_tC_n$ is sum of combination n ${}_tq_x^{''(j_k)}$ from 5 ${}_tq_x^{''(j_k)}$ that ${}_tq_x^{''(D_2)}$ corresponding with ${}_tq_x^{(D_2)}$, exlcude. Let for ${}_tC_2$,

$$ \begin{aligned} {}_tC_2 &= {}_tq_x^{(''D_1)}{}_tq_x^{(''O_1)} + {}_tq_x^{(''D_1)}{}_tq_x^{(''W_1)} + {}_tq_x^{(''D_1)}{}_tq_x^{(''O_2)} + {}_tq_x^{(''D_1)}{}_tq_x^{(''W_2)} \\ &+ {}_tq_x^{(''O_1)}{}_tq_x^{(''O_2)} + {}_tq_x^{(''O_1)}{}_tq_x^{(''W_1)} + {}_tq_x^{(''O_1)}{}_tq_x^{(''W_1)} + {}_tq_x^{(''O_2)}{}_tq_x^{(''W_1)} \\ &+ {}_tq_x^{(''O_2)}{}_tq_x^{(''W_2)} + {}_tq_x^{(''W_1)}{}_tq_x^{(''W_1)}. \end{aligned} $$

2.4. **Probability of Cause.** The cause probability is calculated per cause factor at each unit age, i.e., the number of cause events per factor at each age divided by the total exposure at each age, which is written as follows:

$$ q_x^{''(j_k)} = \frac{d_x^{''(j_k)}}{E_x}, \tag{5} $$

where $d_x^{''(j_k)}$ denotes the number of participants experiencing cause j_k in $[x, x+1)$ and E_x is the number of years contributed up to age x. This exposure is calculated using the Seriatim Method.

2.5. **Extrapolation, Graduation, and Credibility.** The calculation result of $q_x^{''(j_k)}$ in Equation (4) still has empty values, so extrapolation and graduation are needed to get a smooth probability value. Extrapolation and graduation are only performed on the cause of death compared to TMI IV. Generally, extrapolation on mortality rates using the Gompertz-Makeham model is written as,

$$q_x^{''(D_k)} = 1 - \exp\left[-\left(A + \frac{Bc^x(c-1)}{\ln c}\right)\right], \tag{6}$$

with $B > 0$, $A \geq -B$, $c > 1$, and $x \geq 0$. After extrapolation, there are usually death rates that increase too much from the previous ages. Therefore, the mortality rate is graded to obtain a smooth mortality rate using the Whittaker-Henderson Method. This method minimizes a function of M which is written,

$$M = \sum_{x=1}^{n} w_x \left[q_x^{''(D_k)} - \left(q_x^{''(\hat{D}_k)}\right)^2\right] + \lambda \sum_{x=1}^{n-d} \left(\Delta^d q_x^{''(\hat{D}_k)}\right). \tag{7}$$

The selection of smoothing parameter, λ, and degree of difference, d, uses crude search or grid search which is a numerical method.

The number of claims observed by a company is certainly less than the number of claims observed in the industry study (TMI IV). Therefore, credibility theory is needed to obtain a mortality rate that is weighted by the mortality rate in the industry study,

$$q_x^{''(D_k)} = Z\ q_x^{''(\hat{D}_k)} + (1-Z)\ q_x^{TMI(IV)}. \tag{8}$$

3. RESULTS

3.1. **Data.** The data used in this study is data from one life insurance company in Indonesia. The study period starts from 2010 to 2020 with 2153555 participants. These participants are spread over 9 (nine) regions, namely: $R1, R2, \cdots,$ and $R9$. The causes observed are Natural/Personal Accident Death (NDPA), reimbursement of medical expenses (PAD), and surrender (SRD). Table contains the number of participants spread across the above 9 regions.

3.2. **Construction of Multiple Decrement Table.** The steps to construct the MD Table are as follows:

1. Use Equation (5) to calculate the probability of each cause per factor, $q_x^{''(j_k)}$, $j = NDPA, PAD, SRD$, $k = R1, R2, \cdots, R9$, and $x = 17, 18, \cdots, 74$,
2. Fitting $q_x^{''(NDPA)_k}$ using the Gompertz-Makeham Distribution and graduating as well as applying the credibility of $\forall_{(NDPA_k)}$ using Equations (6), (7), and (8), respectively. Meanwhile, PAD and SRD cause according to the existing results,
3. Form clusters for the probability of a particular cause per factor using k-means clustering so that some new f, g, and h are obtained.

TABLE 1. Data of region participants R1 - R9

	R1					R2					R3				
Age	Number of Participants	Number of Occurrences				Number of Participants	Number of Occurrences				Number of Participants	Number of Occurrences			
		NDPA	PAD	SRD	Total		NDPA	PAD	SRD	Total		NDPA	PAD	SRD	Total
17-22	34,097	4	11	2	17	8,466	9		84	93	4,542	1	9	1	11
23-28	86,063	64	10	53	127	29,735	63	1	736	800	11,869	18	9	46	73
29-34	112,383	85	2	155	242	40,457	73	2	1,600	1,675	17,866	30	4	179	213
35-39	104,981	186		165	351	47,522	281		1,133	1,414	19,481	103		246	349
40-45	396,880	331	1	142	474	47,937	233		1,675	1,908	19,357	89	5	266	360
46-51	141,228	641		130	771	42,053	456		1,571	2,027	15,677	115	1	231	347
52-57	82,711	728		63	791	28,751	634		786	1,420	9,810	173	2	155	330
59-63	20,718	357		87	444	8,328	295		150	445	5,701	229		230	459
64-69	4,071	299		30	329	1,169	151		32	183	1,636	215		143	358
70-74	786	132		4	136	282	45		4	49	529	147		36	183
Total	**983,918**	**2,827**	**24**	**831**	**3,682**	**254,700**	**2,240**	**3**	**7,771**	**10,014**	**106,468**	**1,120**	**30**	**1,533**	**2,683**

	R4					R5					R6				
Age	Number of Participants	Number of Occurrences				Number of Participants	Number of Occurrences				Number of Participants	Number of Occurrences			
		NDPA	PAD	SRD	Total		NDPA	PAD	SRD	Total		NDPA	PAD	SRD	Total
17-22	8,510	6		39	45	7,816	20	20	7	47	13,020	11	6	6	23
23-28	32,451	31	1	443	475	8,867	19	7	102	128	5,058	6	1	94	101
29-34	50,663	75		1,029	1,104	16,698	20	2	314	336	9,009	18		307	325
35-39	58,442	370		1,490	1,860	20,932	185		540	725	11,539	113		411	524
40-45	59,477	277		1,695	1,972	24,281	111		438	549	10,675	108		348	456
46-51	52,699	532		1,539	2,071	16,999	187	1	482	670	8,775	106		279	385
52-57	33,286	666		1,036	1,702	12,833	314	1	461	776	5,950	145		121	266
59-63	13,646	589		648	1,237	9,632	557		612	1,169	6,758	434	1	472	907
64-69	2,235	352		105	457	2,667	421	1	233	655	2,297	393		129	522
70-74	453	153		29	182	399	141		20	161	354	205		16	221
Total	**311,862**	**3,051**	**1**	**8,053**	**11,105**	**121,124**	**1,975**	**32**	**3,209**	**5,216**	**73,435**	**1,539**	**8**	**2,183**	**3,730**

	R7					R8					R9				
Age	Number of Participants	Number of Occurrences				Number of Participants	Number of Occurrences				Number of Participants	Number of Occurrences			
		NDPA	PAD	SRD	Total		NDPA	PAD	SRD	Total		NDPA	PAD	SRD	Total
17-22	19,794	14	42	18	74	3,330	-	3		3	6,660	2	3	2	7
23-28	12,213	11	14	165	190	3,460	-	3	12	15	9,118	13		48	61
29-34	20,314	19	7	476	502	6,714	17	1	56	74	15,087	29		149	178
35-39	22,905	138	2	800	940	10,175	97		143	240	18,241	145		253	398
40-45	21,694	77	1	733	811	8,676	49		104	153	20,545	127		435	562
46-51	21,292	176	4	595	775	7,715	70	1	122	193	19,960	242		761	1,003
52-57	17,839	283	2	452	737	5,782	135		205	340	10,329	265		93	358
59-63	6,362	305		485	790	2,213	92		183	275	6,873	362		407	769
64-69	1,268	248		152	400	636	97		66	163	2,184	239		156	395
70-74	160	68		8	76	66	44		6	50	443	71		11	82
Total	**143,841**	**1,339**	**72**	**3,884**	**5,295**	**48,767**	**601**	**8**	**897**	**1,506**	**109,440**	**1,495**	**3**	**2,315**	**3,813**

4. Calculate $q_x^{(j_k)}$, $\forall_k$ which is the new clustering result using Equation (4).
5. Determine the initial number of individuals, $l_x^{(\tau)}$, and
6. Calculate $l_x^{(\tau)}$, calculate $d_x^{(j_k)} = l_x^{(\tau)} q_x^{(j_k)}$, $d_x^{(NDPA)} = \sum_{k=1}^{f} d_x^{(NDPA_k)}$, $d_x^{(PAD)} = \sum_{k=1}^{g} d_x^{(PAD_k)}$, $d_x^{(SRD)} = \sum_{k=1}^{h} d_x^{(SRD_k)}$, and $d_x^{(\tau)} = \sum_{j=1}^{3} d_x^{(j)}$ then $l_{x+t}^{(\tau)} = l_x^{(\tau)} - d_x^{(\tau)}$. Repeat this step until $x = 74$.

Based on Figure 2, in general, the credibility cause probability is between observation and TMI IV for each region, except for the R1 region. This shows that exposures other than the R1 region tend to be small so their cause probability are greater than TMI IV. The credibility cause probability can be an alternative pricing. Meanwhile, the R1 region has a fairly large exposure compared to other regions and the odds are smaller for a certain age than TMI IV.

As explained in Chapter 1, there is complexity in calculating probabilities of multiple decrement increase as the number of causes and factors. Therefore,

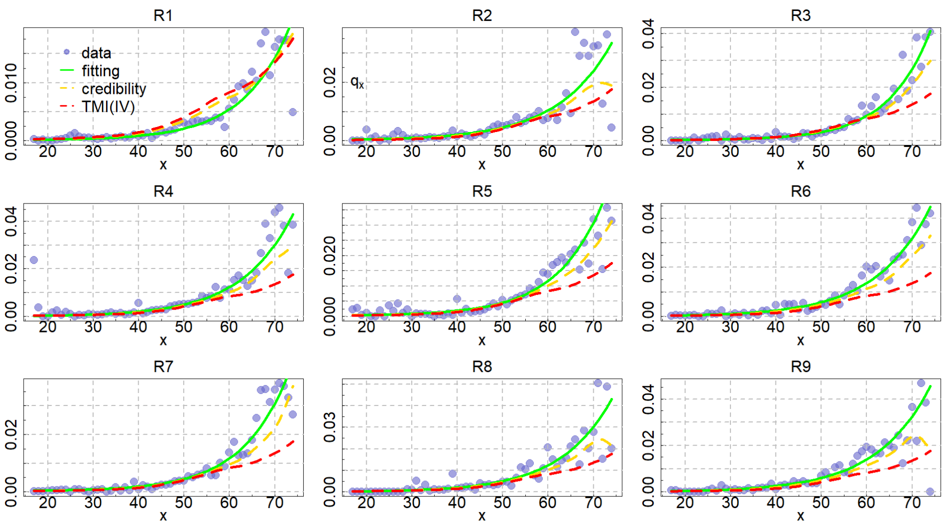

FIGURE 2. Probability of NDPA per age, $q_x^{\prime\prime(NDPA_k)}$, for R1 - R9

the causes probability are clustered using k-means clustering, especially factors with causes other than death (without reducing the original characteristics). This method is one of the non-hierarchical data clustering methods that seeks to partition existing data into two or more clusters. The purpose of this data clustering is to minimize an objective function which generally seeks to minimize diversity within a cluster and maximize diversity between clusters. To calculate the centroid of the kth cluster, on the jth variable, v_{kj} is used,

$$v_{kj} = \frac{\sum_{i=1}^{N_k} x_{ij}}{N_k} \tag{9}$$

with x_{ij} the observation value of the ith object in the kth Cluster for the jth variable and N_k the number of members of the kth cluster [12].

The optimal cluster is determined by the magnitude of the Coefficient of Silhouette which is in $[-1, 1]$. A value close to 1 is the best value. The Silhouette Coefficient is based on geometric considerations of cluster cohesion and separation. Cohesion is used to measure the closeness of objects that are in one cluster and separation is used to measure the closeness between the clusters formed. The Silhouette coefficient of the ith object, x_i, is defined,

$$S_{x_i} = \frac{b_{p,i} - a_{p,i}}{\max(a_{p,i}, b_{p,i})}$$

with $a_{p,i}$ the average distance of the ith object with all objects in the pth cluster (the same cluster as the ith object). As for $b_{p,i}$, the minimum value of $d_{q,i}$, which is the average of the ith object with all objects in the qth cluster (a cluster that is different from the ith object), where $p \neq q$. The value of $b_{p,i}$ shows the average difference of object i to the cluster closest to its neighbors. If S_{x_i} has a high value, the more appropriate the placement of x_i in cluster p. The Silhouette Coefficient (SLH) value is written,

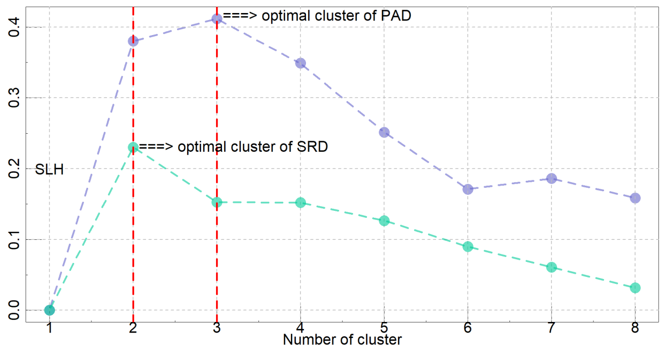

FIGURE 3. SLH of APD (top) and SRD (bottom)

$$SLH = \frac{1}{N}\sum_{i=1}^{N} S_{x_i}$$

with N number of objects. The best clustering is achieved if SLH is maximized which means the distance between objects in the cluster is minimum and the distance between clusters is maximum [13].

In Figure 3, the optimal clusters for PAD causes and SRD causes are 3 (three) and 2 (two) clusters, respectively. Next, K-means clustering is performed for each causes.

TABLE 2. K-means algorithm

Input	Parameter k, number of clusters, and x_{ij}, object with dimension $N \times 2$;
Process	1. Select k objects that will be the initial centroid, v_{kj};
	2. Calculate the distance of x_{ij} and v_{kj} using Euclidean Distance;
	$d(x_i, v_k) = \sqrt{\sum_j^2 (x_{ij} - v_{kj})^2}$, $\forall i, k$;
	3. Allocate x_{ij} to the cluster that has the minimum distance to v_{kj};
	4. Calculate the cluster centroid for each clusters using Equation (9); and
	5. Repeat step 3 through 5 until no object moves to another cluster.
Output	x_{ij} is divided into k clusters

Based on Figure 4, the clustering results of PAD causes are divided into 3 (three) clusters, namely cluster 1 (R3), cluster 2 (R7), and cluster 3 (R1,R2,R4,R5,

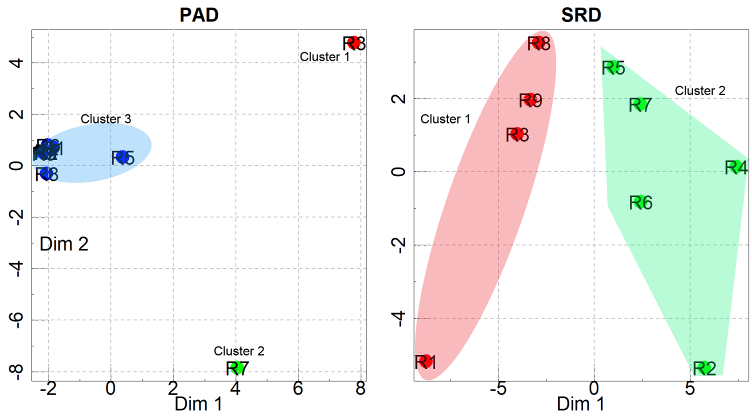

FIGURE 4. Clustering of PAD and SRD for regions

R6,R8,R9). In addition, the clustering results of SRD causes are divided into 2 (two) clusters, namely cluster 1 (R1,R3,R8,R9) and cluster 2 (R2,R4,R5,R6,R7). Thus the causal factors, $NDPA_k$ with $k = R1, R2, \cdots, R9$, PAD_k with $k = C1, C2, C3$, and SRD_k with $k = D1, D2$.

TABLE 3. Probability of multiple decrement

Age	$q_x^{NDPA_{R_1}}$	$q_x^{NDPA_{R_2}}$	$q_x^{NDPA_{R_3}}$	$q_x^{NDPA_{R_4}}$	$q_x^{NDPA_{R_5}}$	$q_x^{NDPA_{R_6}}$	$q_x^{NDPA_{R_7}}$	$q_x^{NDPA_{R_8}}$	$q_x^{NDPA_{R_9}}$	$q_x^{PAD_{C_1}}$	$q_x^{PAD_{C_2}}$	$q_x^{PAD_{C_3}}$	$q_x^{SRD_{D_1}}$	$q_x^{SRD_{D_2}}$
17	0.00029	0.00033	0.00031	0.00028	0.00043	0.00033	0.00035	0.00032	0.00033	0.00000	0.00000	0.00091	0.00000	0.00000
18	0.00032	0.00038	0.00034	0.00031	0.00042	0.00035	0.00037	0.00034	0.00034	0.00000	0.00000	0.00051	0.00000	0.00000
19	0.00034	0.00043	0.00035	0.00033	0.00041	0.00037	0.00039	0.00035	0.00035	0.01331	0.00000	0.00031	0.00000	0.00000
20	0.00035	0.00047	0.00037	0.00035	0.00042	0.00039	0.00042	0.00037	0.00037	0.00304	0.00000	0.00008	0.00013	0.00108
21	0.00036	0.00049	0.00039	0.00037	0.00043	0.00041	0.00043	0.00038	0.00039	0.00234	0.00000	0.00000	0.00000	0.00390
22	0.00037	0.00051	0.00041	0.00039	0.00046	0.00043	0.00045	0.00039	0.00041	0.00000	0.00000	0.00000	0.00038	0.00410
23	0.00037	0.00054	0.00043	0.00042	0.00049	0.00046	0.00046	0.00041	0.00045	0.00227	0.00000	0.00013	0.00065	0.00792
24	0.00037	0.00059	0.00045	0.00044	0.00054	0.00050	0.00048	0.00043	0.00051	0.00062	0.00000	0.00005	0.00067	0.00822
25	0.00038	0.00065	0.00047	0.00047	0.00059	0.00055	0.00051	0.00046	0.00057	0.00000	0.00000	0.00012	0.00095	0.00799
26	0.00040	0.00070	0.00050	0.00051	0.00063	0.00060	0.00054	0.00049	0.00063	0.00043	0.00017	0.00003	0.00057	0.00979
27	0.00044	0.00074	0.00053	0.00055	0.00065	0.00064	0.00057	0.00054	0.00070	0.00076	0.00000	0.00033	0.00111	0.01018
28	0.00048	0.00076	0.00056	0.00060	0.00068	0.00068	0.00061	0.00061	0.00075	0.00067	0.00000	0.00009	0.00098	0.01074
29	0.00052	0.00079	0.00060	0.00066	0.00070	0.00074	0.00065	0.00067	0.00080	0.00000	0.00000	0.00006	0.00129	0.01166
30	0.00057	0.00083	0.00064	0.00071	0.00073	0.00080	0.00070	0.00074	0.00085	0.00027	0.00000	0.00002	0.00100	0.01242
31	0.00062	0.00087	0.00068	0.00078	0.00077	0.00086	0.00075	0.00081	0.00090	0.00024	0.00000	0.00003	0.00107	0.01141
32	0.00067	0.00093	0.00073	0.00084	0.00082	0.00093	0.00080	0.00087	0.00097	0.00022	0.00000	0.00002	0.00143	0.01261
33	0.00071	0.00099	0.00078	0.00091	0.00087	0.00101	0.00086	0.00093	0.00105	0.00020	0.00000	0.00000	0.00143	0.01269
34	0.00077	0.00106	0.00084	0.00099	0.00092	0.00111	0.00093	0.00100	0.00114	0.00000	0.00000	0.00000	0.00118	0.01256
35	0.00083	0.00115	0.00091	0.00107	0.00100	0.00121	0.00101	0.00108	0.00125	0.00000	0.00000	0.00000	0.00122	0.01107
36	0.00089	0.00125	0.00098	0.00117	0.00109	0.00134	0.00110	0.00119	0.00137	0.00000	0.00000	0.00000	0.00117	0.01007
37	0.00097	0.00136	0.00106	0.00128	0.00120	0.00148	0.00120	0.00131	0.00151	0.00000	0.00000	0.00000	0.00120	0.01090
38	0.00105	0.00148	0.00115	0.00141	0.00133	0.00162	0.00131	0.00145	0.00165	0.00000	0.00000	0.00000	0.00151	0.01150
39	0.00115	0.00160	0.00126	0.00155	0.00146	0.00178	0.00143	0.00157	0.00179	0.00000	0.00000	0.00000	0.00185	0.01428
40	0.00127	0.00173	0.00138	0.00170	0.00161	0.00195	0.00157	0.00172	0.00197	0.00012	0.00000	0.00000	0.00106	0.00765
41	0.00140	0.00186	0.00152	0.00185	0.00176	0.00216	0.00171	0.00186	0.00216	0.00016	0.00000	0.00000	0.00118	0.01144
42	0.00155	0.00202	0.00168	0.00203	0.00192	0.00237	0.00188	0.00203	0.00237	0.00033	0.00000	0.00000	0.00117	0.01164
43	0.00172	0.00221	0.00186	0.00223	0.00211	0.00259	0.00207	0.00222	0.00260	0.00017	0.00000	0.00000	0.00041	0.01125
44	0.00191	0.00242	0.00206	0.00246	0.00232	0.00280	0.00229	0.00245	0.00284	0.00000	0.00000	0.00001	0.00172	0.01030
45	0.00210	0.00265	0.00227	0.00271	0.00255	0.00302	0.00252	0.00270	0.00309	0.00000	0.00000	0.00000	0.00477	0.01218
46	0.00232	0.00292	0.00251	0.00299	0.00282	0.00326	0.00279	0.00297	0.00336	0.00000	0.00000	0.00001	0.00654	0.01169
47	0.00255	0.00321	0.00279	0.00330	0.00312	0.00355	0.00309	0.00327	0.00367	0.00000	0.00000	0.00000	0.00435	0.01099
48	0.00280	0.00353	0.00308	0.00363	0.00344	0.00389	0.00341	0.00360	0.00403	0.00000	0.00000	0.00000	0.00125	0.01127
49	0.00307	0.00385	0.00339	0.00398	0.00378	0.00426	0.00375	0.00395	0.00444	0.00018	0.00000	0.00000	0.00112	0.01255
50	0.00337	0.00420	0.00372	0.00436	0.00415	0.00467	0.00411	0.00433	0.00488	0.00000	0.00000	0.00000	0.00118	0.01241
51	0.00370	0.00458	0.00409	0.00478	0.00455	0.00512	0.00452	0.00476	0.00535	0.00000	0.00000	0.00001	0.00098	0.01074
52	0.00407	0.00499	0.00448	0.00523	0.00499	0.00561	0.00495	0.00524	0.00584	0.00022	0.00000	0.00001	0.00091	0.01036
53	0.00446	0.00542	0.00490	0.00571	0.00546	0.00612	0.00540	0.00575	0.00635	0.00023	0.00000	0.00000	0.00108	0.01150
54	0.00489	0.00587	0.00535	0.00622	0.00597	0.00669	0.00589	0.00628	0.00689	0.00000	0.00000	0.00000	0.00118	0.01067
55	0.00532	0.00635	0.00582	0.00676	0.00651	0.00730	0.00640	0.00682	0.00748	0.00000	0.00000	0.00000	0.00136	0.01052
56	0.00576	0.00683	0.00631	0.00733	0.00708	0.00796	0.00693	0.00736	0.00813	0.00000	0.00000	0.00000	0.00121	0.00991
57	0.00617	0.00731	0.00681	0.00790	0.00768	0.00864	0.00748	0.00794	0.00883	0.00000	0.00000	0.00000	0.00111	0.00915
58	0.00653	0.00776	0.00731	0.00849	0.00830	0.00935	0.00802	0.00855	0.00953	0.00000	0.00000	0.00002	0.00274	0.01065
59	0.00683	0.00819	0.00779	0.00907	0.00893	0.01003	0.00855	0.00915	0.01020	0.00000	0.00000	0.00000	0.00653	0.01520
60	0.00710	0.00866	0.00831	0.00968	0.00959	0.01070	0.00910	0.00976	0.01086	0.00000	0.00000	0.00000	0.00850	0.01959
61	0.00736	0.00919	0.00887	0.01034	0.01031	0.01137	0.00968	0.01041	0.01155	0.00000	0.00000	0.00000	0.01259	0.01890
62	0.00767	0.00983	0.00950	0.01105	0.01110	0.01208	0.01036	0.01116	0.01232	0.00000	0.00000	0.00000	0.01261	0.01750
63	0.00805	0.01062	0.01024	0.01189	0.01195	0.01290	0.01119	0.01204	0.01318	0.00000	0.00000	0.00000	0.01132	0.01461
64	0.00849	0.01153	0.01109	0.01292	0.01286	0.01384	0.01219	0.01300	0.01408	0.00000	0.00000	0.00006	0.00935	0.01408
65	0.00899	0.01260	0.01208	0.01419	0.01383	0.01499	0.01335	0.01405	0.01508	0.00000	0.00000	0.00000	0.00903	0.01176
66	0.00953	0.01366	0.01314	0.01564	0.01482	0.01625	0.01454	0.01515	0.01615	0.00000	0.00000	0.00000	0.01318	0.01308
67	0.01020	0.01477	0.01443	0.01732	0.01605	0.01775	0.01585	0.01657	0.01748	0.00000	0.00000	0.00000	0.00901	0.01170
68	0.01092	0.01580	0.01580	0.01895	0.01737	0.01936	0.01718	0.01809	0.01906	0.00000	0.00000	0.00000	0.00795	0.01072
69	0.01173	0.01673	0.01738	0.02054	0.01887	0.02106	0.01870	0.01972	0.02067	0.00000	0.00000	0.00000	0.00544	0.00825
70	0.01257	0.01740	0.01909	0.02186	0.02036	0.02264	0.02045	0.02108	0.02172	0.00000	0.00000	0.00000	0.00594	0.00915
71	0.01351	0.01783	0.02095	0.02303	0.02204	0.02423	0.02274	0.02187	0.02195	0.00000	0.00000	0.00000	0.00332	0.00744
72	0.01450	0.01793	0.02292	0.02413	0.02401	0.02595	0.02573	0.02195	0.02080	0.00000	0.00000	0.00000	0.00372	0.00343
73	0.01548	0.01750	0.02481	0.02539	0.02600	0.02770	0.02938	0.02090	0.01857	0.00000	0.00000	0.00000	0.00439	0.00523
74	0.01659	0.01696	0.02685	0.02683	0.02823	0.02968	0.03327	0.01932	0.01654	0.00000	0.00000	0.00000	0.00000	0.00161

In Table 3, the decrement probability of each cause per factor that has been clustered is obtained. In accordance with Subsection 2.2 that the ASD opportunity is greater than the MD opportunity obtained. As for Table 4, the initial group of 17 years old is simulated as many as 100000 and the remaining 1922 at the age of 74 years.

TABLE 4. Multiple decrement table

Age	l_x^τ	$d_x^{NDPA_{R_1}}$	$d_x^{NDPA_{R_2}}$	$d_x^{NDPA_{R_3}}$	$d_x^{NDPA_{R_4}}$	$d_x^{NDPA_{R_5}}$	$d_x^{NDPA_{R_6}}$	$d_x^{NDPA_{R_7}}$	$d_x^{NDPA_{R_8}}$	$d_x^{NDPA_{R_9}}$	$d_x^{PAD_{C_1}}$	$d_x^{PAD_{C_2}}$	$d_x^{PAD_{C_3}}$	$d_x^{SRD_{D_1}}$	$d_x^{SRD_{D_2}}$
17	100,000	29	33	31	28	43	33	35	32	33	0	0	91	0	0
18	99,613	31	38	33	31	42	35	37	33	34	0	0	51	0	0
19	99,247	33	43	35	33	41	36	39	35	35	1,321	0	31	0	0
20	97,565	35	46	37	34	41	38	41	36	36	297	0	8	13	105
21	96,800	35	48	38	36	42	39	42	37	37	226	0	0	0	377
22	95,842	35	49	39	38	44	41	43	38	40	0	0	0	36	393
23	95,046	35	52	40	40	47	44	44	39	43	215	0	12	61	753
24	93,621	35	55	42	41	51	47	45	40	47	58	0	5	63	770
25	92,322	35	60	44	44	54	51	47	42	52	0	0	11	88	737
26	91,058	37	64	45	46	57	55	49	45	57	40	16	3	52	891
27	89,601	39	66	47	50	58	58	51	49	62	68	0	29	99	912
28	88,013	42	67	49	53	60	60	53	53	66	59	0	8	86	946
29	86,411	45	69	52	57	60	64	56	58	69	0	0	5	112	1,007
30	84,757	48	70	54	61	62	67	59	63	72	23	0	2	85	1,053
31	83,039	51	73	57	65	64	72	62	67	75	20	0	3	89	947
32	81,394	54	76	60	69	67	76	65	71	79	18	0	1	117	1,026
33	79,617	57	79	62	73	69	81	69	74	83	16	0	0	114	1,011
34	77,830	60	83	66	77	72	86	73	78	89	0	0	0	92	977
35	76,078	63	87	69	82	76	92	77	82	95	0	0	0	92	842
36	74,419	66	93	73	87	81	100	82	88	102	0	0	0	87	749
37	72,810	70	99	77	93	88	107	87	96	110	0	0	0	87	794
38	71,102	75	105	82	100	94	115	93	103	117	0	0	0	108	818
39	69,292	80	111	87	107	101	123	99	109	124	0	0	0	128	989
40	67,233	85	116	93	114	108	131	105	115	133	8	0	0	72	515
41	65,637	92	122	100	121	115	142	112	122	142	11	0	0	77	751
42	63,730	99	129	107	129	122	151	120	129	151	21	0	0	75	742
43	61,754	106	136	115	138	130	160	128	137	161	10	0	0	25	695
44	59,812	114	145	123	147	139	167	137	147	170	0	0	1	103	616
45	57,804	122	153	131	156	148	174	146	156	179	0	0	0	275	704
46	55,460	129	162	139	166	156	181	155	165	186	0	0	1	363	649
47	53,010	135	170	148	175	165	188	164	173	195	0	0	0	230	583
48	50,684	142	179	156	184	174	197	173	182	204	0	0	0	63	571
49	48,457	149	187	164	193	183	206	181	191	215	9	0	0	54	608
50	46,116	155	194	172	201	191	215	190	200	225	0	0	0	55	573
51	43,746	162	200	179	209	199	224	198	208	234	0	0	1	43	470
52	41,419	169	207	186	216	207	232	205	217	242	9	0	1	38	429
53	39,062	174	212	192	223	213	239	211	224	248	9	0	0	42	449
54	36,626	179	215	196	228	219	245	216	230	252	0	0	0	43	391
55	34,212	182	217	199	231	223	250	219	233	256	0	0	0	46	360
56	31,795	183	217	201	233	225	253	220	234	259	0	0	0	39	315
57	29,417	181	215	200	233	226	254	220	234	260	0	0	0	33	269
58	27,092	177	210	198	230	225	253	217	232	258	0	0	1	74	289
59	24,729	169	203	193	224	221	248	211	226	252	0	0	0	162	376
60	22,244	158	193	185	215	213	238	202	217	242	0	0	0	189	436
61	19,756	145	181	175	204	204	225	191	206	228	0	0	0	249	373
62	17,374	133	171	165	192	193	210	180	194	214	0	0	0	219	304
63	15,199	122	161	156	181	182	196	170	183	200	0	0	0	172	222
64	13,254	112	153	147	171	170	183	162	172	187	0	0	1	124	187
65	11,485	103	145	139	163	159	172	153	161	173	0	0	0	104	135
66	9,877	94	135	130	155	146	160	144	150	160	0	0	0	130	129
67	8,345	85	123	120	145	134	148	132	138	146	0	0	0	75	98
68	7,000	76	111	111	133	122	136	120	127	133	0	0	0	56	75
69	5,802	68	97	101	119	110	122	108	114	120	0	0	0	32	48
70	4,762	60	83	91	104	97	108	97	100	103	0	0	0	28	44
71	3,847	52	69	81	89	85	93	87	84	84	0	0	0	13	29
72	3,082	45	55	71	74	74	80	79	68	64	0	0	0	11	11
73	2,450	38	43	61	62	64	68	72	51	45	0	0	0	11	13
74	1,922	32	33	52	52	54	57	64	37	32	0	0	0	0	3

4. CONCLUSIONS

The Companies, especially insurance companies, can construct multiple decrement tables according to their experience by considering certain factors, such as region and others. The results of the company study tend to be smaller than the industry (TMI IV 2019). This is due to the smaller exposure of the company compared to the industry. Credibility theory can be utilized to obtain a weighted rate between the industry and the company.

The cause probability without considering other causes (Associated Single Decrement) is greater than the cause probability considering other causes (Multiple Decrement). Factor clustering needs to be done first to reduce the computational complexity of calculating the cause probability. The results of this study can be utilized as pricing and reserving of insurance companies.

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