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ON A GROUP INVOLVING THE AUTOMORPHISM OF THE JANKO GROUP J_2

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Abstract. The Janko sporadic simple group J_2 has an automorphism group 2. Using the electronic Atlas of Wilson [22], the group $J_2{:}2$ has an absolutely irreducible module of dimension 12 over $\mathbb{F}_2$. It follows that a split extension group of the form $2^{12}{:}(J_2{:}2) := \overline{G}$ exists. In this article we study this group, where we compute its conjugacy classes and character table using the coset analysis technique together with Clifford-Fischer Theory. The inertia factor groups of $\overline{G}$ will be determined by analysing the maximal subgroups of $J_2{:}2$ and maximal of the maximal subgroups of $J_2{:}2$ together with various other information. It turns out that the character table of $\overline{G}$ is a 64×64 real valued matrix, while the Fischer matrices are all integer valued matrices with sizes ranging from 1 to 6.

Key words and Phrases: Group extensions, Janko sporadic simple group, inertia groups, Fischer matrices, character table

1. INTRODUCTION

Visiting the history of the classification of finite simple groups, one can see that it is only after a century of the establishment of the last Mathieu group that Z. Janko was able to construct a new sporadic simple group and that was in 1964. This simple group has been named in his honour and is denoted by J_1 and it has order 175560. Then Janko predicted the existence of other sporadic simple groups, namely J_2, J_3 and J_4, which later are all proved to be exist. According to Wilson [21], the original construction of the second Janko group J_2 was due to Marshall Hall (and thus in some other articles, is referred to this group as Hall-Janko group HJ but here we use the more familiar notation J_2). Hall constructed this group as a permutation group acting on 100 points. Starting with the group $PSU(3,3)$, the group J_2 appears as a maximal normal subgroup of index 2 of the automorphism

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group of a graph Γ, associated with $PSU(3,3)$ (for further details on the vertices and how are they connected, see the description given in page 224 of [21]).

The group J_2 has order $604800 = 2^7 \times 3^3 \times 5^2 \times 7$. It has Schur multiplier and outer automorphism groups both isomorphic to $\mathbb{Z}_2$. Therefore the group $J_2{:}2$ exists and in fact it is a maximal subgroup of index 370656 of the Suzuki group Suz. However in this paper we do not look at it as a subgroup of Suz, as the smallest number of points required to represent Suz as a permutation group is 1782. Also to represent Suz in matrix form, the dimensions over finite fields are relatively big. Rather we look at $J_2{:}2$ as a group associated with J_2, where from the electronic Atlas of Wilson [22], we can see that both J_2 and $J_2{:}2$ can be represented in terms of matrices with small dimensions over finite fields, or in terms of permutations on 100 points. More precisely one can see that the group $J_2{:}2$ has a 12 dimensional absolutely irreducible module over $\mathbb{F}_2$. Therefore a split extension group of the form $2^{12}{:}(J_2{:}2) := \overline{G}$ exists. In this article we focus on the group $\overline{G}$, where we will determine its conjugacy classes, the inertia factor groups with the fusions of their conjugacy classes into the classes of $J_2{:}2$, the character tables of these inertia factors and finally the full character table of the full extension $\overline{G}$. The method of the coset analysis together with the Clifford-Fischer Theory are used here. The most interesting part is the determination of the inertia factor groups, where there are three inertia factor groups, namely $H_1 = J_2{:}2$, H_2 and H_3. The main method used to determine the structures of H_2 and H_3, is by analysing the maximal subgroups of $J_2{:}2$ and the maximal subgroups of these maximal subgroups. Sometimes we consider the third level of maximal subgroups of $J_2{:}2$. There are many possibilities for H_2 and H_3 and all possibilities lead to contradictions (using various information from Clifford-Fischer theory and the interplay between the conjugacy classes of $\overline{G}$ obtained using the coset analysis technique and the Fischer matrices); except one possibility, where we ended up with finding that $H_2 = 2^{2+4}{:}D_{12}$ and $H_3 = 2^2{:}S_5$. We used the method of the coset analysis together with Clifford-Fischer theory to construct the character tables of H_2 and H_3, but we organized the columns of the character tables of these inertia factors according to the centralizers sizes. The Fischer matrices of $\overline{G}$ have all been determined in this paper and their sizes range between 1 and 6. The character table of $\overline{G}$ is a 64×64 real valued matrix and it is partitioned into 81 parts corresponding to the three inertia factor groups and the 27 conjugacy classes of $G = J_2{:}2$. If $\overline{G} = N{\cdot}G$ is a group extension (here N is the kernel of the extension and G is isomorphic to $\overline{G}/N$), then the character table of G produced using the coset analysis and Clifford-Fischer Theory is in a special format that can not be obtained by the direct computations using GAP [17] or Magma [13]. Another interesting point is the interplay between the coset analysis and Clifford-Fischer Theory. This can be seen at the size of each Fischer matrix, where it is equal to the number of $\overline{G}$-classes corresponding to $[g_i]_G$ obtained via the coset analysis technique. In other words, computations of the conjugacy classes of $\overline{G}$ using the coset analysis technique will determine the sizes of all Fischer matrices. An application of this interesting point can be seen at the proof of Proposition 3.2. From the Atlas [22], we can see that $J_2{:}2$ has an absolutely irreducible module of dimension 12 over the field $\mathbb{F}_2$. The following two elements g_1 and g_2 are 12×12 matrices over $\mathbb{F}_2$ that generates J_2:2.

$$g_1 = \begin{pmatrix} 0&1&0&0&0&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&0&0 \end{pmatrix}, \; g_2 = \begin{pmatrix} 0&1&1&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0\\ 0&1&1&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&0&0\\ 1&0&0&1&0&1&0&1&0&1&0&0\\ 1&0&1&0&1&0&0&1&0&0&0&0\\ 1&0&1&0&1&1&1&0&1&0&0&0\\ 1&1&1&0&0&0&0&0&0&1&1&0\\ 0&0&0&1&1&1&1&1&0&1&1&0\\ 1&1&0&0&1&1&1&0&0&0&1&1 \end{pmatrix},$$

where $o(g_1) = 2$, $o(g_2) = 5$ and $o(g_1 g_2) = 14$.

Using the generators g_1 and g_2 of J_2:2 together with few GAP commands we were able to construct our split extension group $\overline{G} = 2^{12}$:$(J_2$:$2)$ in terms of 13×13 matrices over $\mathbb{F}_2$. The following two elements $\overline{g}_1$ and $\overline{g}_2$ generate $\overline{G}$.

$$\overline{g}_1 = \begin{pmatrix} 1&1&0&1&0&1&1&0&1&1&1&0&0\\ 1&1&0&1&0&1&1&0&0&1&1&0&0\\ 0&1&1&0&1&0&1&1&0&1&0&0&0\\ 0&1&1&1&1&1&0&1&1&0&1&0&0\\ 1&0&1&0&0&0&0&0&1&1&0&0&0\\ 0&0&0&0&0&1&0&1&1&0&1&0&0\\ 1&1&0&1&1&1&1&1&1&0&1&1&0\\ 0&0&1&1&1&1&1&1&0&1&0&1&0\\ 1&0&0&1&1&0&1&0&1&1&0&0&0\\ 1&1&0&1&1&1&0&0&0&1&1&0&0\\ 0&1&1&0&0&1&1&0&1&1&1&1&0\\ 0&1&1&0&1&1&0&1&0&1&1&0&0\\ 0&0&0&1&0&1&1&0&0&0&1&0&1 \end{pmatrix}, \; \overline{g}_2 = \begin{pmatrix} 1&1&1&0&1&1&1&0&0&0&0&0&0\\ 1&0&0&1&1&0&1&1&1&1&1&1&0\\ 1&1&0&1&0&1&0&0&1&1&1&1&0\\ 0&1&0&0&0&1&1&0&0&1&1&0&0\\ 0&1&1&0&0&0&1&0&0&0&0&1&0\\ 1&0&1&0&0&1&0&0&0&1&1&1&0\\ 0&1&1&0&1&0&0&1&1&0&1&1&0\\ 0&1&0&0&1&1&0&1&0&1&0&1&0\\ 1&1&1&0&0&0&1&0&1&0&0&1&0\\ 0&0&0&0&1&1&1&0&1&0&1&1&0\\ 1&0&0&0&0&1&0&0&1&1&1&0&0\\ 0&1&1&0&0&0&0&1&0&0&0&0&0\\ 1&1&1&0&1&1&1&1&0&0&1&0&1 \end{pmatrix},$$

where $o(\overline{g}_1) = 14$, $o(\overline{g}_2) = 10$ and $o(\overline{g}_1 \overline{g}_2) = 24$.

To make the computations easier, we used few GAP commands to convert the representation of $\overline{G}$ from matrix into permutation representation. We represented $\overline{G}$ in terms of the set $\{1, 2, \cdots, 4096\}$.

Using GAP, the group $\overline{G}$ possesses two proper normal subgroups of orders 4096 and 2477260800. The normal subgroup of order 4096 is an elementary abelian group isomorphic to N. In GAP one can check for the complements of N in $\overline{G}$, where in our case we obtained two complements both isomorphic to J_2:2 and any of these two complements together with N gives the split extension in consideration.

For the notation used in this article and how the Clifford-Fischer theory and the coset analysis techniques are used, we follow [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 16].

2. Conjugacy classes of $\overline{G} = 2^{12}$:$(J_2$:$2)$

Here we compute the conjugacy classes of $\overline{G}$ using the coset analysis technique (see Basheer [2], Basheer and Moori [3, 4, 6] or Moori [19] and [20] for more details) since we are interested to organize the classes of $\overline{G}$ corresponding to the classes of J_2:2. Note that J_2:2 has 27 conjugacy classes (see the Atlas [22] or Table 5 of this paper). Corresponding to these 27 classes of J_2:2, we obtained 64 classes in $\overline{G}$.

In Table 1, we list the conjugacy classes of $\overline{G}$, where in this table:

- k_i represents the number of orbits $Q_{i1}, Q_{i2}, \cdots, Q_{ik_i}$ for the action of N on the coset $N\overline{g}_i = Ng_i$, where g_i is a representative of a class of J_2:2. In particular, the action of N on the identity coset N produces 4096 orbits each consists of singleton. Thus for $\overline{G}$, we have $k_1 = 4096$.
- f_{ij} is the number of orbits fused together under the action of $C_G(g_i)$ on $Q_1, Q_2, \cdots, Q_k$. In particular, the action of $C_G(1_G) = G = J_2{:}2$ on the orbits $Q_1, Q_2, \cdots, Q_k$ affords three orbits of lengths 1, 1575 and 2520 (with corresponding point stabilizers $2^{2+4}{:}D_{12}$ and $2^2{:}S_5$). Thus $f_{11} = 1$, $f_{12} = 1575$ and $f_{13} = 2520$.
- m_{ij}'s are weights (attached to each class of $\overline{G}$) that will be used later in computing the Fischer matrices of $\overline{G}$. These weights are computed by

$$m_{ij} = [N_{\overline{G}}(N\overline{g}_i) : C_{\overline{G}}(g_{ij})] = |N|\frac{|C_G(g_i)|}{|C_{\overline{G}}(g_{ij})|}, \tag{1}$$

where N is the kernel of an extension $\overline{G}$ that is in consideration.

Table 1. The conjugacy classes of $\overline{G}$

$[g_i]_G$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{\overline{G}}$	$o(g_{ij})$	$\|[g_{ij}]_{\overline{G}}\|$	$\|C_{\overline{G}}(g_{ij})\|$
$g_1 = 1A$	$k_1 = 4096$	$f_{11} = 1$	$m_{11} = 1$	g_{11}	1	1	4954521600
		$f_{12} = 1575$	$m_{12} = 1575$	g_{12}	2	1575	3145728
		$f_{13} = 2520$	$m_{13} = 2520$	g_{13}	2	2520	1966080
$g_2 = 2A$	$k_2 = 256$	$f_{21} = 1$	$m_{21} = 16$	g_{21}	2	5040	983040
		$f_{22} = 15$	$m_{22} = 240$	g_{22}	2	75600	65536
		$f_{23} = 120$	$m_{23} = 1920$	g_{23}	4	604800	8190
		$f_{24} = 120$	$m_{24} = 1920$	g_{24}	4	604800	8190
$g_3 = 2B$	$k_3 = 64$	$f_{31} = 1$	$m_{31} = 64$	g_{31}	2	115200	43008
		$f_{32} = 14$	$m_{32} = 896$	g_{32}	4	1612800	3072
		$f_{33} = 21$	$m_{33} = 1344$	g_{33}	4	2419200	2048
		$f_{34} = 28$	$m_{34} = 1792$	g_{34}	4	3225600	1536
$g_4 = 2C$	$k_4 = 64$	$f_{41} = 1$	$m_{41} = 64$	g_{41}	2	161280	30720
		$f_{42} = 1$	$m_{42} = 64$	g_{42}	4	161280	30720
		$f_{43} = 2$	$m_{43} = 128$	g_{43}	4	3225600	15536
		$f_{44} = 15$	$m_{44} = 960$	g_{44}	4	2419200	2048
		$f_{45} = 15$	$m_{45} = 960$	g_{45}	4	2419200	2048
		$f_{46} = 30$	$m_{46} = 1920$	g_{46}	4	4838400	1024
$g_5 = 3A$	$k_5 = 1$	$f_{51} = 1$	$m_{51} = 4096$	g_{51}	3	2293760	2160
$g_6 = 3B$	$k_6 = 16$	$f_{61} = 1$	$m_{61} = 256$	g_{61}	3	4300800	1152
		$f_{62} = 4$	$m_{62} = 768$	g_{62}	6	12902400	384
		$f_{63} = 12$	$m_{63} = 3072$	g_{63}	6	51609600	96

Continued on next page

Table 1 (continued)

$[g_i]_G$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{\overline{G}}$	$o(g_{ij})$	$\lvert[g_{ij}]_{\overline{G}}\rvert$	$\lvert C_{\overline{G}}(g_{ij})\rvert$
$g_7 = 4A$	$k_7 = 16$	$f_{71} = 1$	$m_{71} = 256$	g_{71}	4	1612800	3072
		$f_{72} = 3$	$m_{72} = 768$	g_{72}	4	4838400	1024
		$f_{73} = 6$	$m_{73} = 1536$	g_{73}	4	9676800	512
		$f_{74} = 6$	$m_{74} = 1536$	g_{74}	4	9676800	512
$g_8 = 4B$	$k_8 = 16$	$f_{81} = 1$	$m_{81} = 256$	g_{81}	4	3225600	1536
		$f_{82} = 3$	$m_{82} = 768$	g_{82}	4	9676800	512
		$f_{83} = 12$	$m_{83} = 3072$	g_{83}	8	38707200	128
$g_9 = 4C$	$k_9 = 8$	$f_{91} = 1$	$m_{91} = 512$	g_{91}	4	12902400	384
		$f_{92} = 1$	$m_{92} = 512$	g_{92}	8	12902400	384
		$f_{93} = 3$	$m_{93} = 1536$	g_{93}	8	77414400	64
		$f_{94} = 3$	$m_{94} = 1536$	g_{94}	8	77414400	64
$g_{10} = 5A$	$k_{10} = 16$	$f_{10,1} = 1$	$m_{10,1} = 256$	$g_{10,1}$	5	1032192	4800
		$f_{10,2} = 15$	$m_{10,2} = 3840$	$g_{10,2}$	10	15482880	320
$g_{11} = 5B$	$k_{11} = 1$	$f_{11,1} = 1$	$m_{11,1} = 4096$	$g_{11,1}$	5	99090432	50
$g_{12} = 6A$	$k_{12} = 1$	$f_{12,1} = 1$	$m_{12,1} = 4096$	$g_{12,1}$	6	103219200	48
$g_{13} = 6B$	$k_{13} = 4$	$f_{13,1} = 1$	$m_{13,1} = 1024$	$g_{13,1}$	6	51609600	96
		$f_{13,2} = 1$	$m_{13,2} = 1024$	$g_{13,2}$	12	51609600	96
		$f_{13,3} = 2$	$m_{13,3} = 2048$	$g_{13,3}$	12	103219200	48
$g_{14} = 6C$	$k_{14} = 4$	$f_{14,1} = 1$	$m_{14,1} = 1024$	$g_{14,1}$	6	103219200	48
		$f_{14,2} = 1$	$m_{14,2} = 1024$	$g_{14,2}$	12	103219200	48
		$f_{14,3} = 2$	$m_{14,3} = 2048$	$g_{14,3}$	12	206438400	24
$g_{15} = 7A$	$k_{15} = 1$	$f_{15,1} = 1$	$m_{15,1} = 4096$	$g_{15,1}$	7	353894400	14
$g_{16} = 8A$	$k_{16} = 4$	$f_{16,1} = 1$	$m_{16,1} = 1024$	$g_{16,1}$	8	12902400	384
		$f_{16,2} = 3$	$m_{16,2} = 3072$	$g_{16,2}$	8	38707200	128
$g_{17} = 8B$	$k_{17} = 4$	$f_{17,1} = 1$	$m_{17,1} = 1024$	$g_{17,1}$	8	38707200	128
		$f_{17,2} = 1$	$m_{17,2} = 1024$	$g_{17,2}$	8	38707200	128
		$f_{17,3} = 2$	$m_{17,3} = 2048$	$g_{17,3}$	8	77414400	64
$g_{18} = 8C$	$k_{18} = 4$	$f_{18,1} = 1$	$m_{18,1} = 1024$	$g_{18,1}$	8	77414400	64
		$f_{18,2} = 1$	$m_{18,2} = 1024$	$g_{18,2}$	8	77414400	64
		$f_{18,3} = 2$	$m_{18,3} = 2048$	$g_{18,3}$	8	154828800	32
$g_{19} = 10A$	$k_{19} = 4$	$f_{19,1} = 1$	$m_{19,1} = 1024$	$g_{19,1}$	10	61931520	80
		$f_{18,2} = 1$	$m_{19,2} = 1024$	$g_{19,2}$	20	61931520	80
		$f_{19,3} = 1$	$m_{19,3} = 1024$	$g_{19,3}$	20	61931520	80
		$f_{19,4} = 1$	$m_{19,4} = 1024$	$g_{19,4}$	20	61931520	80
$g_{20} = 10B$	$k_{20} = 1$	$f_{20,1} = 1$	$m_{20,1} = 4096$	$g_{20,1}$	10	495452160	10
$g_{21} = 12A$	$k_{21} = 1$	$f_{21,1} = 1$	$m_{21,1} = 4096$	$g_{21,1}$	12	206438400	24
$g_{22} = 12B$	$k_{22} = 1$	$f_{22,1} = 1$	$m_{22,1} = 4096$	$g_{22,1}$	12	412876800	12
$g_{23} = 12C$	$k_{23} = 1$	$f_{23,1} = 1$	$m_{23,1} = 2048$	$g_{23,1}$	12	206438400	24
		$f_{23,1} = 1$	$m_{23,1} = 2048$	$g_{23,1}$	24	206438400	24

Continued on next page

Table 1 (continued)

$[g_i]_G$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{\overline{G}}$	$o(g_{ij})$	$\lvert[g_{ij}]_{\overline{G}}\rvert$	$\lvert C_{\overline{G}}(g_{ij})\rvert$
$g_{24} = 14A$	$k_{24} = 1$	$f_{24,1} = 1$	$m_{24,1} = 4096$	$g_{24,1}$	14	353894400	14
$g_{25} = 15A$	$k_{25} = 1$	$f_{25,1} = 1$	$m_{25,1} = 4096$	$g_{25,1}$	15	330301440	15
$g_{26} = 24A$	$k_{26} = 1$	$f_{26,1} = 1$	$m_{26,1} = 4096$	$g_{26,1}$	24	206438400	24
$g_{27} = 24B$	$k_{27} = 1$	$f_{27,1} = 1$	$m_{27,1} = 4096$	$g_{27,1}$	24	206438400	24

3. Inertia factor groups of $\overline{G} = 2^{12}{:}(J_2{:}2)$

We have seen in Section 2 that the action of $\overline{G}$ on N produced three orbits of lengths 1, 1575 and 2520. By a theorem of Brauer (for example see Theorem 5.1.1 of Basheer [2]), it follows that the action of $\overline{G}$ on $\mathrm{Irr}(N)$ will also produce three orbits of lengths 1, r and s, where $1 + r + s = |\mathrm{Irr}(N)| = 4096$; that is

$$r + s = 4095. \tag{2}$$

The values of r and s will be determined through deep investigation on the maximal subgroups of $J_2{:}2$ or maximal of the maximal subgroups of $J_2{:}2$ together with various information including the sizes of the Fischer matrices, fusions of the the conjugacy classes of some subgroups into the group $J_2{:}2$ and other information. In Table 2 we supply the maximal subgroups of $J_2{:}2$ (see the Atlas [15]), where we need these subgroups in the process of the determination of H_2 and H_3.

Table 2. The maximal subgroups of $G = J_2{:}2$

M_i	$\lvert M_i\rvert$	$[(J_2{:}2) : M_i]$
J_2	604800	2
$U_3(3){:}2$	12096	100
$3^{\cdot}A_6{}^{\cdot}2^2$	4320	280
$2^{1+4}{}^{\cdot}S_5$	3840	315
$2^{2+4}{:}((3 \times S_3){}^{\cdot}2)$	2304	525
$(A_4 \times A_5){:}2$	1440	1061
$(A_5 \times D_{10}){}^{\cdot}2$	1200	1008
$L_3(2){:}2 \times 2$	600	2016
$5^2{:}(4 \times S_3)$	672	1800
S_5	120	10080

Firstly since 1, r and s are the lengths of the orbits on the action of $\overline{G}$ on N (which can be reduced to the action of G on N), it follows that $[G : H_1] = 1$, $[G : H_2] = r$ and $[G : H_3] = s$, where H_1, H_2 and H_3 are the inertia factor groups in $G = J_2{:}2$. It follows that $H_1 = G = J_2{:}2$ and $r, s \mid |G|$; that is $r, s|1209600$. Now 1209600 has 216 positive divisors, where 146 divisors are less than 4095. Out of these 146 divisors, only four pairs (r, s) satisfy Equation (2). These are the pairs:

$$(r, s) \in \{(63, 4032), (315, 3780), (945, 3150), (1575, 2520)\}. \tag{3}$$

Here we do not distinguish between the pair (r, s) and (s, r) and therefore we excluded the other four pairs $(4032, 63), (3780, 315), (3150, 945)$ and $(2520, 1575)$ from our consideration and we restrict ourselves only to those in Equation (3). Another point that we put in mind is that since the extension $\overline{G}$ splits over N and N is an elementary abelian group, it follows that all the character tables of H_1, H_2 and H_3 that we will use to construct the character table of $\overline{G}$ are the ordinary ones. From Tables 1 and 5 we have $|\mathrm{Irr}(\overline{G})| = 64$ and $|\mathrm{Irr}(H_1)| = |\mathrm{Irr}(G)| = |\mathrm{Irr}(J_2{:}2)| = 27$. Since $\sum_{i=1}^{3} |\mathrm{Irr}(H_i)| = |\mathrm{Irr}(\overline{G})| = 64$, we have $|\mathrm{Irr}(H_1)| + |\mathrm{Irr}(H_2)| + |\mathrm{Irr}(H_3)| = |\mathrm{Irr}(\overline{G})| = 64$, that is

$$|\mathrm{Irr}(H_2)| + |\mathrm{Irr}(H_3)| = 37. \tag{4}$$

Our next task is to show that $(r, s) = (1575, 2520)$ and that the action of $\overline{G}$ on $\mathrm{Irr}(N)$ will be dual to the action of $\overline{G}$ on the classes of N. This will be achieved by excluding the other possible pairs by getting a contradiction to some fact in each case.

Proposition 3.1. $(r, s) \neq (63, 4032)$.

Proof. For the purpose of contradiction assume $(r, s) = (63, 4032)$; that is $r = 63$ and $s = 4032$ (or $[J_2{:}2 : H_2] = 63$ and $[J_2{:}2 : H_3] = 4032$) and consequently $|H_2| = 19200$ and $|H_3| = 300$. Since $|H_2| = 19200$ and by looking at the maximal subgroups of $J_2{:}2$, given in Table 2, it follows that $|H_2|$ is bigger than the size of any maximal subgroup of $J_2{:}2$ except J_2. However $|H_2| \nmid |J_2|$ as $|H_2| = 19200$ and $|J_2| = 604800$. Therefore $J_2{:}2$ does not contain a subgroup of order 19200 and consequently (r, s) can not be $(63, 4032)$. □

Proposition 3.2. $(r, s) \neq (315, 3780)$.

Proof. For the purpose of contradiction assume $(r, s) = (315, 3780)$, that is $r = 315$ and $s = 3780$ (or $[J_2{:}2 : H_2] = 315$ and $[J_2{:}2 : H_3] = 3780$) and consequently $|H_2| = 3840$ and $|H_3| = 320$. Now looking at the maximal subgroups of $J_2{:}2$, given in Table 2, it follows that H_2 must necessarily be the group $2^{1+4\cdot}S_5$, which has 23 ordinary irreducible characters. Using Table 2 again, the group H_3 can be either an index 1890 subgroup of the group J_2 or an index 12 subgroup of the group $2^{1+4\cdot}S_5$. Now we consider these two cases. If H_3 is an index 1890 subgroup of J_2, then by looking at the maximal subgroups of J_2 available in the Atlas [15], then H_3 can only be a subgroup of $2^{1+4}{:}A_5$ with index 6. The group $2^{1+4}{:}A_5$ has 4 maximal subgroups of orders 120, 192, 320 and 384. Investigating the structure of the group of order 320, we found it to be of the form $2^{1+4}{:}D_{10}$, which has 14 ordinary irreducible characters. We put this point in mind and we consider the other case, where H_3 is an index 12 subgroup of $2^{1+4\cdot}S_5$. Now the group $2^{1+4\cdot}S_5$ has 5 maximal subgroups with orders 1920, 768, 640, 384 and 240. Therefore H_3 is either an index 6 subgroup of the group of order 1920 or an index 2 subgroup of the group of order 640. The two maximal subgroups of $2^{1+4\cdot}S_5$ of orders 1920 and 640 have structures $2^{1+4}{:}A_5$ and $2^{1+4}{:}(5{:}4)$ respectively. Now if H_3 is an index 6 subgroup of $2^{1+4}{:}A_5$, then as previously, it will have the structure $2^{1+4}{:}D_{10}$. From another side, the group 2^{1+4}:(5:4) has 3 maximal subgroups of orders 40, 128 and 320. The maximal subgroup of 2^{1+4}:(5:4) of order 320 has the structure $2^{1+4}{:}D_{10}$. Thus in all these cases we can see that H_3 is a group of the form $2^{1+4}{:}D_{10}$. Therefore as far as concerned for the case $(r, s) = (315, 3780)$ we have $H_2 = 2^{1+4\cdot}S_5$, which has 23 ordinary irreducible characters and $H_3 = 2^{1+4}{:}D_{10}$, which has 14 ordinary irreducible characters. Although $|\mathrm{Irr}(H_2)| + |\mathrm{Irr}(H_3)| = 37$, which consists with Equation (4) but we show that $(H_2, H_3) = (2^{1+4\cdot}S_5, 2^{1+4}{:}D_{10})$ is not the required pair of the inertia factor groups. In Tables 3 and 4 we supply the character tables of $2^{1+4\cdot}S_5$ and $2^{1+4}{:}D_{10}$ respectively, together with the fusions of the conjugacy classes of these groups into the conjugacy classes of $J_2{:}2$. The interplay between the coset analysis and Clifford-Fischer Theory was mentioned in some details in [7]. In particular, the size of the Fischer matrix correspond to a conjugacy class $[g]_G$ is equal to $c(g)$, where $c(g)$ is the number of conjugacy classes of the full extension $\overline{G}$ that correspond to the conjugacy class $[g]_G$ obtained using the coset analysis technique. Now from Table 1 we can see that $\overline{G} = 2^{12}{:}(J_2{:}2)$ has four conjugacy classes correspond to the class $[g_2]_{J_2:2} = [2A]_{J_2:2}$. Therefore the Fischer matrix $\mathcal{F}_2$ will be a 4×4 matrix. We also know that the rows of any Fischer matrix $\mathcal{F}_i$ (corresponds to the class $[g_i]_G$) are partitioned into submatrices correspond to the inertia factors, where there is possible fusions from the conjugacy classes of these inertia factors into the class $[g_i]_G$. For the Fischer matrix $\mathcal{F}_2$, which we found to be of size 4, we have one row corresponds to the first inertia factor $H_1 = J_2{:}2$. From Table 3, we can see that there are two conjugacy classes of H_2 that fuse to the class $g_2 = 2A$. Also from Table 4, we can see that there are two conjugacy classes of H_3 that fuse to the class $g_2 = 2A$. Therefore in total the three inertia factors contribute with 5 rows to the Fischer matrix $\mathcal{F}_2$, meaning that $\mathcal{F}_2$ is a 5×5 matrix, which contradicts the fact that $\mathcal{F}_2$ is a 4×4 matrix. We deduce that $(H_2, H_3) = (2^{1+4\cdot}S_5, 2^{1+4}{:}D_{10})$ is not the required pair of the inertia factor groups and hence (r, s) can not be $(315, 3780)$. □

Proposition 3.3. $(r, s) \neq (945, 3150)$.

Proof. For the purpose of contradiction assume $(r, s) = (945, 3150)$; that is $r = 945$ and $s = 3150$ (or $[J_2{:}2 : H_2] = 945$ and $[J_2{:}2 : H_3] = 3150$) and consequently $|H_2| = 1280$ and $|H_3| = 384$. Since $|H_2| = 1280$ and by looking at the maximal subgroups of $J_2{:}2$, given in Table 2, it follows that H_2 can only be a subgroup of $2^{1+4\cdot}S_5$ of index 3 and consequently H_2 must necessarily be a maximal subgroup of $2^{1+4\cdot}S_5$ (since the index is a prime number). However the group $2^{1+4\cdot}S_5$ has 5 maximal subgroups of orders 240, 384, 640, 768 and 1920 and therefore has no maximal subgroup of index 3. It follows that (r, s) can not be $(945, 3150)$. □

Proposition 3.4. *The action of* $J_2{:}2$ *on* $\mathrm{Irr}(2^{12})$ *is dual to the action of* $J_2{:}2$ *on the conjugacy classes of* $N = 2^{12}$.

Proof. We have seen from Section 2 that the action of $J_2{:}2$ on the conjugacy classes of $N = 2^{12}$ produced 3 orbits of lengths 1, 1575 and 2520. From Equation (2) we have $r + s = 4095$, where r and s are the lengths of the second the

TABLE 3. The character table of $2^{1+4\cdot}S_5$

	$1a$	$2a$	$2b$	$2c$	$2d$	$3a$	$4a$	$4b$	$4c$	$4d$	$5a$	$6a$	$6b$	$8a$	$8b$	$8c$	$8d$	$8e$	$10a$	$12a$	$12b$	$24a$	$24b$
$\hookrightarrow J_2{:}2$	$1A$	$2A$	$2A$	$2B$	$2C$	$3A$	$4A$	$4C$	$4A$	$4C$	$5B$	$6A$	$6A$	$8B$	$8A$	$8C$	$8A$	$8B$	$10A$	$12A$	$12B$	$24A$	$24B$
$\lvert C_{G_1}(g)\rvert$	3840	3840	384	32	32	48	192	96	32	8	10	48	12	32	32	16	16	16	10	24	12	24	24
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	-1	1	1	1	-1	1	-1	1	1	1	-1	-1	1	-1	-1	1	1	-1	-1	-1
χ_3	4	4	4	2	0	1	4	2	0	0	-1	1	1	2	2	0	0	0	-1	1	-1	-1	-1
χ_4	4	4	4	-2	0	1	4	-2	0	0	-1	1	1	-2	-2	0	0	0	-1	1	1	1	1
χ_5	5	5	5	-1	1	-1	5	-1	1	1	0	-1	-1	-1	-1	1	1	1	0	-1	-1	-1	-1
χ_6	5	5	5	1	1	-1	5	1	1	-1	0	-1	-1	1	1	1	-1	-1	0	-1	1	1	1
χ_7	5	5	-3	1	1	2	1	-3	1	-1	0	2	0	-1	3	-1	1	1	0	-2	0	0	0
χ_8	5	5	-3	-1	1	2	1	3	1	1	0	2	0	1	-3	-1	-1	-1	0	-2	0	0	0
χ_9	6	6	6	0	-2	0	6	0	-2	0	1	0	0	0	0	-2	0	0	1	0	0	0	0
χ_{10}	10	10	-6	0	2	-2	2	0	2	0	0	-2	0	0	0	-2	0	0	0	2	0	0	0
χ_{11}	10	10	2	0	2	1	-2	-4	-2	0	0	1	-1	2	-2	0	0	0	0	1	-1	1	1
χ_{12}	10	10	2	2	-2	1	-2	-2	2	0	0	1	-1	0	-4	0	0	0	0	1	1	-1	-1
χ_{13}	10	10	2	0	2	1	-2	4	-2	0	0	1	-1	-2	2	0	0	0	0	1	1	-1	-1
χ_{14}	10	10	2	-2	-2	1	-2	2	2	0	0	1	-1	0	4	0	0	0	0	1	-1	1	1
χ_{15}	15	15	-9	1	-1	0	3	-3	-1	1	0	0	0	-1	3	1	-1	-1	0	0	0	0	0
χ_{16}	15	15	-9	-1	-1	0	3	3	-1	-1	0	0	0	1	-3	1	1	1	0	0	0	0	0
χ_{17}	16	-16	0	0	0	4	0	0	0	0	1	-4	0	0	0	0	0	0	-1	0	0	0	0
χ_{18}	16	-16	0	0	0	-2	0	0	0	0	1	2	0	0	0	0	0	0	-1	0	0	$-\sqrt{6}$	$\sqrt{6}$
χ_{19}	16	-16	0	0	0	-2	0	0	0	0	1	2	0	0	0	0	0	0	-1	0	0	$\sqrt{6}$	$-\sqrt{6}$
χ_{20}	20	20	4	-2	0	-1	-4	-2	0	0	0	-1	1	2	2	0	0	0	0	-1	1	-1	-1
χ_{21}	20	20	4	2	0	-1	-4	2	0	0	0	-1	1	-2	-2	0	0	0	0	-1	-1	1	1
χ_{22}	24	-24	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	-2	2	1	0	0	0	0
χ_{23}	24	-24	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	2	-2	1	0	0	0	0

TABLE 4. The character table of 2^{1+4}:D_{10}

	$1a$	$2a$	$2b$	$2c$	$4a$	$4b$	$4c$	$4d$	$5a$	$5b$	$8a$	$8b$	$10a$	$10b$
$\hookrightarrow J_2{:}2$	$1A$	$2A$	$2A$	$2C$	$4A$	$4A$	$4A$	$4A$	$5B$	$5B$	$8C$	$8C$	$10B$	$10B$
$\lvert C_{G_2}(g)\rvert$	320	320	32	8	32	32	16	16	10	10	8	8	10	10
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	-1	1	1	-1	-1	1	1	-1	-1	1	1
χ_3	2	2	2	0	2	2	0	0	$-\frac{1}{2}-\frac{\sqrt{5}}{2}$	$-\frac{1}{2}+\frac{\sqrt{5}}{2}$	0	0	$-\frac{1}{2}-\frac{\sqrt{5}}{2}$	$-\frac{1}{2}+\frac{\sqrt{5}}{2}$
χ_4	2	2	2	0	2	2	0	0	$-\frac{1}{2}+\frac{\sqrt{5}}{2}$	$-\frac{1}{2}-\frac{\sqrt{5}}{2}$	0	0	$-\frac{1}{2}+\frac{\sqrt{5}}{2}$	$-\frac{1}{2}-\frac{\sqrt{5}}{2}$
χ_5	4	-4	0	0	0	0	$2i$	$-2i$	-1	-1	0	0	1	1
χ_6	4	-4	0	0	0	0	$-2i$	$2i$	-1	-1	0	0	1	1
χ_7	5	5	-3	1	1	1	1	1	0	0	-1	-1	0	0
χ_8	5	5	-3	-1	1	1	-1	-1	0	0	1	1	0	0
χ_9	5	5	1	1	1	-3	-1	-1	0	0	-1	1	0	0
χ_{10}	5	5	1	1	-3	1	-1	-1	0	0	1	-1	0	0
χ_{11}	5	5	1	-1	-3	1	1	1	0	0	-1	1	0	0
χ_{12}	5	5	1	-1	1	-3	1	1	0	0	1	-1	0	0
χ_{13}	8	-8	0	0	0	0	0	0	$\frac{1}{2}+\frac{\sqrt{5}}{2}$	$\frac{1}{2}-\frac{\sqrt{5}}{2}$	0	0	$-\frac{1}{2}-\frac{\sqrt{5}}{2}$	$-\frac{1}{2}+\frac{\sqrt{5}}{2}$
χ_{14}	8	-8	0	0	0	0	0	0	$\frac{1}{2}-\frac{\sqrt{5}}{2}$	$\frac{1}{2}+\frac{\sqrt{5}}{2}$	0	0	$-\frac{1}{2}+\frac{\sqrt{5}}{2}$	$-\frac{1}{2}-\frac{\sqrt{5}}{2}$

third orbits on the action of J_2:2 on $\mathrm{Irr}(2^{12})$. Further by Equation (3) we have $(r, s) \in \{(63, 4032), (315, 3780), (945, 3150), (1575, 2520)\}$.We also proved in Propositions 3.1, 3.2 and 3.3 that $(r, s) \notin \{(63, 4032), (315, 3780), (945, 3150)\}$. Therefore $(r, s) = (1575, 2520)$ and it follows that the action of J_2:2 on $\mathrm{Irr}(2^{12})$ is dual to the action of J_2:2 on the conjugacy classes of $N = 2^{12}$ as claimed. □

Proposition 3.5. *The inertia factor groups have the forms* 2^{2+4}:D_{12} *and* 2^2:S_5.

Proof. We found that the orbit lengths on the action of J_2:2 on $\mathrm{Irr}(2^{12})$ are 1, 1575 and 2520. It follows that $[G : H_1] = 1$, $[G : H_2] = 1575$ and $[G : H_3] = 2520$ and consequently $H_1 = G = J_2$:2, $|H_2| = 768$ and $|H_3| = 480$. By Equation (4) we also have $|\mathrm{Irr}(H_2)| + |\mathrm{Irr}(H_3)| = 37$. Now we investigate the maximal subgroups of $J_2{:}2$ to locate H_2 and H_3. Since $|H_2| = 768$ and by looking at the maximal subgroups of $J_2{:}2$, given in Table 2, it follows that H_2 is either an index 5 subgroup of $2^{1+4\cdot}S_5$ or an index 3 subgroup of $2^{2+4}{:}((3 \times S_3)^{\cdot}2)$. If $H_2 \leq 2^{1+4\cdot}S_5$ such that $[2^{1+4\cdot}S_5 : H_2] = 5$, then H_2 must be a maximal subgroup in it since the index is a prime number. Now $2^{1+4\cdot}S_5$ has 5 maximal subgroups of orders 240, 384, 640, 768 and 1920. The maximal subgroup of order 768 has the structure $2^{1+4}{:}S_4$ and it has 23 ordinary irreducible characters. Also if $H_2 \leq 2^{2+4}{:}((3 \times S_3)^{\cdot}2)$ such that $[2^{2+4}{:}((3 \times S_3)^{\cdot}2) : H_2] = 3$, then H_2 must be a maximal subgroup in it since the index is a prime number. Now $2^{2+4}{:}((3 \times S_3)^{\cdot}2)$ has 6 maximal subgroups of orders 1152 (triple), 768 (twice) and 144. The two maximal subgroups of order 768 have structures $2^{1+4}{:}S_4$ and $2^{2+4}{:}D_{12}$, where $|\mathrm{Irr}(2^{1+4}{:}S_4)| = 23$ and $|\mathrm{Irr}(2^{2+4}{:}D_{12})| = 18$. Thus we have

$$H_2 \in \{2^{1+4}{:}S_4, 2^{2+4}{:}D_{12}\} \quad \text{where} \quad |\mathrm{Irr}(2^{1+4}{:}S_4)| = 23 \quad \textit{and} \quad |\mathrm{Irr}(2^{2+4}{:}D_{12})| = 18. \tag{5}$$

Next we consider H_3. Since $|H_3| = 480$ and by looking at the maximal subgroups of $J_2{:}2$, given in Table 2, it follows that H_3 is either:

- an index 1260 subgroup of J_2,
- an index 9 subgroup of $3^{\cdot}A_6{}^{\cdot}2^2$,
- an index 8 subgroup of $2^{1+4\cdot}S_5$ or
- an index 3 subgroup of $(A_4 \times A_5){:}2$.

Now we consider each of the above cases. If H_3 is an index 1260 subgroup of J_2, then by checking the orders of all the maximal subgroups of J_2, available in the Atlas [15], it must be a subgroup of $2^{1+4}{:}A_5$ of index 4. However the maximal subgroups of $2^{1+4}{:}A_5$ have orders 120, 192, 320 and 384 and therefore $H_3 \not\leq 2^{1+4}{:}A_5$ and consequently H_3 can not be a subgroup of J_2. Next we consider the case that H_3 is an index 9 subgroup of $3^{\cdot}A_6{}^{\cdot}2^2$. The group $3^{\cdot}A_6{}^{\cdot}2^2$ has 6 maximal subgroups of orders 2160 (triple), 432, 120 and 96. Since 480 does not divide any of the orders of the maximal subgroups of $3^{\cdot}A_6{}^{\cdot}2^2$, we deduce that $H_3 \not\leq 3^{\cdot}A_6{}^{\cdot}2^2$. Next we consider the case that H_3 is an index 8 subgroup of $2^{1+4\cdot}S_5$. The only possibility for H_3 to be a subgroup of $2^{1+4\cdot}S_5$ is that H_3 be a subgroup of $2^{1+4}{:}A_5$ of index 4. We have seen above that the maximal subgroups of $2^{1+4}{:}A_5$ are of orders 120, 192, 320 and 384 and therefore $H_3 \not\leq 2^{1+4}{:}A_5$ and consequently H_3 can not be a subgroup of $2^{1+4\cdot}S_5$. Finally we turn to the last case where we consider H_3 be a subgroup of $(A_4 \times A_5){:}2$ of index 3. Now the group $(A_4 \times A_5){:}2$ has 6 maximal subgroups of orders 144, 240, 288, 360, 480 and 720. The maximal subgroup of order 480 has the structure $2^2{:}S_5$ and it has 19 ordinary irreducible characters. Using this together with Equation (5) we obtain that $(H_2, H_3) = (2^{2+4}{:}D_{12}, 2^2{:}S_5)$ are the required pair of inertia factor groups since it consists with Equation (4) and we exhausted all the other possible cases, where we obtained contradictions in each case expect in the case $(H_2, H_3) = (2^{2+4}{:}D_{12}, 2^2{:}S_5)$. Hence the result. □

Next we construct the character tables of H_1, H_2 and H_3 and we show the fusions of their conjugacy classes into the classes of $H_1 = G = J_2{:}2$. The character table of the simple Janko group J_2 is available in the Atlas and thus the extension J_2:2 can easily be constructed using Clifford-Fischer Theory. However we used the two generators g_1 and g_2 given in Section 1 together with GAP to construct the character table of J_2:2, which we show below as Table 5.

TABLE 5. The character table of $G = J_2$:2

	$1A$	$2A$	$2B$	$2C$	$3A$	$3B$	$4A$	$4B$	$4C$	$5A$	$5B$	$6A$	$6B$
$\lvert C_G(g)\rvert$	1209600	3840	672	480	2160	72	192	96	24	300	50	48	24
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	−1	1	1	1	1	−1	−1	1	1	1	1
χ_3	28	−4	0	4	10	−2	4	0	0	3	3	2	−2
χ_4	36	4	6	0	0	0	4	−2	0	−4	1	1	0
χ_5	36	4	−6	0	9	0	4	2	0	−4	1	1	0
χ_6	42	10	0	−6	6	0	2	0	0	7	2	−2	0
χ_7	63	15	−7	−1	0	3	3	−3	1	3	−2	0	−1
χ_8	63	15	7	−1	0	3	3	3	−1	3	−2	0	−1
χ_9	90	10	−6	6	9	0	−2	−2	0	5	0	1	0
χ_{10}	90	10	6	6	9	0	−2	2	0	5	0	1	0
χ_{11}	126	14	0	6	−9	0	2	−4	0	1	1	−1	0
χ_{12}	126	14	0	6	−9	0	2	4	0	1	1	−1	0
χ_{13}	140	−20	0	−4	14	2	4	0	0	5	0	−2	2
χ_{14}	160	0	−8	4	16	1	0	0	−2	−5	0	0	1
χ_{15}	160	0	8	4	16	1	0	0	2	−5	0	0	1
χ_{16}	175	15	7	−5	−5	1	−1	−1	1	0	0	3	1
χ_{17}	175	15	−7	−5	−5	1	−1	1	−1	0	0	3	1
χ_{18}	225	−15	1	5	0	3	−3	−3	−1	0	0	0	−1
χ_{19}	225	−15	−1	5	0	3	−3	3	1	0	0	0	−1
χ_{20}	288	0	−8	4	0	−3	0	0	2	3	−2	0	1
χ_{21}	288	0	8	4	0	−3	0	0	−2	3	−2	0	1
χ_{22}	300	−20	6	0	−15	0	4	−2	0	0	0	1	0
χ_{23}	300	−20	−6	0	−15	0	4	2	0	0	0	1	0
χ_{24}	336	16	0	0	−6	0	0	0	0	−4	1	−2	0
χ_{25}	336	16	0	0	−6	0	0	0	0	−4	1	−2	0
χ_{26}	378	−6	0	−6	0	0	−6	0	0	3	3	0	0
χ_{27}	448	0	0	−8	16	−2	0	0	0	−2	−2	0	−2

Table 5 (continued)

	$6C$	$7A$	$8A$	$8B$	$8C$	$10A$	$10B$	$12A$	$12B$	$12C$	$14A$	$15A$	$24A$	$24B$
$\lvert C_{\overline{G}}(g)\rvert$	12	14	96	32	16	20	10	24	12	12	14	15	24	24
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	−1	1	−1	−1	1	1	1	1	−1	−1	−1	1	−1	−1
χ_3	0	0	0	0	0	−1	1	−2	0	0	0	0	0	0
χ_4	0	1	2	2	0	0	−1	1	1	0	−1	−1	−1	−1
χ_5	0	1	−2	−2	0	0	−1	1	−1	0	1	−1	1	1
χ_6	0	0	0	0	−2	−1	0	2	0	0	0	1	0	0
χ_7	−1	0	3	−1	1	−1	0	0	0	1	0	0	0	0
χ_8	1	0	−3	1	1	−1	0	0	0	−1	0	0	0	0
χ_9	0	−1	−4	0	0	1	0	1	1	0	1	−1	−1	−1
χ_{10}	0	−1	4	0	0	1	0	1	−1	0	−1	−1	1	1
χ_{11}	0	0	−2	2	0	1	−1	−1	−1	0	0	1	1	1
χ_{12}	0	0	2	−2	0	1	−1	−1	1	0	0	1	−1	−1
χ_{13}	0	0	0	0	0	1	0	−2	0	0	0	−1	0	0
χ_{14}	1	−1	0	0	0	−1	0	0	0	1	−1	1	0	0
χ_{15}	−1	−1	0	0	0	−1	0	0	0	−1	1	1	0	0
χ_{16}	1	0	−1	−1	−1	0	0	−1	−1	1	0	0	−1	−1
χ_{17}	−1	0	1	1	−1	0	0	−1	1	−1	0	0	1	1
χ_{18}	1	1	3	−1	−1	0	0	0	0	−1	1	0	0	0
χ_{19}	−1	1	−3	1	−1	0	0	0	0	1	−1	0	0	0
χ_{20}	1	1	0	0	0	−1	0	0	0	−1	−1	0	0	0
χ_{21}	−1	1	0	0	0	−1	0	0	0	1	1	0	0	0
χ_{22}	0	−1	−2	−2	0	0	0	1	1	0	−1	0	1	1
χ_{23}	0	−1	2	2	0	0	0	1	−1	0	1	0	−1	−1
χ_{24}	0	0	0	0	0	0	1	0	0	0	0	−1	$-\sqrt{6}$	$\sqrt{6}$
χ_{25}	0	0	0	0	0	0	1	0	0	0	0	−1	$\sqrt{6}$	$-\sqrt{6}$
χ_{26}	0	0	0	0	2	−1	−1	0	0	0	0	0	0	0
χ_{27}	0	0	0	0	0	2	0	0	0	0	0	1	0	0

As subgroups of $G = J_2{:}2$ that generated by g_1 and g_2 given in Section 1, the two inertia factor groups $H_2 = 2^{2+4}{:}D_{12}$ and $H_3 = 2^2{:}S_5$ are generated as follows: $H_2 = \langle \alpha_1, \alpha_2 \rangle$ and $H_3 = \langle \beta_1, \beta_2 \rangle$, where

$$\alpha_1 = \begin{pmatrix} 0&1&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0 \end{pmatrix}, \ \alpha_2 = \begin{pmatrix} 1&0&0&0&0&1&1&1&1&0&0&1 \\ 1&0&1&1&0&1&1&1&0&1&1&1 \\ 0&0&1&1&1&1&0&0&0&0&1&1 \\ 0&0&0&1&1&0&0&1&0&1&1&1 \\ 1&1&1&0&0&0&0&0&0&1&0&0 \\ 0&1&1&1&1&0&1&1&0&1&0&0 \\ 0&0&1&1&0&0&0&1&1&0&1&1 \\ 1&1&0&1&1&0&1&0&0&0&1&1 \\ 0&0&1&1&0&1&0&0&1&0&1&1 \\ 1&1&1&1&1&0&1&1&1&0&1&1 \\ 1&1&0&0&0&0&1&1&0&1&1&0 \\ 0&1&0&1&1&1&0&0&1&0&0&1 \end{pmatrix},$$

$$\beta_1 = \begin{pmatrix} 0&1&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0 \end{pmatrix}, \ \beta_2 = \begin{pmatrix} 0&1&0&0&0&1&1&0&1&0&0&0 \\ 1&0&0&1&0&1&0&1&0&0&1&0 \\ 1&0&0&0&1&1&0&0&0&1&0&1 \\ 0&0&1&0&0&0&1&1&1&0&1&0 \\ 0&0&0&1&1&0&1&1&0&0&0&0 \\ 0&1&0&1&1&1&0&0&1&1&0&1 \\ 0&0&1&0&1&1&0&1&0&0&0&1 \\ 0&1&0&0&1&1&0&0&0&1&1&1 \\ 1&1&0&0&0&0&0&1&1&1&1&0 \\ 0&1&0&0&0&1&0&0&0&0&1&1 \\ 1&1&1&1&0&1&1&0&0&1&0&1 \\ 1&1&0&0&0&1&0&0&0&0&0&1 \end{pmatrix}.$$

Now we have used the Clifford-Fischer Theory recursively to construct the character tables of H_2 and H_3. For H_2, the action of D_{12} on the set $\mathrm{Irr}(2^{2+4})$ produced 5 orbits of lengths 1, 3, 3, 6 and 6 with corresponding inertia factor groups D_{12}, 2^2 (twice) and $\mathbb{Z}_2$ (twice). Also for H_3, the action of S_5 on the set $\mathrm{Irr}(2^2)$ produced 3 orbits of lengths 1, 1 and 2 with corresponding inertia factor groups S_5 (twice) and A_5. In this paper we list the full character tables of H_2 and H_3, but not in the format of Clifford-Fischer, rather we organized the columns of the character tables according to the orders and the sizes of the centralizers.

Recall that H_2 and H_3 are not maximal subgroups of $J_2{:}2$, but they are maximal of some maximal subgroups of $J_2{:}2$ (H_2 is maximal subgroup of $2^{2+4}{:}((3 \times S_3)^{\cdot}2)$ and H_3 is maximal subgroup of $(A_4 \times A_5){:}2$, where both $2^{2+4}{:}((3 \times S_3)^{\cdot}2)$ and $(A_4 \times A_5){:}2$ are maximal subgroups of $J_2{:}2$). We determined the fusions of the conjugacy classes of H_2 and H_3 into the classes $J_2{:}2$ using the permutation characters of $J_2{:}2$ on $2^{2+4}{:}((3 \times S_3)^{\cdot}2)$ and $(A_4 \times A_5){:}2$; the permutation characters of $2^{2+4}{:}((3 \times S_3)^{\cdot}2)$ and $(A_4 \times A_5){:}2$ on H_2 and H_3 respectively; together with the size of centralizers. We found the following proposition to be very useful in the process of determining the fusions.

Proposition 3.6. *Let $K_1 \leq K_2 \leq K_3$ and let ψ be a class function on K_1. Then $(\psi\uparrow_{K_1}^{K_2})\uparrow_{K_2}^{K_3} = \psi\uparrow_{K_1}^{K_3}$. More generally if $K_1 \leq K_2 \leq \cdots \leq K_n$ is a nested sequence of subgroups of K_n and ψ is a class function on K_1, then $(\psi\uparrow_{K_1}^{K_2})\uparrow_{K_2}^{K_3} \cdots \uparrow_{K_{n-1}}^{K_n} = \psi\uparrow_{K_1}^{K_n}$.*

Proof. See Proposition 3.5.6 of Basheer [2]. □

In Tables 6 and 7 we supply the full character tables of the inertia factor groups H_2 and H_3 together with the fusions of their conjugacy classes into the classes of $J_2{:}2$.

TABLE 6. The character table of $H_2 = 2^{2+4}{:}D_{12}$

	$1a$	$2a$	$2b$	$2c$	$2d$	$2e$	$3a$	$4a$	$4b$	$4c$	$4d$	$4e$	$6a$	$8a$	$8b$	$8c$	$8d$	$8e$
$\|C_{H_2}(g)\|$	768	256	48	32	32	32	6	64	32	32	32	8	6	32	32	16	16	8
$\hookrightarrow J_2{:}2$	$1A$	$2A$	$2B$	$2C$	$2B$	$2A$	$3B$	$4A$	$4A$	$4B$	$4A$	$4C$	$6C$	$8B$	$8A$	$8B$	$8C$	$8C$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	−1	1	−1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	1	1
χ_3	1	1	1	1	−1	−1	1	1	1	−1	−1	−1	1	1	1	−1	−1	−1
χ_4	1	1	−1	1	1	−1	1	1	1	1	−1	1	−1	−1	−1	1	−1	−1
χ_5	2	2	−2	2	0	0	−1	2	2	0	0	0	1	−2	−2	0	0	0
χ_6	2	2	2	2	0	0	−1	2	2	0	0	0	−1	2	2	0	0	0
χ_7	3	3	−3	−1	−1	1	0	3	−1	−1	1	1	0	1	1	−1	1	−1
χ_8	3	3	3	−1	−1	−1	0	3	−1	−1	−1	1	0	−1	−1	−1	−1	1
χ_9	3	3	−3	−1	1	−1	0	3	−1	1	−1	−1	0	1	1	1	−1	1
χ_{10}	3	3	3	−1	1	1	0	3	−1	1	1	−1	0	−1	−1	1	1	−1
χ_{11}	6	6	0	2	−2	0	0	−2	−2	−2	0	0	0	0	0	2	0	0
χ_{12}	6	6	0	2	2	0	0	−2	−2	2	0	0	0	0	0	−2	0	0
χ_{13}	6	6	0	−2	0	−2	0	−2	2	0	−2	0	0	0	0	0	2	0
χ_{14}	6	6	0	−2	0	2	0	−2	2	0	2	0	0	0	0	0	−2	0
χ_{15}	12	−4	0	0	−2	2	0	0	0	2	−2	0	0	−2	2	0	0	0
χ_{16}	12	−4	0	0	−2	−2	0	0	0	2	2	0	0	2	−2	0	0	0
χ_{17}	12	−4	0	0	2	−2	0	0	0	−2	2	0	0	−2	2	0	0	0
χ_{18}	12	−4	0	0	2	2	0	0	0	−2	−2	0	0	2	−2	0	0	0

TABLE 7. The character table of $H_3 = 2^2{:}S_5$

	$1a$	$2a$	$2b$	$2c$	$2d$	$2e$	$2f$	$3a$	$4a$	$4b$	$4c$	$5a$	$6a$	$6b$	$6c$	$10a$	$10b$	$10c$	$12a$
$\|C_{H_3}(g)\|$	480	480	240	32	32	24	16	24	24	8	8	20	24	12	12	20	20	20	12
$\hookrightarrow J_2{:}2$	$1A$	$2C$	$2C$	$2A$	$2C$	$2B$	$2C$	$3B$	$4C$	$4B$	$4C$	$5A$	$6B$	$6C$	$6B$	$10A$	$10A$	$10A$	$12C$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	−1	1	1	−1	−1	−1	1	1	−1	1	1	1	1	−1
χ_3	1	1	−1	1	1	−1	−1	1	1	−1	1	1	1	−1	−1	1	−1	−1	1
χ_4	1	1	−1	1	1	1	−1	1	−1	1	−1	1	1	1	−1	1	−1	−1	−1
χ_5	2	−2	0	2	−2	0	0	2	0	0	0	2	−2	0	0	−2	0	0	0
χ_6	4	4	4	0	0	2	0	1	2	0	0	−1	1	−1	1	−1	−1	−1	−1
χ_7	4	4	−4	0	0	2	0	1	−2	0	0	−1	1	−1	−1	−1	1	1	1
χ_8	4	4	4	0	0	−2	0	1	−2	0	0	−1	1	1	1	−1	−1	−1	1
χ_9	4	4	−4	0	0	−2	0	1	2	0	0	−1	1	1	−1	−1	1	1	−1
χ_{10}	5	5	−5	1	1	−1	−1	−1	1	1	−1	0	−1	−1	1	0	0	0	1
χ_{11}	5	5	5	1	1	−1	1	−1	−1	1	1	0	−1	−1	−1	0	0	0	−1
χ_{12}	5	5	5	1	1	1	1	−1	1	−1	−1	0	−1	1	−1	0	0	0	1
χ_{13}	5	5	−5	1	1	1	−1	−1	−1	−1	1	0	−1	1	1	0	0	0	−1
χ_{14}	6	6	6	−2	−2	0	−2	0	0	0	0	1	0	0	0	1	1	1	0
χ_{15}	6	6	−6	−2	−2	0	2	0	0	0	0	1	0	0	0	1	−1	−1	0
χ_{16}	6	−6	0	−2	2	0	0	0	0	0	0	1	0	0	0	−1	$-\sqrt{5}$	$\sqrt{5}$	0
χ_{17}	6	−6	0	−2	2	0	0	0	0	0	0	1	0	0	0	−1	$\sqrt{5}$	$-\sqrt{5}$	0
χ_{18}	8	−8	0	0	0	0	0	2	0	0	0	−2	−2	0	0	2	0	0	0
χ_{19}	10	−10	0	2	−2	0	0	−2	0	0	0	0	2	0	0	0	0	0	0

4. Fischer matrices of $\overline{G} = 2^{12}{:}(J_2{:}2)$

We now calculate the Fischer matrices of $\overline{G} = 2^{12}{:}(J_2{:}2)$. We recall from Section 3 of Basheer and Moori [3] that we label the top and bottom of the columns of the Fischer matrix $\mathcal{F}_i$, corresponding to g_i, by the sizes of the centralizers of $g_{ij},\ 1 \leq j \leq c(g_i)$, in $\overline{G}$ and m_{ij} respectively. Also the rows of $\mathcal{F}_i$ are partitioned into parts $\mathcal{F}_{ik},\ 1 \leq k \leq t$, corresponding to the inertia factors $H_1, H_2, \cdots, H_t$, where each $\mathcal{F}_{ik}$ consists of $c(g_{ik})$ rows correspond to the α_k^{-1}-regular classes (those are the H_k-classes that fuse to class $[g_i]_G$). Thus every row of $\mathcal{F}_i$ is labeled by the pair (k, m), where $1 \leq k \leq t$ and $1 \leq m \leq c(g_{ik})$. We have the values of $|C_{\overline{G}}(g_{ij})|$ and $m_{ij},\ 1 \leq i \leq 27,\ 1 \leq j \leq c(g_i)$ in Table 1. Also the fusions of the conjugacy classes of H_2 and H_3 into the conjugacy classes of G are given in Tables 6 and 7 respectively. Since we know that the size of the Fischer matrix $\mathcal{F}_i$ is $c(g_i)$, it follows from Table 1 that the sizes of the Fischer matrices of $\overline{G}$ range between 1 and 6 for every $i \in \{1, 2, \cdots, 27\}$.

The Fischer matrices satisfy interesting arithmetical properties (see Proposition 3.6 of [3]). We have used these properties to calculate some of the entries of these matrices and to build systems of algebraic equations. We solved these systems of equations using the symbolic mathematical package Maxima [18] and hence we have computed all the Fischer matrices of $\overline{G}$, which we list below.

$\mathcal{F}_1$

g_1		g_{11}	g_{12}	g_{13}
$o(g_{1j})$		1	2	2
$\lvert C_{\overline{G}}(g_{1j})\rvert$		4954521600	3145728	1966080
(k,m)	$\lvert C_{H_k}(g_{1km})\rvert$			
$(1,1)$	1209600	1	1	1
$(2,1)$	768	1575	39	-25
$(3,1)$	480	2520	-40	24
m_{1j}		1	1575	2520

$\mathcal{F}_2$

g_2		g_{21}	g_{22}	g_{23}	g_{24}
$o(g_{2j})$		2	2	4	4
$\lvert C_{\overline{G}}(g_{2j})\rvert$		983040	65536	8192	8192
(k,m)	$\lvert C_{H_k}(g_{2km})\rvert$				
$(1,1)$	3840	1	1	1	1
$(2,1)$	256	15	15	-1	-1
$(2,2)$	32	120	-8	-8	8
$(3,1)$	32	120	-8	8	-8
m_{2j}		16	240	1920	1920

$\mathcal{F}_3$

g_3		g_{31}	g_{32}	g_{33}	g_{34}
$o(g_{3j})$		2	4	4	4
$\lvert C_{\overline{G}}(g_{3j})\rvert$		43008	3072	2048	1536
(k,m)	$\lvert C_{H_k}(g_{3km})\rvert$				
$(1,1)$	672	1	1	1	1
$(2,1)$	48	14	6	-2	-2
$(2,2)$	32	21	-3	5	-3
$(3,1)$	24	28	-4	-4	4
m_{3j}		64	896	1344	1792

$\mathcal{F}_4$

g_4		g_{41}	g_{42}	g_{43}	g_{44}	g_{45}	g_{46}
$o(g_{4j})$		2	4	4	4	4	4
$\lvert C_{\overline{G}}(g_{4j})\rvert$		30720	30720	15360	2048	2048	1024
(k,m)	$\lvert C_{H_k}(g_{4km})\rvert$						
$(1,1)$	480	1	1	1	1	1	1
$(2,1)$	32	15	15	15	-1	-1	-1
$(3,1)$	480	1	1	-1	1	1	-1
$(3,2)$	240	2	-2	0	2	-2	0
$(3,3)$	32	15	15	-15	-1	-1	1
$(3,4)$	16	30	-30	0	-2	2	0
m_{4j}		64	64	128	960	960	1920

$\mathcal{F}_5$

g_5		g_{51}
$o(g_{5j})$		3
$\lvert C_{\overline{G}}(g_{5j})\rvert$		2160
(k,m)	$\lvert C_{H_k}(g_{4km})\rvert$	
$(1,1)$	2160	1
m_{4j}		4096

$\mathcal{F}_6$

g_6		g_{61}	g_{62}	g_{63}
$o(g_{6j})$		3	6	6
$\lvert C_{\overline{G}}(g_{6j})\rvert$		1152	384	96
(k,m)	$\lvert C_{H_k}(g_{6km})\rvert$			
$(1,1)$	72	1	1	1
$(2,1)$	6	12	-4	0
$(3,1)$	24	3	3	-1
m_{6j}		256	768	3072

$\mathcal{F}_7$

g_7		g_{71}	g_{72}	g_{73}	g_{74}
$o(g_{7j})$		4	4	4	4
$\lvert C_{\overline{G}}(g_{7j})\rvert$		3072	1024	512	512
(k,m)	$\lvert C_{H_k}(g_{7km})\rvert$				
$(1,1)$	192	1	1	1	1
$(2,1)$	64	3	3	-1	-1
$(2,2)$	32	6	-2	2	-2
$(3,1)$	32	6	-2	-2	2
m_{7j}		256	768	1536	1536

$\mathcal{F}_8$

g_8		g_{81}	g_{82}	g_{83}
$o(g_{8j})$		4	4	8
$\lvert C_{\overline{G}}(g_{8j})\rvert$		1536	512	128
(k,m)	$\lvert C_{H_k}(g_{8km})\rvert$			
$(1,1)$	96	1	1	1
$(2,1)$	32	3	3	-1
$(3,1)$	8	12	-4	0
m_{8j}		256	768	3072

$\mathcal{F}_9$

g_9		g_{91}	g_{92}	g_{93}	g_{94}
$o(g_{9j})$		4	8	8	8
$\lvert C_{\overline{G}}(g_{9j})\rvert$		192	192	64	64
(k,m)	$\lvert C_{H_k}(g_{9km})\rvert$				
$(1,1)$	24	1	1	1	1
$(2,1)$	8	3	3	-1	-1
$(3,1)$	24	1	-1	-1	1
$(3,2)$	8	3	-3	1	-1
m_{9j}		512	512	1536	1536

$\mathcal{F}_{10}$

g_{10}		$g_{10,1}$	$g_{10,2}$
$o(g_{10j})$		5	10
$\lvert C_{\overline{G}}(g_{10j})\rvert$		4800	320
(k,m)	$\lvert C_{H_k}(g_{10km})\rvert$		
$(1,1)$	300	1	1
$(3,1)$	20	15	-1
m_{10j}		256	3840

$\mathcal{F}_{11}$

g_{11}		$g_{11,1}$
$o(g_{11j})$		5
$\lvert C_{\overline{G}}(g_{11j})\rvert$		50
(k,m)	$\lvert C_{H_k}(g_{11km})\rvert$	
$(1,1)$	50	1
m_{11j}		4096

$\mathcal{F}_{12}$

g_{12}		$g_{12,1}$
$o(g_{12j})$		6
$\lvert C_{\overline{G}}(g_{12j})\rvert$		48
(k,m)	$\lvert C_{H_k}(g_{12km})\rvert$	
$(1,1)$	48	1
m_{12j}		4096

$\mathcal{F}_{13}$

g_{13}		$g_{13,1}$	$g_{13,2}$	$g_{13,3}$
$o(g_{13j})$		6	12	12
$\lvert C_{\overline{G}}(g_{13j})\rvert$		96	96	48
(k,m)	$\lvert C_{H_k}(g_{13km})\rvert$			
$(1,1)$	24	1	1	1
$(2,1)$	24	1	1	-1
$(2,2)$	12	2	-2	0
m_{13j}		1024	1024	2048

$\mathcal{F}_{14}$

g_{14}		$g_{14,1}$	$g_{14,2}$	$g_{14,3}$
$o(g_{14j})$		6	12	12
$\lvert C_{\overline{G}}(g_{14j})\rvert$		48	48	24
(k,m)	$\lvert C_{H_k}(g_{14km})\rvert$			
$(1,1)$	12	1	1	1
$(2,1)$	6	2	-2	0
$(2,2)$	12	1	1	-1
m_{14j}		1024	1024	2048

$\mathcal{F}_{15}$

g_{15}		$g_{15,1}$
$o(g_{15j})$		7
$\lvert C_{\overline{G}}(g_{15j})\rvert$		14
(k,m)	$\lvert C_{H_k}(g_{15km})\rvert$	
$(1,1)$	14	1
m_{15j}		4096

$\mathcal{F}_{16}$

g_{16}		$g_{16,1}$	$g_{16,2}$
$o(g_{16j})$		8	8
$\lvert C_{\overline{G}}(g_{16j})\rvert$		384	128
(k,m)	$\lvert C_{H_k}(g_{16km})\rvert$		
$(1,1)$	96	1	1
$(2,1)$	32	3	-1
m_{16j}		1024	3072

$\mathcal{F}_{17}$

g_{17}		$g_{17,1}$	$g_{17,2}$	$g_{17,3}$
$o(g_{17j})$		8	8	8
$\lvert C_{\overline{G}}(g_{17j})\rvert$		128	128	64
(k,m)	$\lvert C_{H_k}(g_{17km})\rvert$			
$(1,1)$	32	1	1	1
$(2,1)$	32	1	1	-1
$(2,2)$	16	2	-2	0
m_{17j}		1024	1024	2048

$\mathcal{F}_{18}$

g_{18}		$g_{18,1}$	$g_{18,2}$	$g_{18,3}$
$o(g_{18j})$		8	8	8
$\lvert C_{\overline{G}}(g_{18j})\rvert$		64	64	32
(k,m)	$\lvert C_{H_k}(g_{18km})\rvert$			
$(1,1)$	16	1	1	1
$(2,1)$	16	1	1	-1
$(2,2)$	8	2	-2	0
m_{18j}		1024	1024	2048

$\mathcal{F}_{19}$

g_{19}		$g_{19,1}$	$g_{19,2}$	$g_{19,3}$	$g_{19,4}$
$o(g_{19j})$		10	20	20	20
$\lvert C_{\overline{G}}(g_{19j})\rvert$		80	80	80	80
(k,m)	$\lvert C_{H_k}(g_{19km})\rvert$				
$(1,1)$	20	1	1	1	1
$(3,1)$	20	1	1	-1	-1
$(3,2)$	20	1	-1	1	-1
$(3,3)$	20	1	-1	-1	1
m_{19j}		1024	1024	1024	1024

$\mathcal{F}_{20}$

g_{20}		$g_{20,1}$
$o(g_{20j})$		10
$\lvert C_{\overline{G}}(g_{20j})\rvert$		10
(k,m)	$\lvert C_{H_k}(g_{20km})\rvert$	
$(1,1)$	10	1
m_{20j}		4096

$\mathcal{F}_{21}$

g_{21}		$g_{21,1}$
$o(g_{21j})$		12
$\lvert C_{\overline{G}}(g_{21j})\rvert$		24
(k,m)	$\lvert C_{H_k}(g_{21km})\rvert$	
$(1,1)$	24	1
m_{21j}		4096

$\mathcal{F}_{22}$

g_{22}		$g_{22,1}$
$o(g_{22j})$		12
$\lvert C_{\overline{G}}(g_{22j})\rvert$		12
(k,m)	$\lvert C_{H_k}(g_{22km})\rvert$	
$(1,1)$	12	1
m_{22j}		4096

$\mathcal{F}_{23}$

g_{23}		$g_{23,1}$	$g_{23,2}$
$o(g_{23j})$		12	24
$\lvert C_{\overline{G}}(g_{23j})\rvert$		24	24
(k,m)	$\lvert C_{H_k}(g_{23km})\rvert$		
$(1,1)$	12	1	1
$(3,1)$	12	1	-1
m_{23j}		2048	2048

$\mathcal{F}_{24}$

g_{24}		$g_{24,1}$
$o(g_{24j})$		14
$\|C_{\overline{G}}(g_{24j})\|$		14
(k, m)	$\|C_{H_k}(g_{24km})\|$	
$(1, 1)$	14	1
m_{24j}		4096

$\mathcal{F}_{25}$

g_{25}		$g_{25,1}$
$o(g_{25j})$		15
$\|C_{\overline{G}}(g_{24j})\|$		15
(k, m)	$\|C_{H_k}(g_{25km})\|$	
$(1, 1)$	15	1
m_{25j}		4096

$\mathcal{F}_{26}$

g_{26}		$g_{26,1}$
$o(g_{26j})$		24
$\|C_{\overline{G}}(g_{26j})\|$		24
(k, m)	$\|C_{H_k}(g_{26km})\|$	
$(1, 1)$	24	1
m_{26j}		4096

$\mathcal{F}_{27}$

g_{27}		$g_{27,1}$
$o(g_{27j})$		24
$\|C_{\overline{G}}(g_{27j})\|$		24
(k, m)	$\|C_{H_k}(g_{27km})\|$	
$(1, 1)$	24	1
m_{27j}		4096

5. Character table of $\overline{G} = 2^{12}{:}(J_2{:}2)$

In Sections 2, 3 and 4, we have determined:

- The conjugacy classes of $\overline{G} = 2^{12}{:}(J_2{:}2)$ (Table 1),
- The inertia factor groups H_1, H_2 and H_3.
- The character tables of all the inertia factor groups of G (Tables 5, 6 and 7). In Tables 6 and 7 we also supplied the fusions of the conjugacy classes of H_2 and H_3 into the conjugacy classes of G.
- The Fischer matrices of $\overline{G}$ (Section 4).

Following [2, 3] it is with no difficulty the full character table of $\overline{G}$ can be constructed in the format of Clifford-Fischer theory. It will be composed of 81 parts corresponding to the 27 cosets and the three inertia factor groups. The full character table of $\overline{G}$ is 64×64 $\mathbb{R}$-valued matrix and we list it in the format of Clifford-Fischer Theory in Table 8. We conclude by remarking that the accuracy of this character table has been tested using GAP.

TABLE 8. The character table of $\overline{G} = 2^{12}$:(J_2:2)

$[g_i]_G$	1A			2A				2B				2C					
$[g_{ij}]_{\overline{G}}$	1a	2a	2b	2c	2d	4a	4b	2e	4c	4d	4e	2f	4f	4g	4h	4i	4j
$\lvert C_{\overline{G}}(g_{ij})\rvert$	4954521600	3145728	1966080	983040	65536	8192	8192	43008	3072	2048	1536	30720	30720	15360	2048	2048	1024
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1	1	−1	−1	−1	−1	1	1	1	1	1	1
χ_3	28	28	28	−4	−4	−4	−4	0	0	0	0	4	4	4	4	4	4
χ_4	36	36	36	4	4	4	4	6	6	6	6	0	0	0	0	0	0
χ_5	36	36	36	4	4	4	4	−6	−6	−6	−6	0	0	0	0	0	0
χ_6	42	42	42	10	10	10	10	0	0	0	0	−6	−6	−6	−6	−6	−6
χ_7	63	63	63	15	15	15	15	7	7	7	7	−1	−1	−1	−1	−1	−1
χ_8	63	63	63	15	15	15	15	−7	−7	−7	−7	−1	−1	−1	−1	−1	−1
χ_9	90	90	90	10	10	10	10	6	6	6	6	6	6	6	6	6	6
χ_{10}	90	90	90	10	10	10	10	−6	−6	−6	−6	6	6	6	6	6	6
χ_{11}	126	126	126	14	14	14	14	0	0	0	0	6	6	6	6	6	6
χ_{12}	126	126	126	14	14	14	14	0	0	0	0	6	6	6	6	6	6
χ_{13}	140	140	140	−20	−20	−20	−20	0	0	0	0	−4	−4	−4	−4	−4	−4
χ_{14}	160	160	160	0	0	0	0	8	8	8	8	4	4	4	4	4	4
χ_{15}	160	160	160	0	0	0	0	−8	−8	−8	−8	4	4	4	4	4	4
χ_{16}	175	175	175	15	15	15	15	7	7	7	7	−5	−5	−5	−5	−5	−5
χ_{17}	175	175	175	15	15	15	15	−7	−7	−7	−7	−5	−5	−5	−5	−5	−5
χ_{18}	225	225	225	−15	−15	−15	−15	1	1	1	1	5	5	5	5	5	5
χ_{19}	225	225	225	−15	−15	−15	−15	−1	−1	−1	−1	5	5	5	5	5	5
χ_{20}	288	288	288	0	0	0	0	8	8	8	8	4	4	4	4	4	4
χ_{21}	288	288	288	0	0	0	0	−8	−8	−8	−8	4	4	4	4	4	4
χ_{22}	300	300	300	−20	−20	−20	−20	6	6	6	6	0	0	0	0	0	0
χ_{23}	300	300	300	−20	−20	−20	−20	−6	−6	−6	−6	0	0	0	0	0	0
χ_{24}	336	336	336	16	16	16	16	0	0	0	0	0	0	0	0	0	0
χ_{25}	336	336	336	16	16	16	16	0	0	0	0	0	0	0	0	0	0
χ_{26}	378	378	378	−6	−6	−6	−6	0	0	0	0	−6	−6	−6	−6	−6	−6
χ_{27}	448	448	448	0	0	0	0	0	0	0	0	−8	−8	−8	−8	−8	−8
χ_{28}	1575	39	−25	135	7	−9	7	35	3	3	−5	15	15	15	−1	−1	−1
χ_{29}	1575	39	−25	135	7	−9	7	−35	−3	−3	5	15	15	15	−1	−1	−1
χ_{30}	1575	39	−25	−105	23	7	−9	7	−9	7	−1	15	15	15	−1	−1	−1
χ_{31}	1575	39	−25	−105	23	7	−9	−7	9	−7	1	15	15	15	−1	−1	−1
χ_{32}	3150	78	−50	30	30	−2	−2	28	12	−4	−4	30	30	30	−2	−2	−2
χ_{33}	3150	78	−50	30	30	−2	−2	−28	−12	4	4	30	30	30	−2	−2	−2
χ_{34}	4725	117	−75	165	37	−11	5	63	15	−1	−9	−15	−15	−15	1	1	1
χ_{35}	4725	117	−75	165	37	−11	5	−63	−15	1	9	−15	−15	−15	1	1	1
χ_{36}	4725	117	−75	−75	53	5	−11	21	21	−11	−3	−15	−15	−15	1	1	1
χ_{37}	4725	117	−75	−75	53	5	−11	−21	−21	11	3	−15	−15	−15	1	1	1
χ_{38}	9450	234	−150	90	90	−6	−6	42	−6	10	−6	30	30	30	−2	−2	−2
χ_{39}	9450	234	−150	90	90	−6	−6	−42	6	−10	6	30	30	30	−2	−2	−2
χ_{40}	9450	234	−150	330	74	−22	10	0	0	0	0	−30	−30	−30	2	2	2
χ_{41}	9450	234	−150	−150	106	10	−22	0	0	0	0	−30	−30	−30	2	2	2
χ_{42}	18900	468	−300	180	−76	−12	20	42	−6	10	−6	0	0	0	0	0	0
χ_{43}	18900	468	−300	180	−76	−12	20	−42	6	−10	6	0	0	0	0	0	0
χ_{44}	18900	468	−300	−300	−44	20	−12	42	−6	10	−6	0	0	0	0	0	0
χ_{45}	18900	468	−300	−300	−44	20	−12	−42	6	−10	6	0	0	0	0	0	0
χ_{46}	2520	−40	24	120	−8	8	−8	28	−4	−4	4	−16	48	−16	0	0	0
χ_{47}	2520	−40	24	120	−8	8	−8	28	−4	−4	4	48	−16	−16	0	0	0
χ_{48}	2520	−40	24	120	−8	8	−8	−28	4	4	−4	−16	48	−16	0	0	0
χ_{49}	2520	−40	24	120	−8	8	−8	−28	4	4	−4	48	−16	−16	0	0	0
χ_{50}	5040	−80	48	240	−16	16	−16	0	0	0	0	−32	−32	32	0	0	0
χ_{51}	10080	−160	96	0	0	0	0	56	−8	−8	8	−4	12	−4	−4	12	−4
χ_{52}	10080	−160	96	0	0	0	0	56	−8	−8	8	12	−4	−4	12	−4	−4
χ_{53}	10080	−160	96	0	0	0	0	−56	8	8	−8	−4	12	−4	−4	12	−4
χ_{54}	10080	−160	96	0	0	0	0	−56	8	8	−8	12	−4	−4	12	−4	−4
χ_{55}	12600	−200	120	120	−8	8	−8	−28	4	4	−4	−20	60	−20	−4	12	−4
χ_{56}	12600	−200	120	120	−8	8	−8	−28	4	4	−4	60	−20	−20	12	−4	−4
χ_{57}	12600	−200	120	120	−8	8	−8	28	−4	−4	4	−20	60	−20	−4	12	−4
χ_{58}	12600	−200	120	120	−8	8	−8	28	−4	−4	4	60	−20	−20	12	−4	−4
χ_{59}	15120	−240	144	−240	16	−16	16	0	0	0	0	−72	24	24	24	−8	−8
χ_{60}	15120	−240	144	−240	16	−16	16	0	0	0	0	24	−72	24	−8	24	−8
χ_{61}	15120	−240	144	−240	16	−16	16	0	0	0	0	24	24	−24	−8	−8	8
χ_{62}	15120	−240	144	−240	16	−16	16	0	0	0	0	24	24	−24	−8	−8	8
χ_{63}	20160	−320	192	0	0	0	0	0	0	0	0	−8	−8	8	−8	−8	8
χ_{64}	25200	−400	240	240	−16	16	−16	0	0	0	0	−40	−40	40	−8	−8	8

Continued on next page

Table 8 (continued)

$[g_i]_G$	$3A$	$3B$			$4A$				$4B$			$4C$				$5A$		$5B$	$6A$	$6B$		
$[g_{ij}]_{\overline{G}}$	$3a$	$3b$	$6a$	$6b$	$4k$	$4l$	$4m$	$4n$	$4o$	$4p$	$8a$	$4q$	$8b$	$8c$	$8d$	$5a$	$10a$	$5b$	$6c$	$6d$	$12a$	$12b$
$\lvert C_{\overline{G}}(g_{ij})\rvert$	2160	1152	384	96	3072	1024	512	512	1536	512	128	192	192	64	64	4800	320	50	48	96	96	48
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1	1
χ_3	10	−2	−2	−2	4	4	4	4	0	0	0	0	0	0	0	3	3	3	2	−2	−2	−2
χ_4	9	0	0	0	4	4	4	4	−2	−2	−2	0	0	0	0	−4	−4	1	1	0	0	0
χ_5	9	0	0	0	4	4	4	4	2	2	2	0	0	0	0	−4	−4	1	1	0	0	0
χ_6	6	0	0	0	2	2	2	2	0	0	0	0	0	0	0	7	7	2	−2	0	0	0
χ_7	0	3	3	3	3	3	3	3	3	3	3	−1	−1	−1	−1	3	3	−2	0	−1	−1	−1
χ_8	0	3	3	3	3	3	3	3	−3	−3	−3	1	1	1	1	3	3	−2	0	−1	−1	−1
χ_9	9	0	0	0	−2	−2	−2	−2	2	2	2	0	0	0	0	5	5	0	1	0	0	0
χ_{10}	9	0	0	0	−2	−2	−2	−2	−2	−2	−2	0	0	0	0	5	5	0	1	0	0	0
χ_{11}	−9	0	0	0	2	2	2	2	−4	−4	−4	0	0	0	0	1	1	1	−1	0	0	0
χ_{12}	−9	0	0	0	2	2	2	2	4	4	4	0	0	0	0	1	1	1	−1	0	0	0
χ_{13}	14	2	2	2	4	4	4	4	0	0	0	0	0	0	0	5	5	0	−2	2	2	2
χ_{14}	16	1	1	1	0	0	0	0	0	0	0	2	2	2	2	−5	−5	0	0	1	1	1
χ_{15}	16	1	1	1	0	0	0	0	0	0	0	−2	−2	−2	−2	−5	−5	0	0	1	1	1
χ_{16}	−5	1	1	1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	0	0	0	3	1	1	1
χ_{17}	−5	1	1	1	−1	−1	−1	−1	1	1	1	−1	−1	−1	−1	0	0	0	3	1	1	1
χ_{18}	0	3	3	3	−3	−3	−3	−3	−3	−3	−3	−1	−1	−1	−1	0	0	0	0	−1	−1	−1
χ_{19}	0	3	3	3	−3	−3	−3	−3	3	3	3	1	1	1	1	0	0	0	0	−1	−1	−1
χ_{20}	0	−3	−3	−3	0	0	0	0	0	0	0	−2	−2	−2	−2	3	3	−2	0	1	1	1
χ_{21}	0	−3	−3	−3	0	0	0	0	0	0	0	2	2	2	2	3	3	−2	0	1	1	1
χ_{22}	−15	0	0	0	4	4	4	4	−2	−2	−2	0	0	0	0	0	0	0	1	0	0	0
χ_{23}	−15	0	0	0	4	4	4	4	2	2	2	0	0	0	0	0	0	0	1	0	0	0
χ_{24}	−6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−4	−4	1	−2	0	0	0
χ_{25}	−6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−4	−4	1	−2	0	0	0
χ_{26}	0	0	0	0	−6	−6	−6	−6	0	0	0	0	0	0	0	3	3	3	0	0	0	0
χ_{27}	16	−2	−2	−2	0	0	0	0	0	0	0	0	0	0	0	−2	−2	−2	0	−2	−2	−2
χ_{28}	0	12	−4	0	15	−1	−1	−1	3	3	−1	3	3	−1	−1	0	0	0	0	0	0	0
χ_{29}	0	12	−4	0	15	−1	−1	−1	−3	−3	1	−3	−3	1	1	0	0	0	0	0	0	0
χ_{30}	0	12	−4	0	3	3	−5	3	3	3	−1	3	3	−1	−1	0	0	0	0	0	0	0
χ_{31}	0	12	−4	0	3	3	−5	3	−3	−3	1	−3	−3	1	1	0	0	0	0	0	0	0
χ_{32}	0	−12	4	0	18	2	−6	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{33}	0	−12	4	0	18	2	−6	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{34}	0	0	0	0	9	9	1	−7	3	3	−1	−3	−3	1	1	0	0	0	0	0	0	0
χ_{35}	0	0	0	0	9	9	1	−7	−3	−3	1	3	3	−1	−1	0	0	0	0	0	0	0
χ_{36}	0	0	0	0	−3	13	−3	−3	−3	−3	1	3	3	−1	−1	0	0	0	0	0	0	0
χ_{37}	0	0	0	0	−3	13	−3	−3	3	3	−1	−3	−3	1	1	0	0	0	0	0	0	0
χ_{38}	0	0	0	0	−18	−2	6	−2	6	6	−2	0	0	0	0	0	0	0	0	0	0	0
χ_{39}	0	0	0	0	−18	−2	6	−2	−6	−6	2	0	0	0	0	0	0	0	0	0	0	0
χ_{40}	0	0	0	0	18	−14	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{41}	0	0	0	0	−6	−6	−6	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{42}	0	0	0	0	−12	4	−4	4	−6	−6	2	0	0	0	0	0	0	0	0	0	0	0
χ_{43}	0	0	0	0	−12	4	−4	4	6	6	−2	0	0	0	0	0	0	0	0	0	0	0
χ_{44}	0	0	0	0	12	−4	4	−4	−6	−6	2	0	0	0	0	0	0	0	0	0	0	0
χ_{45}	0	0	0	0	12	−4	4	−4	6	6	−2	0	0	0	0	0	0	0	0	0	0	0
χ_{46}	0	3	3	−1	0	0	0	0	12	−4	0	−4	4	0	0	15	−1	0	0	−1	3	−1
χ_{47}	0	3	3	−1	0	0	0	0	12	−4	0	4	−4	0	0	15	−1	0	0	3	−1	−1
χ_{48}	0	3	3	−1	0	0	0	0	−12	4	0	4	−4	0	0	15	−1	0	0	−1	3	−1
χ_{49}	0	3	3	−1	0	0	0	0	−12	4	0	−4	4	0	0	15	−1	0	0	3	−1	−1
χ_{50}	0	6	6	−2	0	0	0	0	0	0	0	0	0	0	0	30	−2	0	0	−2	−2	2
χ_{51}	0	3	3	−1	0	0	0	0	0	0	0	−2	2	−2	2	−15	1	0	0	−1	3	−1
χ_{52}	0	3	3	−1	0	0	0	0	0	0	0	2	−2	2	−2	−15	1	0	0	3	−1	−1
χ_{53}	0	3	3	−1	0	0	0	0	0	0	0	2	−2	2	−2	−15	1	0	0	−1	3	−1
χ_{54}	0	3	3	−1	0	0	0	0	0	0	0	−2	2	−2	2	−15	1	0	0	3	−1	−1
χ_{55}	0	−3	−3	1	0	0	0	0	12	−4	0	−2	2	2	−2	0	0	0	0	1	−3	1
χ_{56}	0	−3	−3	1	0	0	0	0	12	−4	0	2	−2	−2	2	0	0	0	0	−3	1	1
χ_{57}	0	−3	−3	1	0	0	0	0	−12	4	0	2	−2	−2	2	0	0	0	0	1	−3	1
χ_{58}	0	−3	−3	1	0	0	0	0	−12	4	0	−2	2	2	−2	0	0	0	0	−3	1	1
χ_{59}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	15	−1	0	0	0	0	0
χ_{60}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	15	−1	0	0	0	0	0
χ_{61}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	15	−1	0	0	0	0	0
χ_{62}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	15	−1	0	0	0	0	0
χ_{63}	0	6	6	−2	0	0	0	0	0	0	0	0	0	0	0	−30	2	0	0	−2	−2	2
χ_{64}	0	−6	−6	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	−2

Continued on next page

Table 8 (continued)

$[g_i]_G$	$6C$			$7A$	$8A$		$8B$			$8C$			$10A$				$10B$	$12A$	$12B$	$12C$		$14A$	$15A$	$24A$	$24B$
$[g_{ij}]_{\overline{G}}$	$6e$	$12c$	$12d$	$7a$	$8e$	$8f$	$8g$	$8h$	$8i$	$8j$	$8k$	$8l$	$10b$	$20a$	$20b$	$20c$	$10c$	$12e$	$12f$	$12g$	$24c$	$14a$	$15a$	$24a$	$24b$
$\lvert C_{\overline{G}}(g_{ij})\rvert$	48	48	24	14	384	128	128	128	64	64	64	32	80	80	80	80	10	24	12	24	24	14	15	24	24
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	−1	−1	−1	1	−1	−1	−1	−1	−1	1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	1	−1	−1
χ_3	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	−1	−1	1	−2	0	0	0	0	0	0	0
χ_4	0	0	0	1	2	2	2	2	2	0	0	0	0	0	0	0	−1	1	1	0	0	−1	−1	−1	−1
χ_5	0	0	0	1	−2	−2	−2	−2	−2	0	0	0	0	0	0	0	−1	1	−1	0	0	1	−1	1	1
χ_6	0	0	0	0	0	0	0	0	0	−2	−2	−2	−1	−1	−1	−1	0	2	0	0	0	0	1	0	0
χ_7	1	1	1	0	−3	−3	1	1	1	1	1	1	−1	−1	−1	−1	0	0	0	−1	−1	0	0	0	0
χ_8	−1	−1	−1	0	3	3	−1	−1	−1	1	1	1	−1	−1	−1	−1	0	0	0	1	1	0	0	0	0
χ_9	0	0	0	−1	4	4	0	0	0	0	0	0	1	1	1	1	0	1	−1	0	0	−1	−1	1	1
χ_{10}	0	0	0	−1	−4	−4	0	0	0	0	0	0	1	1	1	1	0	1	1	0	0	1	−1	−1	−1
χ_{11}	0	0	0	0	−2	−2	2	2	2	0	0	0	1	1	1	1	−1	−1	−1	0	0	0	1	1	1
χ_{12}	0	0	0	0	2	2	−2	−2	−2	0	0	0	1	1	1	1	−1	−1	1	0	0	0	1	−1	−1
χ_{13}	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	−2	0	0	0	0	−1	0	0
χ_{14}	−1	−1	−1	−1	0	0	0	0	0	0	0	0	−1	−1	−1	−1	0	0	0	−1	−1	1	1	0	0
χ_{15}	1	1	1	−1	0	0	0	0	0	0	0	0	−1	−1	−1	−1	0	0	0	1	1	−1	1	0	0
χ_{16}	1	1	1	0	−1	−1	−1	−1	−1	−1	−1	−1	0	0	0	0	0	−1	−1	1	1	0	0	−1	−1
χ_{17}	−1	−1	−1	0	1	1	1	1	1	−1	−1	−1	0	0	0	0	0	−1	1	−1	−1	0	0	1	1
χ_{18}	1	1	1	1	3	3	−1	−1	−1	−1	−1	−1	0	0	0	0	0	0	0	−1	−1	1	0	0	0
χ_{19}	−1	−1	−1	1	−3	−3	1	1	1	−1	−1	−1	0	0	0	0	0	0	0	1	1	−1	0	0	0
χ_{20}	−1	−1	−1	1	0	0	0	0	0	0	0	0	−1	−1	−1	−1	0	0	0	1	1	1	0	0	0
χ_{21}	1	1	1	1	0	0	0	0	0	0	0	0	−1	−1	−1	−1	0	0	0	−1	−1	−1	0	0	0
χ_{22}	0	0	0	−1	−2	−2	−2	−2	−2	0	0	0	0	0	0	0	0	1	1	0	0	−1	0	1	1
χ_{23}	0	0	0	−1	2	2	2	2	2	0	0	0	0	0	0	0	0	1	−1	0	0	1	0	−1	−1
χ_{24}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	−1	$-\sqrt{6}$	$\sqrt{6}$
χ_{25}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	−1	$\sqrt{6}$	$-\sqrt{6}$
χ_{26}	0	0	0	0	0	0	0	0	0	2	2	2	−1	−1	−1	−1	−1	0	0	0	0	0	0	0	0
χ_{27}	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	0	0	0	0	0	0	1	0	0
χ_{28}	2	−2	0	0	3	−1	3	−1	−1	3	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{29}	−2	2	0	0	−3	1	−3	1	1	3	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{30}	−2	2	0	0	−3	1	1	−3	1	−3	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{31}	2	−2	0	0	3	−1	−1	3	−1	−3	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{32}	−2	2	0	0	6	−2	2	2	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{33}	2	−2	0	0	−6	2	−2	−2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{34}	0	0	0	0	−3	1	1	−3	1	−1	3	−1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{35}	0	0	0	0	3	−1	−1	3	−1	−1	3	−1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{36}	0	0	0	0	−3	1	−3	1	1	1	−3	1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{37}	0	0	0	0	3	−1	3	−1	−1	1	−3	1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{38}	0	0	0	0	0	0	−4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{39}	0	0	0	0	0	0	4	−4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{40}	0	0	0	0	0	0	0	0	0	−2	−2	2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{41}	0	0	0	0	0	0	0	0	0	2	2	−2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{42}	0	0	0	0	−6	2	2	2	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{43}	0	0	0	0	6	−2	−2	−2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{44}	0	0	0	0	6	−2	−2	−2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{45}	0	0	0	0	−6	2	2	2	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{46}	1	1	−1	0	0	0	0	0	0	0	0	0	−1	−1	−1	3	0	0	0	−1	1	0	0	0	0
χ_{47}	1	1	−1	0	0	0	0	0	0	0	0	0	3	−1	−1	−1	0	0	0	1	−1	0	0	0	0
χ_{48}	−1	−1	1	0	0	0	0	0	0	0	0	0	−1	−1	−1	3	0	0	0	1	−1	0	0	0	0
χ_{49}	−1	−1	1	0	0	0	0	0	0	0	0	0	3	−1	−1	−1	0	0	0	−1	1	0	0	0	0
χ_{50}	0	0	0	0	0	0	0	0	0	0	0	0	−2	2	2	−2	0	0	0	0	0	0	0	0	0
χ_{51}	−1	−1	1	0	0	0	0	0	0	0	0	0	1	1	1	−3	0	0	0	1	−1	0	0	0	0
χ_{52}	−1	−1	1	0	0	0	0	0	0	0	0	0	−3	1	1	1	0	0	0	−1	1	0	0	0	0
χ_{53}	1	1	−1	0	0	0	0	0	0	0	0	0	1	1	1	−3	0	0	0	−1	1	0	0	0	0
χ_{54}	1	1	−1	0	0	0	0	0	0	0	0	0	−3	1	1	1	0	0	0	1	−1	0	0	0	0
χ_{55}	−1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−1	0	0	0	0
χ_{56}	−1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	1	0	0	0	0
χ_{57}	1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	1	0	0	0	0
χ_{58}	1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−1	0	0	0	0
χ_{59}	0	0	0	0	0	0	0	0	0	0	0	0	3	−1	−1	−1	0	0	0	0	0	0	0	0	0
χ_{60}	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	−1	3	0	0	0	0	0	0	0	0	0
χ_{61}	0	0	0	0	0	0	0	0	0	0	0	0	−1	A	A^*	−1	0	0	0	0	0	0	0	0	0
χ_{62}	0	0	0	0	0	0	0	0	0	0	0	0	−1	A^*	A	−1	0	0	0	0	0	0	0	0	0
χ_{63}	0	0	0	0	0	0	0	0	0	0	0	0	2	−2	−2	2	0	0	0	0	0	0	0	0	0
χ_{64}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

where in Table 8, $A = 1 - 2\sqrt{5}$ and $A^* = 1 + 2\sqrt{5}$.

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