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On the Locating-Chromatic Number of the Sunflower Graph

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Abstract. Let c be vertex coloring of a connected graph. Define $c : V \to \{1, 2, \ldots, k\}$ such that $c(u) \neq c(v)$ for adjacent vertices u and v in G. Let S_i be a set of vertices assigned by color i where $1 \leq i \leq k$, defined as color class. Let $\Pi = \{S_1, S_2, \ldots, S_k\}$ be an ordered partition of $V(G)$ that is induced by coloring c, then the representation of vertex v with respect to Π is called a color code of v, denoted as $c_\Pi(v)$, defined as $c_\Pi(v) = (d(v, S_1), d(v, S_2), \ldots, d(v, S_k))$, where $d(v, S_i) = min\{d(v, x)|x \in S_i\}$ for $1 \leq i \leq k$. If all distinct vertices of G have distinct color codes, then c is called a k-locating coloring of G. The locating-chromatic number is defined as the minimum k such that graph G admits a k-locating coloring, denoted by $\chi_L(G)$. In this paper, we determine the locating-chromatic number of the sunflower graph SF_n for $n \geq 3$.

Key words and Phrases: Sunflower graph, locating-chromatic number, color code.

1. INTRODUCTION

Let $G = (V(G), E(G))$ be a graph with vertex set $V(G)$ and edge set $E(G)$. For non negative integer k, a sequence of vertices $W = v_0e_1v_1e_2v_2 \ldots e_kv_k$, such that $v_0, v_i \in V(G)$ and $e_i = v_{i-1}v_i \in E(G)$ for $1 \leq i \leq k$ is called a (v_0, v_k)-walk of G. If there is no vertex repeated in the sequence, then it is called (v_0, v_k)-path. For $u, v \in V(G)$, the length of the shortest (u, v)-path is called the distance between u and v. The complete graph, denote as K_n, is a graph in which all vertices are adjacent to each other.

Locating-chromatic number was introduced by Chartrand *et al.* [1], combining the concepts of vertex coloring and partition dimension of a graph. The locating-chromatic number is defined as follows. Let c be a vertex coloring of a connected graph. Define $c : V \to \{1, 2, \ldots, k\}$ such that $c(u) \neq c(v)$ for adjacent vertices u and v in G. Let S_i be a set of vertices assigned by color i where $1 \leq i \leq k$, defined as color class. Let $\Pi = \{S_1, S_2, \ldots, S_k\}$ be an ordered partition of $V(G)$

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that is induced by coloring c, then the representation of vertex v with respect to Π is called a color code of v, denoted as $c_\Pi(v)$, defined as

$$c_\Pi(v) = (d(v, S_1), d(v, S_2), \ldots, d(v, S_k)), \quad (1)$$

where $d(v, S_i) = min\{d(v, x)|x \in S_i\}$ for $1 \leq i \leq k$. Let $c(v) = i$, vertex v is called a dominant vertex if $d(v, S_i) = 0$ and $d(v, S_j) = 1$ for $1 \leq i, j \leq k$. If all distinct vertices of G have distinct color codes, then c is called a k-locating coloring of G. Then, the locating chromatic number of G is defined as the minimum number of k colors such that G has k-locating coloring, denoted by $\chi_L(G)$. Chartrand *et al.* [1] determined the locating-chromatic number of some graph, such as path, cycle, double stars, trees, and multipartite complete graph.

There are some studies about determining locating-coloring number of a graph. In 2013, Purwasih *et al.* [2] determined the locating-chromatic number for a subdivision of a wheel on one cycle edge. In the same year, Welyyanti, et al. [3] determined the locating chromatic number of homogeneous lobster. Then, in 2014, Behtoei *et al.* [4] determined the locating chromatic number of the join of graphs, such as the join of path and cycle graph, complete and cycle graph, and two cycles graph. In 2015, Purwasih *et al.* [5] has determined the bounds on the locating-chromatic number for a subdivision of a graph on one edge. In the same year, Welyyanti *et al.* [6] determined the locating chromatic number for graphs with dominant vertices. In two years later, Welyyanti *et al.* [7] also determined the locating-chromatic number for graphs with two homogenous components. In 2021, Irawan *et al.* [8] has determined the locating-chromatic number of origami graphs. In the same year, Anti *et al.* [9] determined the locating-chromatic number of the join of path and wheel graph. In the next year, Rahmatalia *et al.* [10] determined the locating-chromatic number of path split graph, then Sudarsana *et al.* [11] has also determined the locating chromatic number for m-shadow of a connected graph. Subsequently, in the same year, Fakhri Zikra *et al.* [12] has determined the locating chromatic number of disjoint union of fan graphs. Then, in 2023, Asmiati *et al.* [13] determined the locating chromatic number for certain operation of origami graphs. In the same year, Welyyanti *et al.* [14] determined the locating-chromatic number for certain lobster graph.

In this paper, we study the locating-chromatic number of the sunflower graph SF_n for $n \geq 3$. We achieve this by analyzing the maximum number of color combinations while considering the coloring constraints for each neighbor of a vertex. Using these principles, we establish the exact values for $\chi_L(SF_n)$ and demonstrate how the graph's structural properties influence its locating-chromatic number. Furthermore, we outline the methodology used to derive these results and provide insights into the broader implications of our findings. Our work contributes to the understanding of locating-chromatic numbers in structured graph families and offers directions for further exploration in this area.

2. SUNFLOWER GRAPH

Let W_n be a wheel on $n+1$ vertices. Denote the central vertex as o and the vertices on the n-cycle as $v_0, v_1, \ldots, v_{n-1}$. The sunflower graph SF_n is constructed by adding n vertices $w_0, w_1, \ldots, w_{n-1}$, and then adding n edges $\{v_i w_i | 0 \leq i \leq n-1\}$, $n-1$ edges $\{v_{i+1} w_i | 0 \leq i \leq n-2\}$, and one edge $\{v_0 w_{n-1}\}$ [15]. The sunflower graph SF_n has order $2n+1$ and size $4n$. The sunflower graph SF_n will be shown in Figure 1.

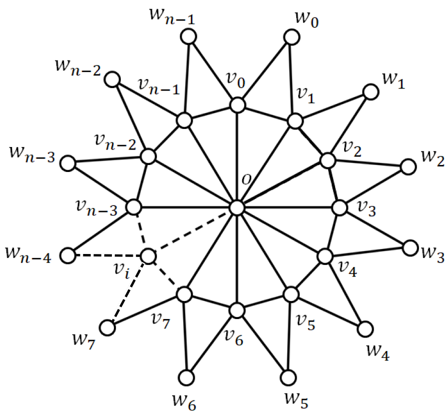

FIGURE 1. SF_n graph for $n \geq 3$

The vertex and edge sets of the sunflower graph SF_n for $n \geq 3$ are as follows:

$$V(SF_n) = \{o\} \cup \{v_i, w_i | 0 \leq i \leq n-1\}, \quad (2)$$

$$\begin{aligned} E(SF_n) = & \{v_j v_{j+1} | 0 \leq j \leq n-2\} \cup \{v_0 v_{n-1}\} \cup \{o v_i | 0 \leq i \leq n-1\} \\ & \cup \{v_i w_i | 0 \leq i \leq n-1\} \cup \{w_j v_{j+1} | 0 \leq j \leq n-2\} \\ & \cup \{w_{n-1} v_0\}, \end{aligned} \quad (3)$$

3. MAIN RESULTS

Let c be a locating-coloring in a connected graph $G(V, E)$. Define $c : V(G) \to \{1, 2, \ldots, k\}$ such that $c(u) \neq c(v)$ if u is not adjacent to v. The following theorem gives the locating-chromatic number of the sunflower graph (SF_n) for $n \geq 3$.

Theorem 3.1. *Let SF_n be a sunflower graph for $n \geq 3$. Then, the locating-chromatic number of SF_n for $n \geq 3$,*

$$\chi_L(SF_n) = \begin{cases} 4, & for\ n = 3, \\ 5, & for\ 4 \leq n \leq 28, \\ 6, & for\ 29 \leq n \leq 75, \\ q, & for\ 1 + \sum_{k=0}^{3} \dfrac{(q-2)!}{(k+1)!(q-(k+4))!} \leq n \leq \sum_{k=0}^{3} \dfrac{(q-1)!}{(k+1)!(q-(k+3))!} \\ & for\ q \geq 7. \end{cases}$$

Proof. Consider the following cases:

Case 1. $n = 3$

First, we determine the lower bound of $\chi_L(SF_3)$. Since graph W_3 is same as graph K_4, and graph W_3 is a subgraph of SF_3, then it is clear that we need at least 4-locating coloring. Consequently, if we use three colors, then three colors will not enough for locating coloring of SF_3. As a result, $\chi_L(SF_3) \geq 4$.

Next, we determine the upper bound of $\chi_L(SF_3)$. Define $c : V \to \{1, 2, 3, 4\}$, as follows:

$$c(v) = \begin{cases} 1, & \text{for } v = v_0, w_1, \\ 2, & \text{for } v = v_1, w_2, \\ 3, & \text{for } v = v_2, w_0, \\ 4, & \text{for } v = o. \end{cases}$$

The coloring on SF_3 will be shown in Figure 2.

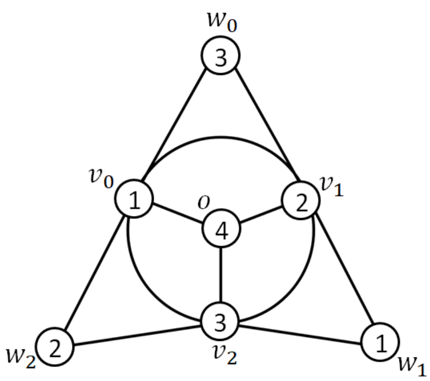

Figure 2. The coloring on SF_3

Then, we have distinct color codes as follows:

Based on the color codes above, all vertices of SF_3 have distinct color codes. As a result, $\chi_L(SF_3) \leq 4$. Thus, $\chi_L(SF_3) = 4$.

$c_\Pi(o) = (1,1,1,0)$, $c_\Pi(v_1) = (1,0,1,1)$, $c_\Pi(w_0) = (1,1,0,2)$, $c_\Pi(w_2) = (1,0,1,2)$.
$c_\Pi(v_0) = (0,1,1,1)$, $c_\Pi(v_2) = (1,1,0,1)$, $c_\Pi(w_1) = (0,1,1,2)$,

Case 2. $4 \leq n \leq 28$

First, we determine the lower bound of $\chi_L(SF_n)$ for $4 \leq n \leq 28$. Assume that there exists 4-locating coloring c on SF_n for $4 \leq n \leq 28$. Without loss of generality, let $c(o) = 4$. Since o is adjacent to v_i for $0 \leq i \leq n-1$, then there are at least three colors needed to be assigned to each v_i. The coloring on v_i will be divided into two subcases:

Subcase 2.1. Vertex v_i for $0 \leq i \leq n-1$ will be assigned by one of two colors

Without loss of generality, define the coloring of v_i, as follows:

$$c(v_i) \quad = \quad \begin{cases} 1, & \text{for even } i, \\ 2, & \text{for odd } i. \end{cases}$$

Vertex w_i for $0 \leq i \leq n-1$ can only be assigned by one of two colors, say color 3 or 4. Let $c(w_i) = 3$. Since $d(w_i, S_1) = d(w_i, S_2) = 1, d(w_i, S_3) = 0$, and $d(w_i, S_4) = 2$, then to avoid two vertices have the same color code, we can assign color 3 for only one vertex, and we assign color 4 to the other vertices. It is clear that w_{i+1} and w_{i-1} have the same color code because $d(w_{i+1}, S_k) = d(w_{i-1}, S_k)$ for every k, $1 \leq k \leq 4$. Therefore, we have at least two vertices that have the same color code.

Subcase 2.2. Vertex v_i for $0 \leq i \leq n-1$ will be assigned by one of three colors

By applying the pigeonhole principle, there are at least two vertices that have the same color, say vertices v_x and v_y.

Without loss of generality, let $c(v_x) = c(v_y) = 1$. Now, we can consider three possibilities: either v_x and v_y are both dominant vertices, one of them is dominant vertex, or neither of them are dominant vertices. Then, those will be divided into these subcases, as follows:

Subcase 2.2.1. Either v_x and v_y are both dominant vertices

We can clearly see that both v_y and v_y are dominant vertices. Hence, $c_\Pi(v_x) = c_\Pi(v_y) = (0,1,1,1)$.

Subcase 2.2.2. One of v_x or v_y is dominant vertex

Let v_y be a non-dominant vertex, then we have $c(v_{y-1}) = c(v_{y+1}) \in \{2,3\}$ and $c(w_y) = c(w_{y-1}) = 4$. Without loss of generality, let $c(v_{y-1}) = c(v_{y+1}) = 2$. Consequently, we have $d(w_y, S_1) = d(w_y, S_2) = d(w_{y-1}, S_1) = d(w_{y-1}, S_2) = 1, d(w_y, S_3) \in \{2,3\}, d(w_{y-1}, S_3) \in \{2,3\}$, and $d(w_y, S_4) = d(w_{y-1}, S_4) = 0$. Since $d(w_y, S_3) \in \{2,3\}$ and $d(w_{y-1}, S_3) \in \{2,3\}$, then we have some possible colorings for $v_{y-2}, v_{y+2}, w_{y-2}$, and w_{y+1}. If $d(w_y, S_3) = 2$, then $c(v_{y+2}) = 3$ or $c(w_{y+1}) = 3$. Alternatively, if $d(w_y, S_3) = 3$, then $c(v_{y+2}) \neq c(w_{y+1}) \neq 3$. Similarly, if $d(w_{y-1}, S_3) = 2$, then $c(v_{y-2}) = 3$ or $c(w_{y-2}) = 3$. On the other hand, if $d(w_{y-1}, S_3) = 3$, then $c(v_{y-2}) \neq c(w_{y-2}) \neq 3$. Now, we can consider three possibilities, as follows:

(1) Let $(c(v_{y+2}) = 3$ or $c(w_{y+1}) = 3)$ and $(c(v_{y-2}) = 3$ or $c(w_{y-2}) = 3)$
Without loss of generality, let $c(v_{y+2}) = c(v_{y-2}) = 3$. Since $d(w_{y-1}, S_3) = 2$, then see that w_y and w_{y-1} have the same color code, which is $(1, 1, 2, 0)$. Hence, $c_\Pi(w_y) = c_\Pi(w_{y-1}) = (1, 1, 2, 0)$.

(2) Let $(c(v_{y+2}) = 3$ or $c(w_{y+1}) = 3)$ and $(c(v_{y-2}) \neq c(w_{y-2}) \neq 3)$, and vice versa
Without loss of generality, let $c(v_{y+2}) = 3$. Since $d(w_y, S_3) = 2$ and $d(w_{y-1}, S_3) = 3$, then we have $c(v_{y-3}) \in \{2, 3\}$ and $c(w_{y-3}) \in \{1, 2, 3, 4\} \setminus \{c(v_{y-2}), c(v_{y-3})\}$. Then, consider the vertex w_{y-2}. We have $d(w_{y-2}, S_3) \in \{2, 3\}$. If $d(w_{y-2}, S_3) = 2$, then $c(v_{y-3}) = 3$ or $c(w_{y-3}) = 3$. In other hand, if $d(w_{y-2}, S_3) = 3$, then $c(v_{y-3}) \neq c(w_{y-3}) \neq 3$. Then, those will be explained in these following cases:
   (a) If $d(w_{y-2}, S_3) = 2$, then $c(v_{y-3}) = 3$ or $c(w_{y-3}) = 3$
   Without loss of generality, let $c(v_{y-3}) = 2$ and $c(w_{y-3}) = 3$. Consequently, we have two vertices that have the same color code, which are w_{y-2} and w_y. Thus, $c_\Pi(w_y) = c_\Pi(w_{y-1}) = (1, 1, 2, 0)$.
   (b) If $d(w_{y-2}, S_3) = 3$, then $c(v_{y-3}) \neq c(w_{y-3}) \neq 3$
   Without loss of generality, let $c(v_{y-3}) = 2$ and $c(w_{y-3}) = 4$. Then, see w_{y-1} and w_{y-2}. Those vertices have the same color code, which is $(1, 1, 3, 0)$. Therefore, $c_\Pi(w_{y-1}) = c_\Pi(w_{y-2}) = (1, 1, 3, 0)$.

(3) Let $c(v_{y+2}) \neq c(w_{y+1}) \neq c(v_{y-2}) \neq c(w_{y-2}) \neq 3$
Without loss of generality, let $c(v_{y+2}) = c(v_{y-2}) = 1, c(w_{y-2}) = c(w_{y+1}) = 4$, and $c(v_z) = 3$ for $0 \leq z \leq n-1, z \notin \{y-2, y-1, y, y+1, y+2\}$. Since $d(w_y, S_3) = d(w_{y-1}, S_3) = 3$, then w_y and w_{y-1} have the same color code, which is $(1, 1, 3, 0)$. Hence, $c_\Pi(w_y) = c_\Pi(w_{y-1}) = (1, 1, 3, 0)$.

Subcase 2.2.3. Neither v_x nor v_y are dominant vertices

To demonstrate that this case also contains at least two vertices with the same color code, we follow a similar approach as in Case 2. Considering v_y as a non-dominant vertex, the argument parallels the previous case, wherein we showed that either v_x or v_y served as non-dominant vertices. Hence, by adopting the same reasoning, we conclude that there are at least two vertices with identical color codes in this possibility as well.

Based on Subcase 2.1 and Subcase 2.2, we have shown that there are at least two vertices with the same color code. Consequently, this finding contradicts the definition of locating-coloring so four colors are insufficient for locating-coloring on SF_n for $4 \leq n \leq 28$. Thus, $\chi_L(SF_n) \geq 5$ for $4 \leq n \leq 28$.

Next, we determine the upper bound of $\chi_L(SF_n)$ for $4 \leq n \leq 28$. Assume that there exists 5-locating coloring c on SF_n for $4 \leq n \leq 28$. Define $c : V \rightarrow \{1, 2, 3, 4, 5\}$ and $\Pi = \{S_1, S_2, S_3, S_4, S_5\}$ be a partition on $V(SF_n)$. Without loss of generality, let $c(v_i) = a, c(v_{i+1}) = b$, and $c(o) = 5$. Then, we have $d(v_i, S_a) = 0, d(v_i, S_b) = 1, d(v_i, S_5) = 1$, and $d(v_i, S_k) \in \{1, 2\}$ for $1 \leq a, b, k \leq 4$. Since the color codes of each vertex are distinct, then we can count every possible color code by arranging the coordinates except $d(v_i, S_5)$, as follows:

TABLE 1. The number of possible distinct color codes on v_i if $\chi_L(SF_n) = 5$

$\chi_L(SF_n)$	$c_\Pi(v_i)$				The number of possible distinct color codes on v_i
	$d(v_i, S_a)$	$d(v_i, S_b)$	$d(v_i, S_k)$	$d(v_i, S_5)$	
5	0	1	1,1	1	$\frac{4!}{1!3!0!} = 4$
	0	1	1,2	1	$\frac{4!}{1!2!1!} = 12$
	0	1	2,2	1	$\frac{4!}{1!1!2!} = 12$
Total					28

According to Table 1, consider the first row where $d(v_i, S_a) = 0, d(v_i, S_b) = 1, d(v_i, S_k) = 1$, and $d(v_i, S_5) = 1$ for $1 \leq a, b, k \leq 4$. By arranging the coordinate except for $d(v_i, S_5) = 1$, we obtain four possible distinct color codes for v_i, such as $(0, 1, 1, 1, 1), (1, 0, 1, 1, 1), (1, 1, 0, 1, 1)$, and $(1, 1, 1, 0, 1)$. It is same for the other rows as well. Then, the number of possible distinct color codes on v_i, are 28. Based on the definition of locating-coloring, it is ensured that every vertex has a unique color code. Consequently, since there are 28 possible distinct color codes on v_i from SF_n, then there can be a maximum of 28 vertices on v_i, and each vertex has a different color code. Since the number of v_i is same as the number of w_i in SF_n, then there can also be a maximum of 28 vertices on w_i. As a result, five colors are still sufficient for locating-coloring in SF_n when $n \leq 28$.

Based on the upper bound and the lower bound of $\chi_L(SF_n)$, we have $\chi_L(SF_n) = 5$ for $4 \leq n \leq 28$.

Consider a 5-locating coloring of SF_{28}. Define $c : V \to \{1, 2, 3, 4, 5\}$, as follows:

$$
\begin{aligned}
c(o) &= 5, \\
c(v_i) &= \begin{cases} 1, & \text{for } i = 0, 4, 6, 12, 14, 20, 22 \\ 2, & \text{for } i = 3, 5, 7, 16, 18, 25, 27, \\ 3, & \text{for } i = 1, 9, 11, 13, 15, 24, 26, \\ 4, & \text{for } i = 2, 8, 10, 17, 19, 21, 23, \end{cases} \\
c(w_i) &= \begin{cases} 1, & \text{for } i = 2, \\ 2, & \text{for } i = 1, 19, \\ 3, & \text{for } i = 3, \\ 4, & \text{for } i = 0, 11, \\ 5, & \text{for } i \text{ otherwise.} \end{cases}
\end{aligned}
$$

The coloring on SF_{28} will be shown in Figure 3

Then, we have distinct color codes, as follows:

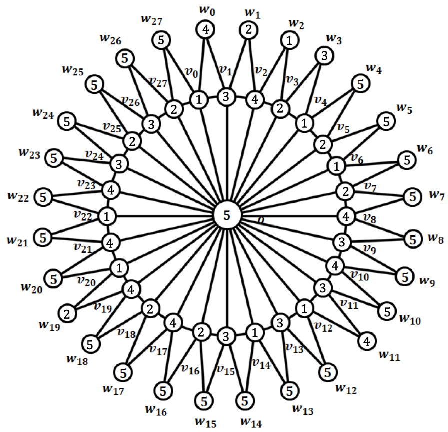

FIGURE 3. The 5-locating coloring on SF_{28}

Table 2: The color codes of $V(SF_{28})$

$c_{\Pi}(o) = (1,1,1,1,0)$,	$c_{\Pi}(v_{18}) = (2,0,2,1,1)$,	$c_{\Pi}(w_9) = (3,3,1,1,0)$,
$c_{\Pi}(v_0) = (0,1,1,1,1)$,	$c_{\Pi}(v_{19}) = (1,1,2,0,1)$,	$c_{\Pi}(w_{10}) = (2,3,1,1,0)$,
$c_{\Pi}(v_1) = (1,1,0,1,1)$,	$c_{\Pi}(v_{20}) = (0,1,2,1,1)$,	$c_{\Pi}(w_{11}) = (1,3,1,0,2)$,
$c_{\Pi}(v_2) = (1,1,1,0,1)$,	$c_{\Pi}(v_{21}) = (1,2,2,0,1)$,	$c_{\Pi}(w_{12}) = (1,3,1,2,0)$,
$c_{\Pi}(v_3) = (1,0,1,1,1)$,	$c_{\Pi}(v_{22}) = (0,2,2,1,1)$,	$c_{\Pi}(w_{13}) = (1,3,1,3,0)$,
$c_{\Pi}(v_4) = (0,1,1,2,1)$,	$c_{\Pi}(v_{23}) = (1,2,1,0,1)$,	$c_{\Pi}(w_{14}) = (1,2,1,3,0)$,
$c_{\Pi}(v_5) = (1,0,2,2,1)$,	$c_{\Pi}(v_{24}) = (2,1,0,1,1)$,	$c_{\Pi}(w_{15}) = (2,1,1,2,0)$,
$c_{\Pi}(v_6) = (0,1,2,2,1)$,	$c_{\Pi}(v_{25}) = (2,0,1,2,1)$,	$c_{\Pi}(w_{16}) = (3,1,2,1,0)$,
$c_{\Pi}(v_7) = (1,0,2,1,1)$,	$c_{\Pi}(v_{26}) = (2,1,0,2,1)$,	$c_{\Pi}(w_{17}) = (3,1,3,1,0)$,
$c_{\Pi}(v_8) = (2,1,1,0,1)$,	$c_{\Pi}(v_{27}) = (1,0,1,2,1)$,	$c_{\Pi}(w_{18}) = (2,1,3,1,0)$,
$c_{\Pi}(v_9) = (2,2,0,1,1)$,	$c_{\Pi}(w_0) = (1,2,1,0,2)$,	$c_{\Pi}(w_{19}) = (1,0,3,1,2)$,
$c_{\Pi}(v_{10}) = (2,2,1,0,1)$,	$c_{\Pi}(w_1) = (2,0,1,1,2)$,	$c_{\Pi}(w_{20}) = (1,2,3,1,0)$,
$c_{\Pi}(v_{11}) = (1,2,0,1,1)$,	$c_{\Pi}(w_2) = (0,1,2,1,2)$,	$c_{\Pi}(w_{21}) = (1,3,3,1,0)$,
$c_{\Pi}(v_{12}) = (0,2,1,1,1)$,	$c_{\Pi}(w_3) = (1,1,0,2,2)$,	$c_{\Pi}(w_{22}) = (1,3,2,1,0)$,
$c_{\Pi}(v_{13}) = (1,2,0,1,1)$,	$c_{\Pi}(w_4) = (1,1,2,3,0)$,	$c_{\Pi}(w_{23}) = (2,2,1,1,0)$,
$c_{\Pi}(v_{14}) = (0,2,1,2,1)$,	$c_{\Pi}(w_5) = (1,1,3,3,0)$,	$c_{\Pi}(w_{24}) = (3,1,1,2,0)$,
$c_{\Pi}(v_{15}) = (1,1,0,2,1)$,	$c_{\Pi}(w_6) = (1,1,3,2,0)$,	$c_{\Pi}(w_{25}) = (3,1,1,3,0)$,
$c_{\Pi}(v_{16}) = (2,0,1,1,1)$,	$c_{\Pi}(w_7) = (2,1,2,1,0)$,	$c_{\Pi}(w_{26}) = (2,1,1,3,0)$,
$c_{\Pi}(v_{17}) = (2,1,2,0,1)$,	$c_{\Pi}(w_8) = (3,2,1,1,0)$,	$c_{\Pi}(w_{27}) = (1,1,2,2,0)$,

Based on the color codes above, all vertices of SF_{28} have different color codes. Thus, $\chi_L(SF_{28}) = 5$.

Then, we determine the locating-chromatic number for $n \geq 29$ by determining the number of possible color codes for $\chi_L(SF_n) = q$ for $q \geq 6$ that is divided into two cases, as follows.

Case 1. $\chi_L(SF_n) = 6$ for $29 \leq n \leq 75$

Let c be a locating-coloring in SF_n. Define $c : V \to \{1,2,3,4,5,6\}$ and $\Pi = \{S_1, S_2, S_3, S_4, S_5, S_6\}$ be a partition on $V(SF_n)$. Without loss of generality, let $c(v_i) = a, c(v_{i+1}) = b$, and $c(o) = 6$. Then, we have $d(v_i, S_a) = 0, d(v_i, S_b) = 1, d(v_i, S_6) = 1$, and $d(v_i, S_k) \in \{1,2\}$ for $1 \leq a, b, k \leq 5$. Since the color codes of each vertex are distinct, then we can count every possible color code by arranging the coordinates except $d(v_i, S_6)$, as follows:

TABLE 3. The number of possible distinct color codes on v_i if $\chi_L(SF_n) = 6$

$\chi_L(SF_n)$	$c_\Pi(v_i)$				The number of possible distinct color codes on v_i
	$d(v_i, S_a)$	$d(v_i, S_b)$	$d(v_i, S_k)$	$d(v_i, S_6)$	
6	0	1	1,1,1	1	$\frac{5!}{1!4!0!} = 5$
	0	1	1,1,2	1	$\frac{5!}{1!3!1!} = 20$
	0	1	1,2,2	1	$\frac{5!}{1!2!2!} = 30$
	0	1	2,2,2	1	$\frac{5!}{1!1!3!} = 20$
Total					75

According to Table 3, consider the first row where $d(v_i, S_a) = 0, d(v_i, S_b) = 1, d(v_i, S_k) = 1$, and $d(v_i, S_6) = 1$ for $1 \leq a, b, k \leq 5$. By arranging the coordinate except for $d(v_i, S_6) = 1$, we obtain five possible distinct color codes for v_i, such as $(0,1,1,1,1,1), (1,0,1,1,1,1), (1,1,0,1,1,1), (1,1,1,0,1,1)$ and $(1,1,1,1,0,1)$. It is same for the other rows as well. Then, the number of possible distinct color codes on v_i, are 75. Based on the definition of locating-coloring, it is ensured that every vertex has a unique color code. Consequently, since there are 75 possible distinct color codes on v_i from SF_n, then there can be a maximum of 75 vertices on v_i, and each vertex has a different color code. Since the number of v_i is same as the number of w_i in SF_n, then there can also be a maximum of 75 vertices on w_i. As a result, six colors are still sufficient for locating-coloring in SF_n when $n \leq 75$. Based on previous case, since $\chi_L(SF_n) = 5$ for $4 \leq n \leq 28$, then $\chi_L(SF_n) = 6$ for $29 \leq n \leq 75$. Hence, $\chi_L(SF_n) = 6$ for $29 \leq n \leq 75$.

Case 2. $\chi_L(SF_n) = q$ for
$$1 + \sum_{k=0}^{3} \frac{(q-2)!}{(k+1)!(q-(k+4))!} \leq n \leq \sum_{k=0}^{3} \frac{(q-1)!}{(k+1)!(q-(k+3))!}, \text{ for } q \geq 7$$

Let c be a locating-coloring in SF_n. Define $c : V \to \{1, 2, \ldots, q\}$ and $\Pi = \{S_1, S_2, \ldots, S_q\}$ be a partition on $V(SF_n)$. Without loss of generality, let $c(v_i) = a, c(v_{i+1}) = b$, and $c(o) = q$. Then, we have $d(v_i, S_a) = 0, d(v_i, S_b) = 1, d(v_i, S_q) = 1$, and $d(v_i, S_k) \in \{1,2\}$ for $1 \leq a, b, k \leq q-1$. Since the color codes of each vertex are distinct, then we can count every possible color code by arranging the coordinates except $d(v_i, S_q)$, as follows:

Table 4. The number of possible distinct color codes on v_i if $\chi_L(SF_n) = q$

$\chi_L(SF_n)$	$c_\Pi(v_i)$				The number of possible distinct color codes on v_i
	$d(v_i, S_a)$	$d(v_i, S_b)$	$d(v_i, S_k)$	$d(v_i, S_q)$	
7	0	1	1,1,1,2	1	$\frac{6!}{1!4!1!}$
	0	1	1,1,2,2	1	$\frac{6!}{1!3!2!}$
	0	1	1,2,2,2	1	$\frac{6!}{1!2!3!}$
	0	1	2,2,2,2	1	$\frac{6!}{1!1!4!}$
8	0	1	1,1,1,2,2	1	$\frac{7!}{1!4!2!}$
	0	1	1,1,2,2,2	1	$\frac{7!}{1!3!3!}$
	0	1	1,2,2,2,2	1	$\frac{7!}{1!2!4!}$
	0	1	2,2,2,2,2	1	$\frac{7!}{1!1!5!}$
⋮	⋮	⋮	⋮	⋮	⋮
q	0	1	1,1,1,2,...,2	1	$\frac{(q-1)!}{1!4!(q-6)!}$
	0	1	1,1,2,2,...,2	1	$\frac{(q-1)!}{1!3!(q-5)!}$
	0	1	1,2,2,2,...,2	1	$\frac{(q-1)!}{1!2!(q-4)!}$
	0	1	2,2,2,2,...,2	1	$\frac{(q-1)!}{1!1!(q-3)!}$

Based on Table 4, consider the first row where $\chi_L(SF_n) = 7, d(v_i, S_a) = 0, d(v_i, S_b) = 1, d(v_i, S_k) = 1$, and $d(v_i, S_7) \in \{1, 2\}$ for $1 \leq a, b, k \leq 6$. By arranging the coordinate except for $d(v_i, S_7) = 1$, we obtain $\frac{6!}{1!4!1!}$ or 30 possible distinct color codes for v_i. It is same as the other rows as well. The number of possible distinct color codes on v_i is obtained by summing $\dfrac{6!}{1!4!1!} + \dfrac{6!}{1!3!2!} + \dfrac{6!}{1!2!3!} + \dfrac{6!}{1!1!4!}$. Then, we have 180 distinct color codes on v_i if $\chi_L(SF_n) = 7$. Similarly, this holds true for $\chi_L(SF_n) = q$ where $q \geq 7$.

Then, consider the number of color codes when $\chi_L(SF_n) = q$, for $q \geq 7$. By induction, the number of possible distinct color codes on v_i when $\chi_L(SF_n) = q$, are

$$\begin{aligned} & \frac{(q-1)!}{1!3!(q-5)!} + \frac{(q-1)!}{1!2!(q-4)!} + \frac{(q-1)!}{1!2!(q-4)!} + \frac{(q-1)!}{1!1!(q-3)!} \\ = & \sum_{k=0}^{3} \frac{(q-1)!}{(k+1)!(q-(k+3))!} \end{aligned} \tag{4}$$

Based on the definition of locating-coloring, it is ensured that every vertex has a unique color code. Consequently, since there are $\sum_{k=0}^{3} \dfrac{(q-1)!}{(k+1)!(q-(k+3))!}$ possible distinct color codes on v_i from SF_n, then there can be a maximum of

$\sum_{k=0}^{3} \dfrac{(q-1)!}{(k+1)!(q-(k+3))!}$ vertices on v_i, and each vertex has a different color code. Since the number of v_i is same as the number of w_i in SF_n, then there can also be a maximum of $\displaystyle\sum_{k=0}^{3} \frac{(q-1)!}{(k+1)!(q-(k+3))!}$ vertices on w_i. As a result, q colors for $q \geq 7$ are still sufficient for q-locating-coloring in SF_n when $n \leq \sum_{k=0}^{3} \dfrac{(q-1)!}{(k+1)!(q-(k+3))!}$.

Then, let $\chi_L(SF_n) = q-1$, for $q \geq 7$. Based on Equation 4, the maximum number of possible distinct color codes on v_i and w_i are

$$\sum_{k=0}^{3} \frac{((q-1)-1)!}{(k+1)!((q-1)-(k+3))!} = \sum_{k=0}^{3} \frac{(q-2)!}{(k+1)!(q-(k+4))} \tag{5}$$

Therefore, based on Equation 4 and Equation 5, we can conclude that q colors are still sufficient for q-locating coloring in SF_n if

$$1 + \sum_{k=0}^{3} \frac{(q-2)!}{(k+1)!(q-(k+4))} \leq n \leq \sum_{k=0}^{3} \frac{(q-1)!}{(k+1)!(q-(k+3))}.$$

Thus, we have $\chi_L(SF_n) = q$ for $1 + \sum_{k=0}^{3} \dfrac{(q-2)!}{(k+1)!(q-(k+4))!} \leq n \leq \sum_{k=0}^{3} \dfrac{(q-1)!}{(k+1)!(q-(k+3))!}$, for $q \geq 7$. ■

4. CONCLUDING REMARKS

In this paper, we have determined that the locating-chromatic number of the sunflower graph, as follows:

$$\chi_L(SF_n) = \begin{cases} 4, & \text{for } n = 3, \\ 5, & \text{for } 4 \leq n \leq 28, \\ 6, & \text{for } 29 \leq n \leq 75, \\ q, & \text{for } 1 + \sum_{k=0}^{3} \dfrac{(q-2)!}{(k+1)!(q-(k+4))!} \leq n \leq \sum_{k=0}^{3} \dfrac{(q-1)!}{(k+1)!(q-(k+3))!} \\ & \text{for } q \geq 7. \end{cases}$$

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