J. Indones. Math. Soc.
Vol. 29, No. 02 (2023), pp. 235–244.

$\mathfrak{f}_q$-DERIVATION OF BP-ALGEBRAS

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Abstract. First, this article presents the definition of left-right derivation and right-left derivation in BP-algebra, and their characteristic are explored. Then, we define the concept of inside and outside $\mathfrak{f}_q$-derivation of BP-algebras. Finally, their properties are explored. Furthermore, the notion of $\mathfrak{f}_q$-derivation within BP-algebra is synonymous with B-algebra; however, they do exhibit variations in their respective characteristics.

Key words and Phrases: *left-right derivation, right-left derivation, inside $\mathfrak{f}_q$-derivation, outside $\mathfrak{f}_q$-derivation, BP-algebra*

1. INTRODUCTION

Negger and Kim [9] introduced the notion of B-algebra $(H;*,0)$ in their research. This type of algebra adheres to the following principles : (I) $k*k=0$, (II) $k*0=k$, and (III) $(k*l)*m=k*(m*(0*l))$ for each $k,l,m \in H$. Then, Kim and Park [10] explored a unique variation of B-algebra referred to as 0-commutative algebra. This type of algebra adheres to the axiom : $k*(0*l)=l*(0*k)$ for all $k,l \in H$, where H represents a specific set. Furthermore, Ahn and Han [1] constructed a new algebra related to B-algebra called BP-algebra $(M;*,0)$, which satisfies the axioms : (I) $k*k=0$, (II) $k*(k*l)=l$, and (III) $(k*m)*(l*m)=k*l$, for every $k,l,m \in M$. The exists a connection between B-algebra and BP-algebra, where in every 0-commutative B-algebra can be classified as a BP-algebra. Additionally, a BP-algebra that fulfills the condition $(k*l)*m=k*(m*l)$ can be identified as a B-algebra. Various ideas have been explored within the realm of BP-algebra including the notions of the external direct product [4] and BP-space concepts [7].

The investigation of derivations initially originated in the study of rings and near rings [3]. Al-Shehrie [2] extended this concept to B-algebra. Subsequently, Muangkarn et al. [8], Gemawati et al. [5], and Yattaqi et al. [11] have introduced the notion of $\mathfrak{f}_q$-derivation in some algebras, which constitutes a distinct

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2020 Mathematics Subject Classification: 06F35, 03G25.
Received: 23-06-2023, accepted: 25-08-2023.

form of derivation. They explored the application of $\mathfrak{f}_q$-derivation by establishing a mapping that incorporates endomorphisms. Gemawati et al.[6] have also explored additional critical concepts within the realm of abstract algebra, including various classifications of ideals in a given algebra.

This article introduces the notion of derivation in BP-algebra and examines its properties. Subsequently, the idea of $\mathfrak{f}_q$-derivation within BP-algebra is thoroughly examined, and several associated properties are investigated.

2. PRELIMINARIES

The following provides the basic concepts needed in the construction of the concept of derivation and $\mathfrak{f}_q$-derivation in BP-algebras.

Definition 2.1. [9] *Let H be a non-empty set representing a B-algebra $(H;*,0)$ satisfying the following conditions:*

(B1) $k*k=0$,
(B2) $k*0=k$,
(B3) $(k*l)*m=k*(m*(0*l))$,

for every $k,l,m \in H$.

Lemma 2.2. [9] *In B-algebra $(H;*,0)$, the following properties hold:*

(i) $0*(0*k)=k$,
(ii) $(k*l)*(0*l)=k$,
(iii) $l*m=l*(0*(0*m))$,
(iv) $k*(l*m)=(k*(0*m))*l$,
(v) *If* $k*m=l*m$, *then* $k=l$,
(vi) *If* $k*l=0$, *then* $k=l$,

for each $k,l,m \in H$.

Definition 2.3. [10] *A B-algebra $(H;*,0)$ is 0-commutative if fulfill $k*(0*l)=l*(0*k)$ for every $k,l \in H$.*

Example 2.4. *Let $P=\{0,a,1\}$ is a set defined in Table 1.*

TABLE 1. Table for $(P;*,0)$

$*$	0	a	1
0	0	1	a
a	a	0	1
1	1	a	0

Based on Table 1, *we can observe that the B-algebra $(B;*,0)$ satisfies the property of* 0*-commutative.*

To discuss the concept of derivation in B-algebra, lets consider $(H;*,0)$ as a B-algebra. The operation " $\wedge$ " is defined in B-algebra, that is, $k \wedge l = l * (l * k)$ for all $k, l \in H$.

Definition 2.5. [2] *For a given B-algebra $(H;*,0)$, a mapping δ from H to itself is considered a left-right derivation in H if it fulfills the condition:*

$$\delta(k * l) = (\delta(k) * l) \wedge (k * \delta(l))$$

for every $k, l \in H$. Then, δ is referred to as a right-left derivation in H if it satisfies

$$\delta(k * l) = (k * \delta(l)) \wedge (\delta(k) * l).$$

A mapping δ is called a derivation of H if it acts as both a left-right derivation and a right-left derivation in H simultaneously.

Definition 2.6. [1] *BP-algebra is defined as a non-empty set $(D;*,0)$ satisfying the following axioms:*

(BP1) $k * k = 0$,
(BP2) $k * (k * l) = l$,
(BP3) $(k * m) * (l * m) = k * m$,

for all $k, l, m \in D$.

Example 2.7. *Let $M = \{0, b, c, 1\}$ is a set defined in Table 2.*

Table 2. Table for $(M;*,0)$

$*$	0	b	c	1
0	0	b	c	1
b	b	0	1	c
c	c	1	0	b
1	1	c	b	0

The structure $(M;*,0)$ represents a BP-algebra.

Theorem 2.8. [1] *If $(H;*,0)$ is a BP-algebra, then for every $k, l \in H$:*

(i) $0 * (0 * k) = k$,
(ii) $0 * (l * k) = k * l$,
(iii) $k * 0 = k$,
(iv) *If* $k * l = 0$, *then* $l * k = 0$,
(v) *If* $0 * k = 0 * l$, *then* $k = l$,
(vi) *If* $0 * k = l$, *then* $0 * l = k$,
(vii) *If* $0 * k = k$, *then* $k * l = l * k$.

Muangkarn et al. [8] examines the concept of the $\mathfrak{f}_q$-derivation in B-algebra.

Definition 2.9. *Let $(H; *, 0)$ be a B-algebra. A self-map $\mathfrak{f}$ of H is called an endomorphism if $\mathfrak{f}(k * l) = \mathfrak{f}(k) * \mathfrak{f}(l)$ for all $k, l \in H$.*

Let $\mathfrak{f}$ be an endomorphism of B-algebra $(A; *, 0)$ and $q \in A$. The self-map $\delta_q^{\mathfrak{f}}$ on A is defined by $\delta_q^{\mathfrak{f}}(a) = \mathfrak{f}(a) * q$ for all $a \in A$.

Definition 2.10. [8] *Let $\mathfrak{f}$ be an endomorphism of B-algebra $(A; *, 0)$. A self-map $\delta_q^{\mathfrak{f}}$ of A for all $q \in A$ is called an inside $\mathfrak{f}_q$-derivation of A if $\delta_q^{\mathfrak{f}}(a*b) = \delta_q^{\mathfrak{f}}(a)*\mathfrak{f}(b)$ for all $a, b \in A$. If $\delta_q^{\mathfrak{f}}(a*b) = \mathfrak{f}(a)*\delta_q^{\mathfrak{f}}(b)$, then we say that $\delta_q^{\mathfrak{f}}$ is an outside $\mathfrak{f}_q$-derivation of A. An $\mathfrak{f}_q$-derivation of A if it is both an inside and outside $\mathfrak{f}_q$-derivation of A.*

3. Derivation of BP-algebra

In this section, a left-right and a right-left derivation in BP-algebras are defined. Then, some of its properties are obtained.

Let $(M; *, 0)$ be a BP-algebra, we denote $k \wedge l = l * (l * k)$ for all $k, l \in M$.

Definition 3.1. *Consider a BP-algebra $(M; *, 0)$. A left-right derivation of M is a self-map, denoted as δ, that satisfies the identity $\delta(k*l) = (\delta(k)*l) \wedge (k*\delta(l))$ for all $k, l \in M$. In addition, if M satisfies the identity $\delta(k*l) = (k*\delta(l)) \wedge (\delta(k)*l)$ for all $k, l \in M$, we refer to δ as a right-left derivation. Furthermore, if δ satisfies both the left-right and right-left derivation, we classify it as a derivation of M.*

Example 3.2. *Consider the set of integers $\mathbb{Z}$ equipped with the subtraction operation $(-)$ and the constant 0. It can be easily demonstrated that $\mathbb{Z}$ forms a BP-algebra. Let δ be a self-map of $\mathbb{Z}$ defined as $\delta(i) = i - 1$ for all $i \in \mathbb{Z}$. We can show that δ is a left-right derivation in $\mathbb{Z}$. However, if we examine the expression $(3-(1-1)) \wedge (3-1-1)$, it equals 3, whereas $\delta(3-1)$ evaluates to 1. Hence, we observe that δ is not a right-left derivation in $\mathbb{Z}$, as it fails to satisfy the right-left derivation identity.*

Example 3.3. *Let $A = \{0, a, 1, 2\}$ is a set defined in Table 3.*

Table 3. Table for $(A; *, 0)$

$*$	0	a	1	2
0	0	2	1	a
a	a	0	2	1
1	1	a	0	2
2	2	1	a	0

Thus, it can be readily demonstrated that A is a BP-algebras. Define a map $\delta : A \to A$ by

$$\delta(k) = \begin{cases} 1 & \text{if } k = 0, \\ 2 & \text{if } k = a, \\ 0 & \text{if } k = 1, \\ a & \text{if } k = 2, \end{cases}$$

We can demonstrate that δ is both a left-right and a right-left derivation of A, which allows us to classify δ as a derivation of A.

Definition 3.4. *Let $(M; *, 0)$ be a BP-algebra. A self-map δ is said to be regular if $\delta(0) = 0$.*

Theorem 3.5. *Let $(M; *, 0)$ be a BP-algebra and δ be a left-right derivation in M, then*

(i) $\delta(k * l) = \delta(k) * l$ *for all* $k, l \in M$,
(ii) $\delta(0) = \delta(k) * k$ *for all* $k \in M$,
(iii) $\delta(k * \delta(k)) = 0$ *for all* $k \in M$,
(iv) If δ is regular, then δ is an identity function.

Proof. Let $(M; *, 0)$ be a BP-algebra and δ be a left-right derivation in M.

(i) Since δ is a left-right derivation in M, then by axiom $BP2$ we have

$$\begin{aligned} \delta(k * l) &= (\delta(k) * l) \wedge (k * \delta(l)) \\ &= (k * \delta(l)) * [(k * \delta(l)) * (\delta(k) * l)] \\ \delta(k * l) &= \delta(k) * l. \end{aligned}$$

Hence, this shows that $\delta(k * l) = \delta(k) * l$ for all $k, l \in M$.
The converse of (i) is held in general.

(ii) By (i) It is obtained that $\delta(k * l) = \delta(k) * l$. By substitution $l = k$ then $\delta(k * k) = \delta(k) * k$, and by axiom $BP1$ we get $\delta(0) = \delta(k) * k$ for all $k \in M$.

(iii) By (i) and axiom $BP1$ we have $\delta(k * \delta(k)) = \delta(k) * \delta(k) = 0$ for all $k \in M$.

(iv) By (i) and Theorem 2.4 (i), and since δ is regular, then for all $k \in M$, we have

$$\delta(k) = \delta(0 * (0 * k)) = \delta(0) * (0 * k) = 0 * (0 * k) = k.$$

Theorem 3.6. *Let $(M; *, 0)$ be a BP-algebra and δ be a right-left derivation in M, then*

(i) $\delta(k * l) = k * \delta(l)$ *for all* $k, l \in M$,
(ii) $\delta(0) = k * \delta(k)$ *for all* $k \in M$,
(iii) $\delta(\delta(k) * k) = 0$ *for all* $k \in M$,
(iv) If δ is regular, then δ is an identity function.

PROOF. Let $(M; *, 0)$ be a BP-algebra and δ be a right-left derivation in M.

(i) Since δ is a right-left derivation in M, then by axiom $BP2$ we get

$$\begin{aligned}\delta(k * l) &= (k * \delta(l)) \wedge (\delta(k) * l)\\ &= (\delta(k) * l) * [(\delta(k) * l) * (k * \delta(l))]\\ \delta(k * l) &= k * \delta(l).\end{aligned}$$

Thus, we have $\delta(k * l) = k * \delta(l)$ for all $k, l \in M$.
The converse of (i) is held in general.

(ii) By (i) it is obtained that $\delta(k * l) = k * \delta(l)$. Substituting $l = k$ yields $\delta(k * k) = k * \delta(k)$, and by axiom $BP1$ we get $\delta(0) = k * \delta(k)$ for all $k \in M$.

(iii) By (i) and axiom $BP1$ we have $\delta(\delta(k) * k) = \delta(k) * \delta(k) = 0$ for all $k \in M$.

(iv) By (i) and Theorem 2.4 (iii), and since δ is regular, then for all $k \in M$ we have

$$\delta(k) = \delta(k * 0) = k * \delta(0) = k * 0 = k.$$

Theorem 3.7. *Let $(M; *, 0)$ be a BP-algebra and δ be a derivation in M. δ is regular if and only if δ is an identity function.*

PROOF. If we consider δ as a left-right derivation in M, Theorem 3.5 (iv) demonstrates that δ function as an identity. On the other hand, if δ is a right-left derivation in M, Theorem 3.6 (iv) establishes that δ also function as an identity. Conversely, if δ is an identity function, it is evident that $\delta(0) = 0$, indicating that δ is a regular.

4. $\mathfrak{f}_q$-DERIVATION OF BP-ALGEBRA

This section introduces the definitions of an inside $\mathfrak{f}_q$-derivation, an outside $\mathfrak{f}_q$-derivation, and an $\mathfrak{f}_q$-derivation in BP-algebras. It further explores the associated properties of inside and outside $\mathfrak{f}_q$-derivations in BP-algebras.

Let $(H; *, 0)$ be a BP-algebra. A self-map $\mathfrak{f}$ of H is called an endomorphism if $\mathfrak{f}(k * l) = \mathfrak{f}(k) * \mathfrak{f}(l)$ for all $k, l \in H$. Let $\mathfrak{f}$ be an endomorphism of BP-algebra $(H; *, 0)$ and $q \in H$. The self-map $\delta_q^{\mathfrak{f}}$ on H is defined by $\delta_q^{\mathfrak{f}}(k) = \mathfrak{f}(k) * q$ for all $k \in H$.

Definition 4.1. *Consider an endomorphism $\mathfrak{f}$ of the B-algebra $(H; *, 0)$. A self-map $\delta_q^{\mathfrak{f}}$ of H for all $q \in H$ is referred to as an inside $\mathfrak{f}_q$-derivation of H if for all $k, l \in H$, $\delta_q^{\mathfrak{f}}(k * l) = \delta_q^{\mathfrak{f}}(k) * \mathfrak{f}(l)$. Furthermore, if $\delta_q^{\mathfrak{f}}(k * l) = \mathfrak{f}(k) * \delta_q^{\mathfrak{f}}(l)$, we classify $\delta_q^{\mathfrak{f}}$ as an outside $\mathfrak{f}_q$-derivation of H. An $\mathfrak{f}_q$-derivation of H satisfies both the inside and outside $\mathfrak{f}_q$-derivation conditions.*

Example 4.2. *Consider the BP-algebra $(\mathbb{Z}; -, 0)$. It can be easily demonstrated that a self-map $\delta_q^{\mathfrak{f}}(k) = \mathfrak{f}(k) - q$ for all $k, q \in \mathbb{Z}$ is an inside $\mathfrak{f}_q$-derivation in $\mathbb{Z}$. However, it is not an outside $\mathfrak{f}_q$-derivation in $\mathbb{Z}$. This is evident when we examine the expression $\mathfrak{f}(k) - \delta_q^{\mathfrak{f}}(l)$. It simplifies to $\mathfrak{f}(k) - (\mathfrak{f}(l) - q)$, which further reduces to $\mathfrak{f}(k) - \mathfrak{f}(l) + q$. As a result, it does not coincide with $\delta_q^{\mathfrak{f}}(k - l) = \mathfrak{f}(k - l) - q = \mathfrak{f}(k) - \mathfrak{f}(l) - q$ for all elements k and l belonging to $\mathbb{Z}$.*

Theorem 4.3. *Let $(H; *, 0)$ be a BP-algebra and $\mathfrak{f}$ be an endomorphism of H, then $\delta_0^{\mathfrak{f}}$ is an $\mathfrak{f}_0$-derivation of H.*

Proof. By Theorem 2.8 (iii) we have

$$\begin{aligned} \delta_0^{\mathfrak{f}}(k * l) &= \mathfrak{f}(k * l) * 0 \\ &= \mathfrak{f}(k * l) \\ &= \mathfrak{f}(k) * \mathfrak{f}(l) \\ &= (\mathfrak{f}(k) * 0) * \mathfrak{f}(l) \\ \delta_0^{\mathfrak{f}}(k * l) &= \delta_0^{\mathfrak{f}}(k) * (l), \end{aligned}$$

for all $k, l \in H$. Hence, $\delta_0^{\mathfrak{f}}$ is an inside $\mathfrak{f}_0$-derivation of H. On the other side, we get

$$\begin{aligned} \delta_0^{\mathfrak{f}}(k * l) &= \mathfrak{f}(k * l) * 0 \\ &= \mathfrak{f}(k * l) \\ &= \mathfrak{f}(k) * \mathfrak{f}(l) \\ &= \mathfrak{f}(k) * (\mathfrak{f}(l) * 0) \\ \delta_0^{\mathfrak{f}}(k * l) &= \mathfrak{f}(k) * \delta_0^{\mathfrak{f}}(l), \end{aligned}$$

for all $k, l \in H$. Hence, $\delta_0^{\mathfrak{f}}$ is an outside $\mathfrak{f}_0$-derivation of H. Thus, $\delta_0^{\mathfrak{f}}$ is an $\mathfrak{f}_0$-derivation of H.

Theorem 4.4. *Let $(H; *, 0)$ be a BP-algebra and $\mathfrak{f}$ be an endomorphism of H.*

(i) If $(H; *, 0)$ is associative, then $\delta_q^{\mathfrak{f}}$ is an outside $\mathfrak{f}_q$-derivation of H for all $q \in H$,

(ii) If $(H; *, 0)$ is associative and $0 * k = k$ for all $k \in H$, then $\delta_q^{\mathfrak{f}}$ is an inside $\mathfrak{f}_q$-derivation of H for all $q \in H$.

Proof.

(i) Since $(H; *, 0)$ is associative, we get

$$\begin{aligned} \delta_q^{\mathfrak{f}}(k * l) &= \mathfrak{f}(k * l) * q \\ &= (\mathfrak{f}(k) * \mathfrak{f}(l)) * q \\ &= \mathfrak{f}(k) * (\mathfrak{f}(l) * q) \\ \delta_q^{\mathfrak{f}}(k * l) &= \mathfrak{f}(k) * \delta_q^{\mathfrak{f}}(l), \end{aligned}$$

for all $k, l \in H$. Hence, $\delta_q^{\mathfrak{f}}$ is an outside $\mathfrak{f}_q$-derivation of H.

(ii) If $0 * k = k$ for all $k \in H$, then by Theorem 2.8 (vii) we have $k * l = l * k$ for all $k, l \in H$. Since $(H; *, 0)$ is associative, we obtain

$$\begin{aligned} \delta_q^{\mathfrak{f}}(k * l) &= \delta_q^{\mathfrak{f}}(l * k) \\ &= \mathfrak{f}(l * k) * q \\ &= (\mathfrak{f}(l) * \mathfrak{f}(k)) * q \\ &= \mathfrak{f}(l) * (\mathfrak{f}(k) * q) \\ &= \mathfrak{f}(l) * \delta_q^{\mathfrak{f}}(k) \\ \delta_q^{\mathfrak{f}}(k * l) &= \delta_q^{\mathfrak{f}}(k) * \mathfrak{f}(l), \end{aligned}$$

for all $k, l \in H$. Hence, $\delta_q^{\mathfrak{f}}$ is an inside $\mathfrak{f}_q$-derivation of H.

Corollary 4.5. *If $(H; *, 0)$ is an associative BP-algebra and $0 * k = k$ for all $k \in H$, then $\delta_q^{\mathfrak{f}}$ is an $\mathfrak{f}_q$-derivation of H for all $q \in H$.*

Proof. It is straightforward to Theorem 4.4.

Lemma 4.6. *Let $(H; *, 0)$ be a BP-algebra and $\mathfrak{f}$ be an endomorphism of H.*

(i) If $\delta_q^{\mathfrak{f}}$ is an inside $\mathfrak{f}_q$-derivation of H for all $q \in H$, then $\delta_q^{\mathfrak{f}}(0) = \delta_q^{\mathfrak{f}}(k) * \mathfrak{f}(k)$ for all $k \in H$,

(ii) If $\delta_q^{\mathfrak{f}}$ is an outside $\mathfrak{f}_q$-derivation of H for all $q \in H$, then $\delta_q^{\mathfrak{f}}(0) = q$.

Proof.

(i) Since $\delta_q^{\mathfrak{f}}$ is an inside $\mathfrak{f}_q$-derivation of H and by axiom $BP1$, for all $k \in H$ we have

$$\begin{aligned} \delta_q^{\mathfrak{f}}(k * k) &= \delta_q^{\mathfrak{f}} * \mathfrak{f}(k) \\ \delta_q^{\mathfrak{f}}(0) &= \delta_q^{\mathfrak{f}}(k) * \mathfrak{f}(k). \end{aligned}$$

(ii) Since $\delta_q^{\mathfrak{f}}$ is an outside $\mathfrak{f}_q$-derivation of H, by axiom $BP1$ and $BP2$, for all $k \in H$ we have

$$\begin{aligned} \delta_q^{\mathfrak{f}}(k * k) &= \mathfrak{f}(k) * \delta_q^{\mathfrak{f}}(k) \\ \delta_q^{\mathfrak{f}}(0) &= \mathfrak{f}(k) * (\mathfrak{f}(k) * q) \\ \delta_q^{\mathfrak{f}}(0) &= q. \end{aligned}$$

Theorem 4.7. *Let $(H; *, 0)$ be a BP-algebra, and $\delta_q^{\mathfrak{f}}$ is an $\mathfrak{f}_q$-derivation of H for all $q \in H$. If $\delta_q^{\mathfrak{f}}$ regular, then $\delta_q^{\mathfrak{f}} = f$.*

PROOF. Since $\delta_q^{\mathfrak{f}}$ is a regular, by Theorem 2.8 (i) for all $k \in H$ we obtain

$$\begin{aligned}\delta_q^{\mathfrak{f}}(k) &= \delta_q^{\mathfrak{f}}(0 * (0 * k)) \\ &= \delta_q^{\mathfrak{f}}(0) * \mathfrak{f}(0 * k) \\ &= 0 * (0 * \mathfrak{f}(k)) \\ \delta_q^{\mathfrak{f}}(k) &= \mathfrak{f}(k).\end{aligned}$$

On the other side, by Theorem 2.8 (i), we have

$$\begin{aligned}\delta_q^{\mathfrak{f}}(k) &= \delta_q^{\mathfrak{f}}(k * 0) \\ &= \mathfrak{f}(k) * \delta_q^{\mathfrak{f}}(0) \\ &= \mathfrak{f}(k) * 0 \\ \delta_q^{\mathfrak{f}}(k) &= \mathfrak{f}(k).\end{aligned}$$

5. CONCLUSION

This paper introduces the concepts of left-right derivation, right-left derivation, and derivation in BP-algebra, and examines their properties. One significant finding is a property resembling the left-right derivation and right-left derivation: if δ is a regular in BP-algebra, then it is also an identity function. This implies that any derivation which is regular in BP-algebra is necessarily an identity function. Additionally, the definition of $\mathfrak{f}_q$-derivation in BP-algebra is equivalent to that in B-algebra, but in general, their properties are different.

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