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# ANALYTICAL SOLUTION OF EQUATIONS GOVERNING ALIGNED PLANE ROTATING MAGNETOHYDRODYNAMIC FLUID THROUGH POROUS MEDIA BY MARTIN'S METHOD

B. K. Vishwakarma[1], Sayantan Sil[2], and Manoj Kumar [3]

**[1]Department of Physics, B B M K University, Dhanbad-828130, Jharkhand, India, biru12maths@gmail.com**
**[2]Department of Physics, P K Roy Memorial College, B B M K University, Dhanbad-828130, Jharkhand, India, sayan12350@gmail.com**
**[3]Department of Physics, Marwari College, Ranchi University, Ranchi-834001, Jharkhand, India, profmanoj@rediffmail.com**

**Abstract.** This investigation is an approach to setup an analytical solution of steady plane aligned magnetohydrodynamic (MHD) fluid flow having infinite electrical conductivity in a rotating frame through porous media by Martins method. The governing non-linear equations of the fluid flow are transformed into a new form called Martins form by employing differential geometry where the curvilinear co-ordinates ($\Phi$, $\Psi$) in the plane of flow shows that, the co-ordinate lines $\Psi$ are the streamlines of flow and the co-ordinate lines $\Phi$ are arbitrary constants. Exact solution is obtained and velocity, vorticity, current density magnetic field and pressure distribution are found out. Also, the diagrams have been plotted to sketch the streamline patterns and to study variation of pressure function with angular velocity.

*Key words and Phrases*: MHD, porous medium, exact solution, rotating frame, stream function.

## 1. INTRODUCTION

At present time rotating fluids considering, the problems of stretching surface, finds huge importance due to its numerous applications such as rotating machinery, gas turbine design, disk cleaners, food processing, rotor-stator system and product applications, providing design and modelling capability for diverse products such

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as jet engines, pumps as well as geophysical flow and many studies have been made on the rotating fluid and several surveys have been carried out on several types of flows. It is also important because of its appearance in many natural phenomena and its application divided into oceanography, meteorology, atmospheric science and limnology. Many researcher [1-10] have studied fluid flow in rotating frame in different flow problems. A large number of applications of fluid flow through a porous medium has applied in industries dealing with polymer processing, metallurgical processing, geographical and allied areas. A porous medium is a solid with pores in it. Pores are void spaces, which must be distributed more or less frequently throughout the material. When a fluid permeates through a porous medium, the actual path of an individual fluid particle cannot be found because of the fluid-rock boundary conditions, which must be considered. Thus, in a porous medium one generally considers the fluid motion in terms of volume or ensemble average of the motion of individual fluid elements over regions of space. This was usually done by famous Darcys law, as a result of this, the viscous term in the equations of fluid motion will be replaced by the resistance term $-\frac{\eta}{k}\vec{V}$, where $\eta$ is the viscosity of the fluid, $k$ is the permeability of the medium and $\vec{V}$ is the seepage velocity of the fluid. Many researchers [11-19] have studied fluid flows through porous medium in different flow problems. The basic equations and Navier-Stokes equations governing the flow of magnetohydrodynamic (MHD) fluid are non-linear partial differential equations and have no general solution. Only a small number of exact solutions have been found since the non-linear term do not vanish normally. The governing equations of motion of MHD fluid flow are second and third order partial differential equations which are non-linear in nature and hence are quite complex to solve. Various transformation techniques serve as the powerful analytical tools for solving non-linear partial differential equations.

Martin [20] employed differential geometry and developed new technique to convert equations into solvable form, by using the curvilinear coordinates $(\Phi, \Psi)$, in the plane of flow and keeping co-ordinate lines $\Psi$ = constant which is taken to be streamlines of flow and the co-ordinate lines $\Phi$= constant, are left arbitrary. Martins approach has been followed in this paper. The von-Mises co-ordinates $(x, \Psi)$ that require the use of $\Phi = x$ in Martins co-ordinates $(\Phi, \Psi)$ has been used in this investigation. Exact solutions by Martin's method were found in [21-37] for various fluid flow problems involving viscous, incompressible, steady, unsteady, rotating, non-rotating MHD and non-MHD fluids. In recent discoveries, MHD [38-43] received a lots of attention towards boundary value conditions.

In this investigation, we have found the exact solutions of steady plane aligned MHD flows of an incompressible rotating viscous fluid through porous media with infinite electrical conductivity using Martins method. There are number of published researched articles which deal with the importance of steady flow over a rotating frame through porous media under the influence of magnetic field. But further investigation in this regard is needed to clarify more, the effect of Martins method which is the main motivation of this paper. The aim of the present work is to analyze the new aspects of the studies mentioned above. This method definitely has a strong impact on the solutions obtained.

## 2. BASIC FLOW EQUATIONS

The general governing equations of the plane, viscous, incompressible fluid of infinite electrical conductivity through porous media in the presence of magnetic field is given by

$$\vec{\nabla}\cdot\vec{V}=0, \tag{1}$$

$$\rho[(\vec{V}\cdot\vec{\nabla})\vec{V}+2\vec{\Omega}\times\vec{V}+\vec{\Omega}\times(\vec{\Omega}\times\vec{r})]=-\vec{\nabla}p+\eta\nabla^2\vec{V}+\mu(\nabla\times\overrightarrow{H})\times\overrightarrow{H}-\frac{\eta}{k}\vec{V}, \tag{2}$$

$$\nabla\times(\overrightarrow{V}\times\overrightarrow{H})=0, \tag{3}$$

$$\nabla\cdot\overrightarrow{H}=0, \tag{4}$$

where $\overrightarrow{V}$ is velocity field vector, $\overrightarrow{H}$ is magnetic field vector, $p$ is dynamic pressure function, $\rho$ is the constant fluid field density, $\overrightarrow{\Omega}$ is angular velocity vector, $\overrightarrow{r}$ is radius vector, $\eta$ is coefficient of dynamic viscosity, $\mu$ is constant magnetic permeability and $k$ is the permeability of the medium. We consider two dimensional flow $\vec{V}=\vec{V}(x,y)$ and all variable are function of $x$ and $y$. Also we introduce vorticity function and Bernoulli function

$$\omega=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}, \qquad \text{(Vorticity function)} \tag{5}$$

$$Z=\frac{\partial H_2}{\partial x}-\frac{\partial H_1}{\partial y}, \qquad \text{(Current density function)} \tag{6}$$

$$B=\frac{1}{2}\rho V^2+p'+\frac{1}{2}\rho|\vec{\Omega}\times\vec{r}|^2, \qquad \text{(Bernoulli function)} \tag{7}$$

$V^2=u^2+v^2$, $\omega=$ vorticity, $Z=$ current density function and $p'$ is the reduced pressure function given by $p'=p-\frac{1}{2}\rho\left|\vec{\Omega}\times\overrightarrow{r}\right|^2$.

The last term being the centrifugal contribution of pressure, $u$, $v$ are the components of velocity vector $\overrightarrow{V}$, $H_1, H_2$ are components of magnetic field vector $\overrightarrow{H}$. Separating into components Equations (1) to (4) are replaced by the following equations;

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{8}$$

$$\frac{\partial B}{\partial x}+\eta\frac{\partial\omega}{\partial y}-2\rho\Omega v-\rho v\omega+\mu H_2 Z+\frac{\mu}{k}u=0, \tag{9}$$

$$\frac{\partial B}{\partial y}-\eta\frac{\partial\omega}{\partial x}+2\rho\Omega u+\rho u\omega-\mu H_1 Z+\frac{\mu}{k}v=0, \tag{10}$$

$$uH_2-vH_1=C, \tag{11}$$

$$\frac{\partial H_1}{\partial x}+\frac{\partial H_2}{\partial y}=0, \tag{12}$$

$$\frac{\partial H_2}{\partial x} - \frac{\partial H_1}{\partial y} = Z, \tag{13}$$

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}, \tag{14}$$

where $C$ is arbitrary constant of integration. We consider aligned flow in which magnetic field is everywhere parallel to velocity field, so that;

$$\overrightarrow{H} = \beta \overrightarrow{V} \qquad \Rightarrow \quad H_1 = \beta u, \qquad H_2 = \beta v, \tag{15}$$

where $\beta$ is some unknown scalar field such that

$$\overrightarrow{V} \cdot \nabla \beta = 0, \tag{16}$$

is the condition satisfied by $\beta$ as obtained from Equation (1), (4) and (15). Using (15) in the above system of equations (8) to (14) we get the following system of partial differential equations

$$\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = 0, \tag{17}$$

$$\frac{\partial B}{\partial x} + \eta \frac{\partial \omega}{\partial y} - 2\rho\Omega v - \rho v \omega + \mu \beta v Z + \frac{\mu}{k} u = 0, \tag{18}$$

$$\frac{\partial B}{\partial y} - \eta \frac{\partial \omega}{\partial x} + 2\rho\Omega u - \rho u \omega - \mu \beta u Z + \frac{\mu}{k} v = 0, \tag{19}$$

$$u \frac{\partial \beta}{\partial x} + v \frac{\partial \beta}{\partial y} = 0, \tag{20}$$

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}, \tag{21}$$

$$\beta \omega + v \frac{\partial \beta}{\partial x} - u \frac{\partial \beta}{\partial y} = Z, \tag{22}$$

above are six partial differential equations in six unknown functions $u(x, y), v(x, y)$, $\beta(x, y)$, $\omega(x, y)$, $Z(x, y)$ and $\beta(x, y)$. Once a solution of these equations are determined, pressure function $p(x, y)$ and the velocity vector $\overrightarrow{V}$ can be determined.

## 3. Some results of differential geometry

The equation of continuity of Equation (17) implies the existence of a stream function $\Psi = \Psi(x, y)$ such that;

$$\frac{\partial \Psi}{\partial x} = -v \quad \text{and} \quad \frac{\partial \Psi}{\partial y} = u. \tag{23}$$

We take $\Phi(x, y)=$ constant to be some arbitrary family of curves which generates with the streamline $\Psi(x, y) =$ constant, curvilinear coordinate so that in the physical plane the independent variables $x$, $y$ can be replaced by $\Phi$, $\Psi$.
Let

$$x = x(\Phi, \Psi), y = y(\Phi, \Psi), \tag{24}$$

define a curvilinear coordinate in the $(x, y)$ plane with the squared element of arc length along any curve given by

$$ds^2 = E(\Phi, \Psi)\, d\Phi^2 + 2F(\Phi, \Psi)\, d\Phi d\Psi + G(\Phi, \Psi)\, d\Psi^2, \tag{25}$$

where

$$\begin{aligned} E &= \left(\frac{\partial x}{\partial \Phi}\right)^2 + \left(\frac{\partial y}{\partial \Phi}\right)^2, \\ F &= \frac{\partial x}{\partial \Phi}\frac{\partial x}{\partial \Psi} + \frac{\partial y}{\partial \Phi}\frac{\partial y}{\partial \Psi}, \\ G &= \left(\frac{\partial x}{\partial \Psi}\right)^2 + \left(\frac{\partial y}{\partial \Psi}\right)^2. \end{aligned} \tag{26}$$

Equation (24) can be solved to obtain $\Phi$=$\Phi(x, y)$, $\Psi = \Psi(x, y)$ such that

$$\frac{\partial x}{\partial \Phi} = J\frac{\partial \Psi}{\partial y}, \qquad \frac{\partial x}{\partial \Psi} = -J\frac{\partial \Phi}{\partial y}, \qquad \frac{\partial y}{\partial \Phi} = -J\frac{\partial \Psi}{\partial x}, \qquad \frac{\partial y}{\partial \Psi} = J\frac{\partial \Phi}{\partial x}. \tag{27}$$

Provided $0 < |J| < \infty$, where $J$ is the Jacobian transformation

$$J = \frac{\partial x}{\partial \Phi}\frac{\partial y}{\partial \Psi} - \frac{\partial x}{\partial \Psi}\frac{\partial y}{\partial \Phi} = \pm\sqrt{EG - F^2} = \pm W. \tag{28}$$

If $\alpha$ be the local angle of inclination of the tangent to the coordinate line $\Psi$= constant, directed in the sense of increasing $\Phi$, we have from differential geometry

$$\begin{gathered} \frac{\partial x}{\partial \Phi} = \sqrt{E}\cos\alpha, \qquad \frac{\partial y}{\partial \Phi} = \sqrt{E}\sin\alpha, \qquad \frac{\partial x}{\partial \Psi} = \frac{F}{\sqrt{E}}\cos\alpha - \frac{J}{\sqrt{E}}\sin\alpha, \\ \frac{\partial y}{\partial \Psi} = \frac{F}{\sqrt{E}}\sin\alpha + \frac{J}{\sqrt{E}}\cos\alpha, \qquad \frac{\partial \alpha}{\partial \Phi} = \frac{J}{E}\Gamma_{11}, \qquad \frac{\partial \alpha}{\partial \Psi} = \frac{J}{E}\Gamma^2{}_{12} \end{gathered} \tag{29}$$

and

$$K = \frac{1}{W}\left[\frac{\partial}{\partial \Psi}\left(\frac{W}{E}\Gamma^2{}_{11}\right) - \frac{\partial}{\partial \Phi}\left(\frac{W}{E}\Gamma^2{}_{12}\right)\right] = 0,$$

where

$$\begin{aligned} \Gamma^2{}_{11} &= \frac{1}{2W^2}\left[-F\frac{\partial F}{\partial \Phi} + 2E\frac{\partial F}{\partial \Phi} - E\frac{\partial E}{\partial \Psi}\right], \\ \Gamma^2{}_{12} &= \frac{1}{2W^2}\left[E\frac{\partial G}{\partial \Phi} - F\frac{\partial E}{\partial \Psi}\right], \end{aligned}$$

here K is the Gaussian curvature.

## 4. MARTINS FORM OF FLOW EQUATIONS

Equation (23), (27) and (29) gives

$$\begin{aligned} \sqrt{E}\cos\alpha &= \frac{\partial x}{\partial \Phi} = J\frac{\partial \Psi}{\partial y} = Ju = JV\cos\theta, \\ \sqrt{E}\sin\alpha &= \frac{\partial y}{\partial \Phi} = -J\frac{\partial \Psi}{\partial x} = Jv = JV\sin\theta, \end{aligned}$$

where $V = \sqrt{u^2 + v^2}$ and $\theta$ is the direction of flow in the physical plane. This pair of equations shows that the fluid flows along the streamlines towards higher or lower parameter values of $\Phi$ according as to whether J>0 or J<0. We consider here fluid flows towards the higher parameter values of $\Phi$ so that J= $W > 0$.

Using (23) in Equations (18) and (19) and considering (27), the equations in ($\Phi$, $\Psi$) co-ordinate are given by

$$\frac{\partial B}{\partial \Phi}\frac{\partial y}{\partial \Psi} - \frac{\partial B}{\partial \Psi}\frac{\partial y}{\partial \Phi} + \eta\left(-\frac{\partial \omega}{\partial \Phi}\frac{\partial x}{\partial \Psi} + \frac{\partial \omega}{\partial \Psi}\frac{\partial x}{\partial \Phi}\right) - \left[\rho\left(2\Omega + \omega\right) - \mu\beta Z\right]\frac{\partial y}{\partial \Phi} + \frac{\eta}{\kappa}\frac{\partial x}{\partial \Phi} = 0, \tag{30}$$

$$\frac{\partial B}{\partial \Phi}\frac{\partial x}{\partial \Psi} - \frac{\partial B}{\partial \Psi}\frac{\partial x}{\partial \Phi} + \eta\left(\frac{\partial \omega}{\partial \Phi}\frac{\partial y}{\partial \Psi} - \frac{\partial \omega}{\partial \Psi}\frac{\partial y}{\partial \Phi}\right) - \left[\rho\left(2\Omega + \omega\right) - \mu\beta Z\right]\frac{\partial x}{\partial \Phi} - \frac{\eta}{\kappa}\frac{\partial y}{\partial \Phi} = 0. \tag{31}$$

Multiplying (30) by $\frac{\partial x}{\partial \Phi}$ and (31) by $\frac{\partial y}{\partial \Phi}$ and subtracting we get

$$J\frac{\partial B}{\partial x} = \eta\left(E\frac{\partial \omega}{\partial \Psi} - J\frac{\partial \omega}{\partial \Phi} + \frac{1}{\kappa}E\right), \tag{32}$$

Again multiplying equation (30) by $\frac{\partial x}{\partial \Psi}$ and (31) by $\frac{\partial y}{\partial \Psi}$ and subtracting

$$J\frac{\partial B}{\partial \Psi} = \eta\left(G\frac{\partial \omega}{\partial \Phi} - E\frac{\partial \omega}{\partial \Psi} - \frac{1}{\kappa}F\right) - J\left[\rho\left(2\Omega + \omega\right) - \mu\beta Z\right]. \tag{33}$$

4.1. **Solenoidal equation.** Using (23) in the Equation (20) and transforming the resulting equations to ($\Phi$, $\Psi$) co-ordinate we get

$$\frac{\partial \Psi}{\partial y}\left[\frac{\partial B}{\partial \Phi}\frac{\partial \Phi}{\partial x} + \frac{\partial \beta}{\partial \Psi}\frac{\partial \Psi}{\partial x}\right] - \frac{\partial \Psi}{\partial x}\left[\frac{\partial \beta}{\partial \Phi}\frac{\partial \Phi}{\partial y} + \frac{\partial \beta}{\partial \Psi}\frac{\partial \Psi}{\partial y}\right] = 0, \tag{34}$$

which on simplification gives

$$\frac{\partial \beta}{\partial \Phi} = 0. \tag{35}$$

**Current density equation**

Employing (23) in Equation (22) we have

$$\beta\omega - \frac{\partial \Psi}{\partial x}\frac{\partial \beta}{\partial x} - \frac{\partial \Psi}{\partial y}\frac{\partial \beta}{\partial y} = Z, \tag{36}$$

using (27) and employing (35) and (26) we get

$$\beta\omega - \frac{E}{J^2}\frac{\partial \beta}{\partial \Psi} = Z. \tag{37}$$

4.2. **Equations of continuity and vorticity.** Martins [20] obtained the necessary and sufficient condition for the flow of a fluid along the coordinate lines $\Psi$ = constant of curvilinear co-ordinate system with $ds^2$ given by (25) to satisfy the principle of conservation of mass to be

$$WV = \sqrt{E}\,, \qquad u + iv = \frac{\sqrt{E}}{W}e^{i\alpha}. \tag{38}$$

He has also proven that the vorticity equation takes the form:

$$\omega = \frac{1}{W}\left[\frac{\partial}{\partial\Phi}\left(\frac{F}{W}\right) - \frac{\partial}{\partial\Psi}\left(\frac{E}{W}\right)\right]. \tag{39}$$

## 5. Exact solution

We consider the given flow problem in the form $(y - f(x))/g(x) =$ constant, where $f(x)$ and $g(x) \neq 0$ are continuously differentiable functions. For a given flow problem with $(y - f(x))/g(x) =$ constant as the family of streamlines, we have

$$y = f(x) + g\left(x\right)\gamma\left(\Psi\right), \tag{40}$$

where $\gamma\left(\Psi\right)$ is an unknown functions satisfying $\gamma^{'}\left(\Psi\right) = 0$. Employing von Mises co-ordinates $\Phi(x, y) = x$ and Equation (40) in Equation (26) and (28) we get

$$\begin{aligned} E &= 1 + \left[f^{'}(x) + g^{'}(x)\gamma(\Psi)\right]^2, \\ F &= \left[g\left(x\right)f^{'}\left(x\right) + g\left(x\right)\gamma\left(\Psi\right)\right]\gamma^{'}\left(\Psi\right), \\ G &= g^2\left(x\right)\gamma^{'2}\left(x\right), \\ J &= W = g\left(x\right)\gamma^{'}\left(\Psi\right). \end{aligned} \tag{41}$$

Thus, we obtain parabolic flows along $y - m_1x^2 - m_2x =$ constant.
Since the family of parabolic curves are the streamlines, it follows that, there exists some functions $\gamma\left(\Psi\right)$ such that

$$y = m_1x^2 + m_2x + \gamma\left(\Psi\right), \quad \gamma^{'}\left(\Psi\right) \neq 0. \tag{42}$$

Comparing (42) with (40), we have
$f\left(x\right) = m_1x^2 + m_2x, \quad g\left(x\right) = 1$. Using these expressions for $f(x)$, $g(x)$ in (41) we get

$$\begin{aligned} E &= 1 + \left(2m_1x + m_2\right)^2, \ F = \left(2m_1x + m_2\right)\gamma^{'}\left(\Psi\right) \\ G &= \gamma^{'2}\left(\Psi\right), \ J = W = \gamma^{'}\left(\Psi\right). \end{aligned} \tag{43}$$

Now, we write the Equations (32), (33), (37) and (39) in von Mises co-ordinates $(\gamma, \Psi)$ we have,

$$\begin{aligned} \frac{\partial B}{\partial x} = \eta\Big[&(1 + (2m_1x + m_2)^2)\Big(-2m_1\frac{\gamma^{''}\Big(\Psi\Big)}{\gamma^{'3}\Big(\Psi\Big)} + \frac{1}{\gamma^{'}\Big(\Psi\Big)}\Big[\frac{\gamma^{''}\Big(\Psi\Big)}{\gamma^{'3}\Big(\Psi\Big)}\Big]^{'} \\ &+(2m_1x + m_2)\frac{1}{\gamma^{'}\Big(\Psi\Big)}\Big[\frac{\gamma^{''}\Big(\Psi\Big)}{\gamma^{'3}\Big(\Psi\Big)}\Big]^{'}\Big) \\ &-4m_1(2m_1x + m_2)\frac{\gamma^{''}\left(\Psi\right)}{\gamma^{'3}\left(\Psi\right)} + \frac{1}{k}(1 + (2m_1x + m_2)^2)\frac{1}{\gamma^{'}\left(\Psi\right)}\Big], \end{aligned} \tag{44}$$

$$\frac{\partial B}{\partial \Psi} = \eta\Big(2m_1(2m_1x+m_2)\frac{\gamma^{''}(\Psi)}{\gamma^{'3}(\Psi)} - (2m_1x+m_2)\Big[\frac{\gamma^{''}(\Psi)}{\gamma^{'3}(\Psi)}\Big]^{'} - (2m_1x+m_2)^3\frac{\gamma^{''}(\Psi)}{\gamma^{'3}(\Psi)}$$
$$-\frac{1}{k}(2m_1x+m_2)\Big) - \Big[\rho\Big(2\Omega + \frac{1}{\gamma^{'3}(\Psi)}\Big\{2m_1\gamma^{'2}(\Psi) + \gamma^{''}(\Psi)$$
$$+(2m_1x+m_2)^2\gamma^{''}(\Psi)\Big\}\Big) - \mu\beta\Big\{\ beta\Big(\frac{1}{\gamma^{'3}(\Psi)}\Big\{2m_1\gamma^{'2}(\Psi) + \gamma^{''}(\Psi)$$
$$+(2m_1x+m_2)^2\gamma^{''}(\Psi)\Big\}\Big) - \ Big(1+(2m_1x+m_2)^2\Big)\frac{\gamma^{''}(\Psi)}{\gamma^{'3}(\Psi)}\Big\}\Big], \tag{45}$$

$$Z = \beta(\Psi)\,\omega - \left[\frac{1+(2m_1x+m_2)^2}{\gamma^{'2}(\Psi)}\right]\beta^{'}(\Psi), \tag{46}$$

and

$$\omega = \frac{1}{\gamma^{'3}(\Psi)}\left[2m_1\gamma^{'2}(\Psi) + \gamma^{''}(\Psi) + (2m_1x+m_2)^2\gamma^{''}(\Psi)\right]. \tag{47}$$

Now, eliminating $\omega$ and $Z$ from the integrability condition $\frac{\partial^2 B}{\partial x \partial \Psi} = \frac{\partial^2 B}{\partial \Psi \partial x}$ using (44) to (47) we have

$$\sum_{n=0}^{4} a_n(\Psi)\,[2m_1x+m_2]^n = 0, \tag{48}$$

where

$$a_0(\Psi) = a_4(\Psi) + 2\eta m_1\left[\frac{\gamma^{''}(\Psi)}{\gamma^{'3}(\Psi)}\right]^{'} + \frac{1}{k}\frac{1}{\gamma^{'2}(\Psi)} - \frac{2m_1\eta}{k},$$

$$a_1(\Psi) = 4m_1\left[\mu\beta^2\frac{\gamma^{''}(\Psi)}{\gamma^{'3}(\Psi)} - \mu\beta\beta^{'}\frac{1}{\gamma^{'2}(\Psi)} - \rho\frac{\gamma^{''}(\Psi)}{\gamma^{'3}(\Psi)}\right]$$

$$a_2(\Psi) = -2\eta a_4(\Psi) + 2\eta m_1\left[\frac{\gamma^{''}(\Psi)}{\gamma^{'3}(\Psi)}\right]^{'}, a_3(\Psi) = 0,$$

and

$$a_4(\Psi) = \eta\left[\frac{1}{\gamma^{'}(\Psi)}\left(\frac{\gamma^{''}(\Psi)}{\gamma^{'3}(\Psi)}\right)^{'}\right]^{'}.$$

Since x and $\Psi$ are independent variables the identity (48) can only hold if $a_0(\Psi)$, $a_1(\Psi)$, $a_2(\Psi)$, $a_3(\Psi)$, and $a_4(\Psi)$, vanish identically. Using the consequences $a_4(\Psi)=0$, $a_2(\Psi)=0$ in $a_0(\Psi)=0$, $a_1(\Psi)=0$, we find

$$\gamma(\Psi) = c_1\Psi + c_2 \ and \ \beta(\Psi) = \beta_0 \tag{49}$$

where $c_1 \neq 0$ and $\beta_0 \neq 0$ are arbitrary constants. From (49) and (42), we get

$$c_1\Psi(x,y) + c_2 = y - m_1x^2 - m_2x. \tag{50}$$

**Streamline Profile**

The family of streamlines flow of the given problem can be shown in the Figure 1 which shows that streamline of flow equation are concentric parabola.

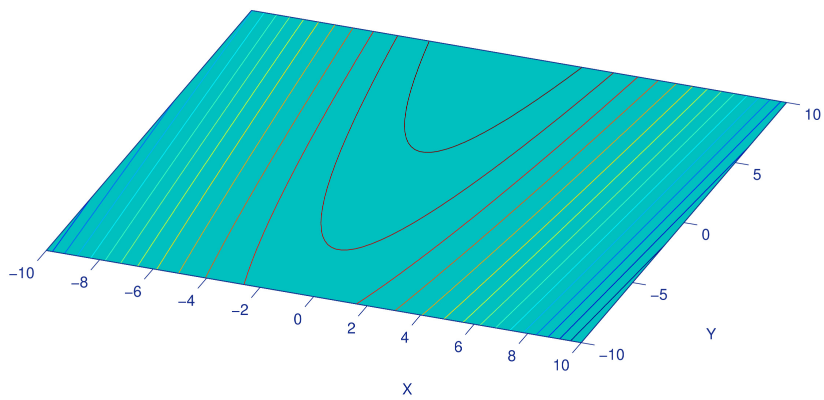


FIGURE 1. Streamline for the stream function $y - x^2 - x = Constant$

**Velocity Profile**

Velocity profile of the given problem is as shown in Figure 2 which shows parallel straight lines.

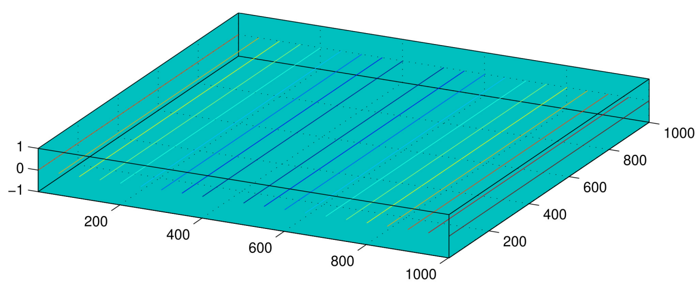


FIGURE 2. Velocity Profile

For the chosen parabolic flow pattern, using (49), and (50) in (23), (15), (5), (6), (7) and (17) to (22) we find

$$u = \frac{1}{c_1}, \tag{51}$$

$$v = \left(\frac{2m_1x + m_2}{c_1}\right), \tag{52}$$

$$H_1 = \frac{\beta_0}{c_1^2}, \tag{53}$$

$$H_2 = \beta_0\left(\frac{2m_1x + m_2}{c_1}\right), \tag{54}$$

$$\begin{aligned} p &= \frac{2m_1}{c_1^2}\left[\mu\beta_0^2 - \rho\left(1 + \frac{c_1\Omega}{m_1}\right)\right]y - \frac{\rho}{2c_1^2}\left(1 + m_2^2\right) \\ &- 2\left[\left(\frac{m_1\mu\beta_0^2}{c_1^2}\right) - \left(\frac{\rho\Omega}{c_1}\right)\right]\left(m_1x^2 + m_2x\right) - \frac{\eta}{k}(x + y) + p_0, \end{aligned} \tag{55}$$

$$\omega = \frac{2m_1}{c_1}, \tag{56}$$

$$Z = \frac{2m_1}{c_1}\beta_0, \tag{57}$$

where $p_0$ is arbitrary constant.

## 6. RESULT AND DISCUSSION

In this present investigation, we have considered steady plane aligned MHD fluid flow having infinite electrical conductivity in a rotating frame through porous media. By applying the concept of differential geometry the governing non-linear partial differential equations of the fluid flow are converted into a new form called Martins form and then an approach for the determination of exact solution has been carried out. The expressions for velocity, magnetic field, vorticity, current density and pressure distribution are found out. Also, graphs have been plotted to sketch the streamline pattern, velocity profile and variation of pressure function with the change in angular velocity for different values of $\frac{\eta}{k}$ and fluid density $\rho$. The main outcomes of the present investigation are listed below:

(1) The component of velocity $u$, component of magnetic field $H_1$, the vorticity function $\omega$ and the current density function $Z$ are found to be constants.
(2) The expression for velocity does not involve the term for permeability of the porous medium $k$ and the angular velocity $\Omega$ of the rotating frame.
(3) The pressure function depends on the magnetic permeability $\mu$, fluid density $\rho$, angular velocity $\Omega$ of the rotating frame and the porosity of the medium $k$.

(4) Pressure versus angular velocity $\Omega$ below 10 rad/sec (Figure 3 to Figure 6) for different values of $\frac{\eta}{k}$ with constant fluid density $\rho$ up to 3000 $kg/m^3$ shows that the pressure variation with angular velocity is almost constant for higher values of permeability of the porous medium $k$, but decreases considerably for lower values of permeability $k$.

(5) Pressure versus angular velocity (Figure 7 to Figure 11) for different values of $\frac{\eta}{k}$ for higher values of fluid density $\rho$ more than 3000 $kg/m^3$ shows that decrease of pressure to a minimum value and then considerable increase with increase in angular velocity $\Omega$. Increase of pressure function is quite sharp for higher values of permeability of the porous medium $k$.

(6) The pressure function versus angular velocity (Figure 12 to Figure 18) for different fluid density $\rho$ with constant value of $\frac{\eta}{k}$ (for higher values of $k$) show decrease of pressure function to a minimum value and then increase with increase in angular velocity $\Omega$. This is similar for other values of fluid density $\rho$.

(7) Pressure function versus angular velocity (Figure 19 to Figure 28) for different values of fluid density for constant $\frac{\eta}{k}$ (for lower values of $k$) shows decrease of pressure function almost linearly with angular velocity for relatively smaller angular velocity $\Omega$ irrespective of the value of fluid density. Beyond the value of $\frac{\eta}{k} = 20 \times 10^6 Pa.s/m^2$ (low value of permeability $k$), variation of pressure with smaller values of angular velocity $\Omega$ is almost same for any value of the fluid density and pressure function for different values of fluid density overlaps with each other which indicates that the linear decrease of pressure with smaller angular velocity $\Omega$ for very low values of permeability $k$ is independent of the fluid density. On increasing the angular velocity $\Omega$ there is decrease in pressure linearly up to a certain value beyond that decrease is non-linear to a minimum value and then there is increase in pressure with increasing angular velocity.

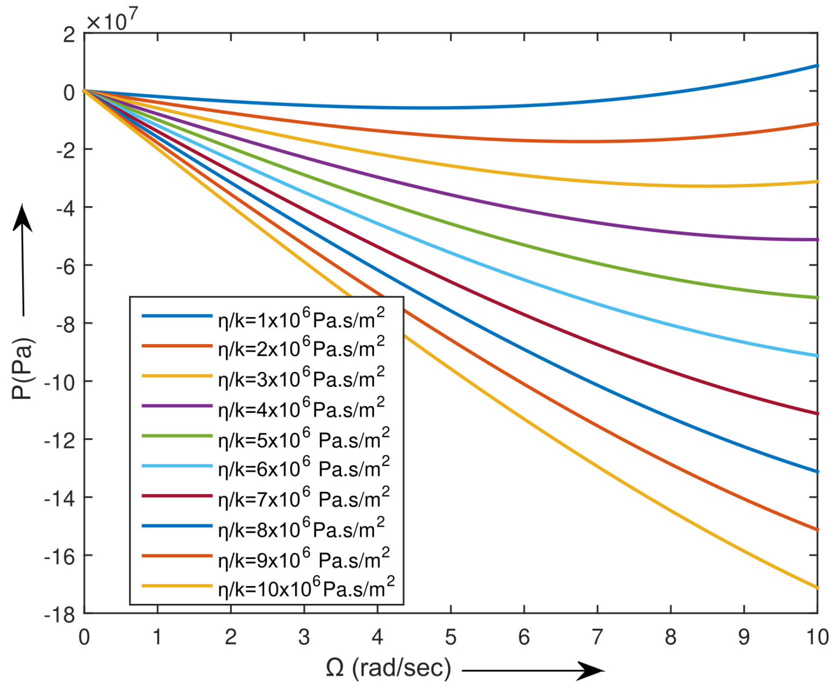


FIGURE 3. Pressure for density 1200 with variable $\frac{\eta}{k}$

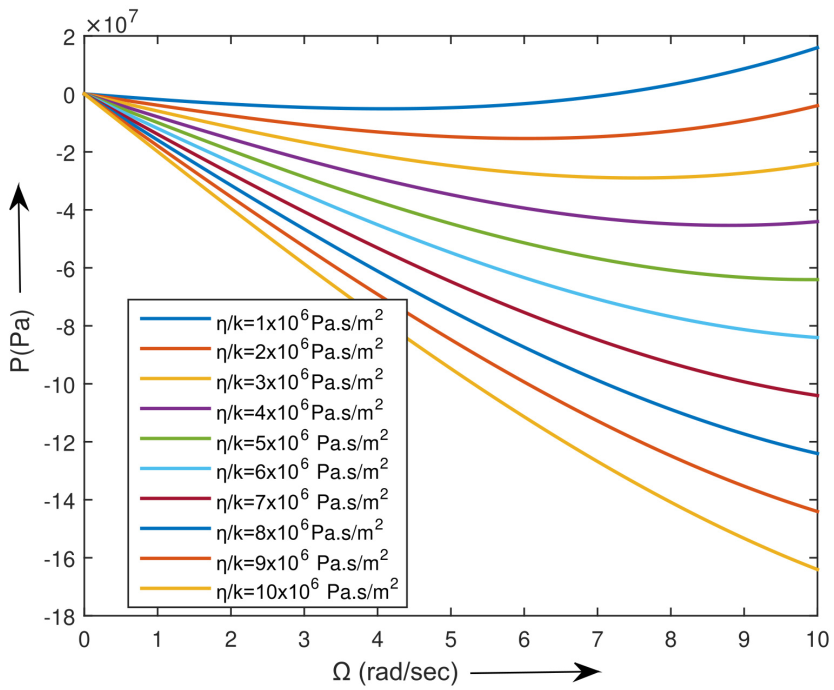


FIGURE 4. Pressure for density 1500 with variable $\frac{\eta}{k}$

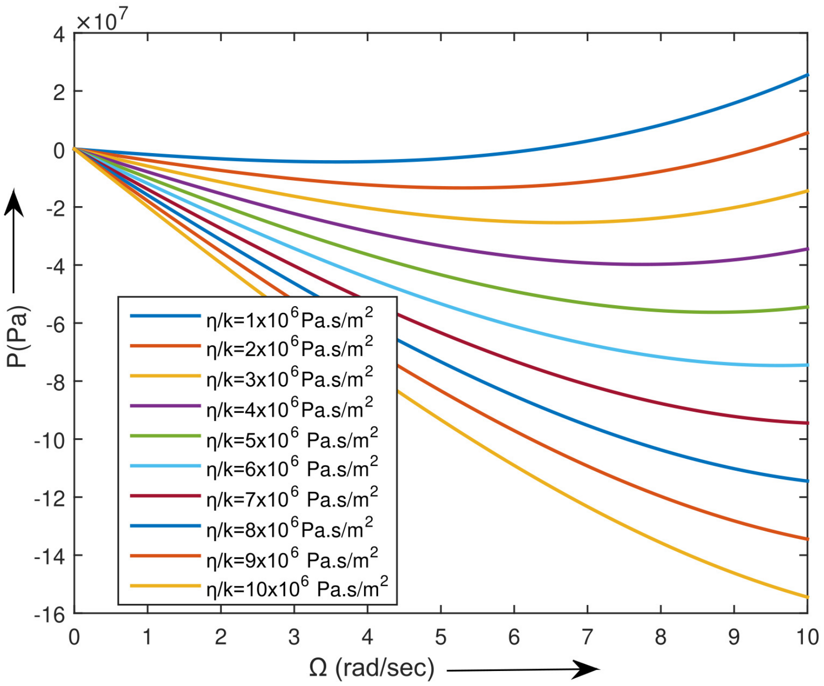


Figure 5. Pressure for density 1900 with variable $\frac{\eta}{k}$

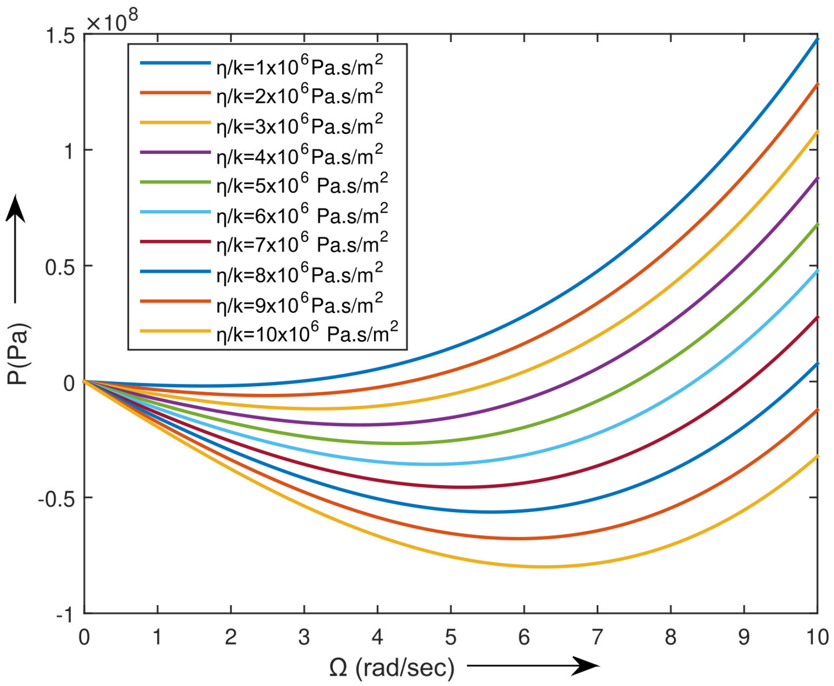


Figure 6. Pressure for density 7000 with variable $\frac{\eta}{k}$

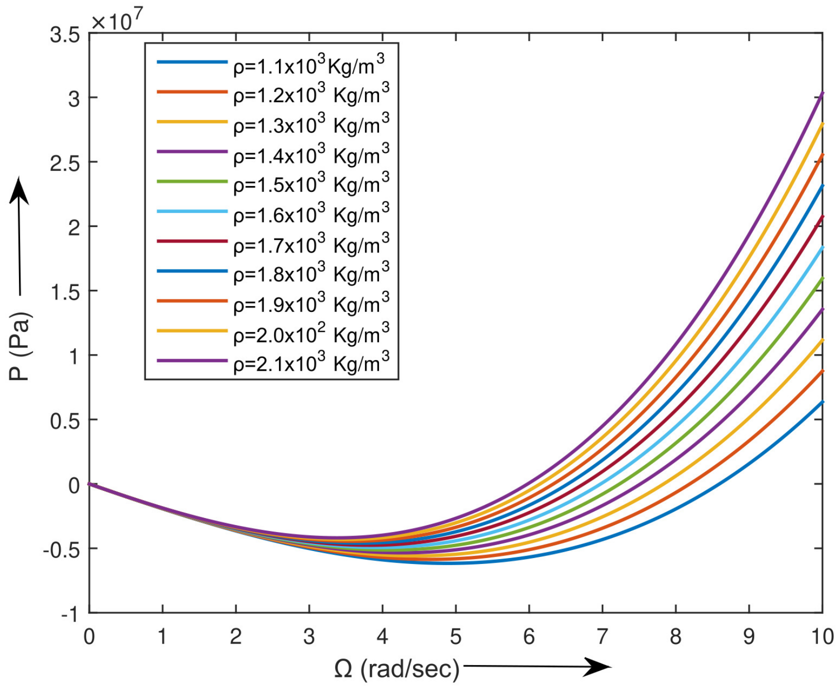


FIGURE 7. Pressure for variable density for $\frac{\eta}{k} = 1 \times 10^6$

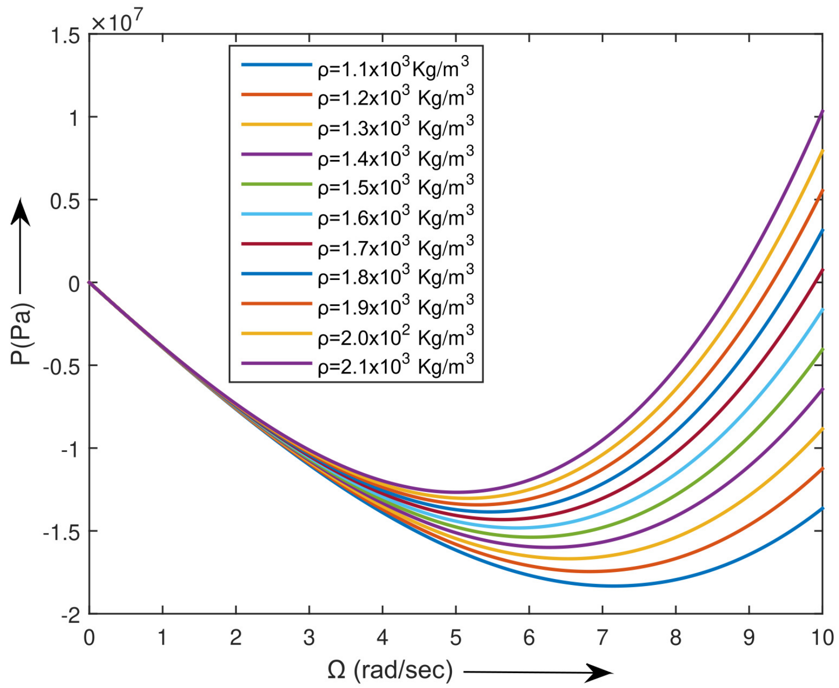


FIGURE 8. Pressure for variable density for $\frac{\eta}{k} = 2 \times 10^6$

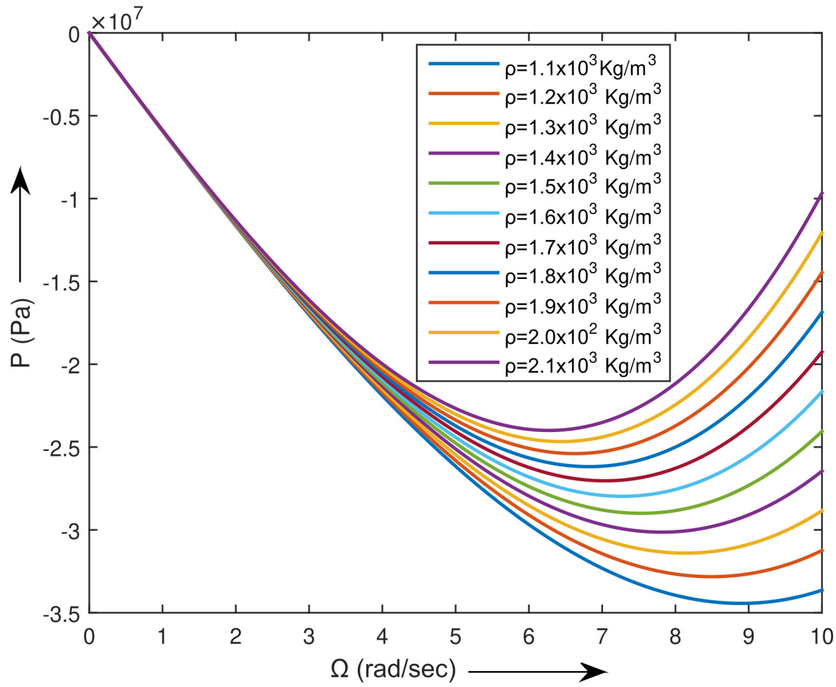


FIGURE 9. Pressure for variable density for $\frac{\eta}{k} = 3 \times 10^6$

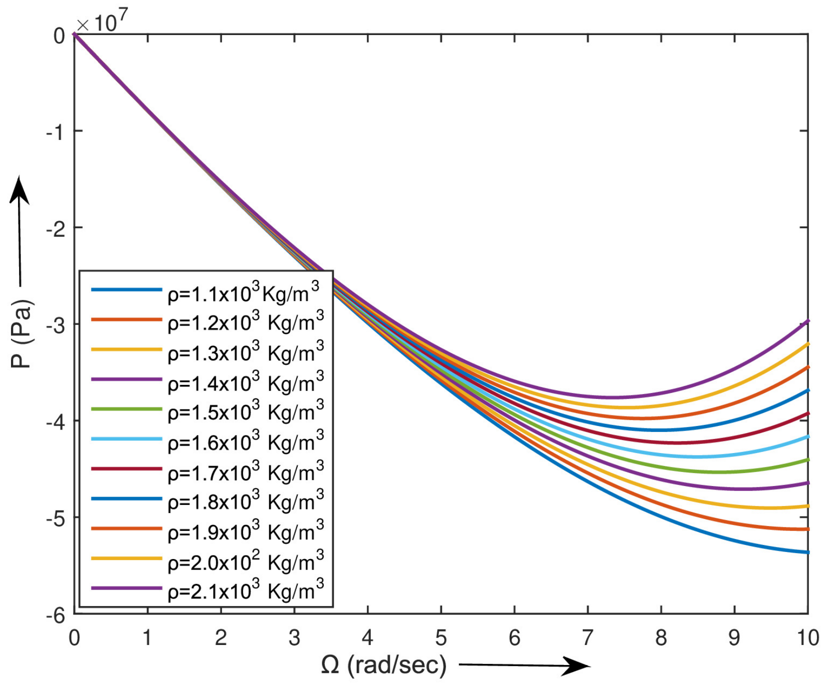


FIGURE 10. Pressure for variable density for $\frac{\eta}{k} = 4 \times 10^6$

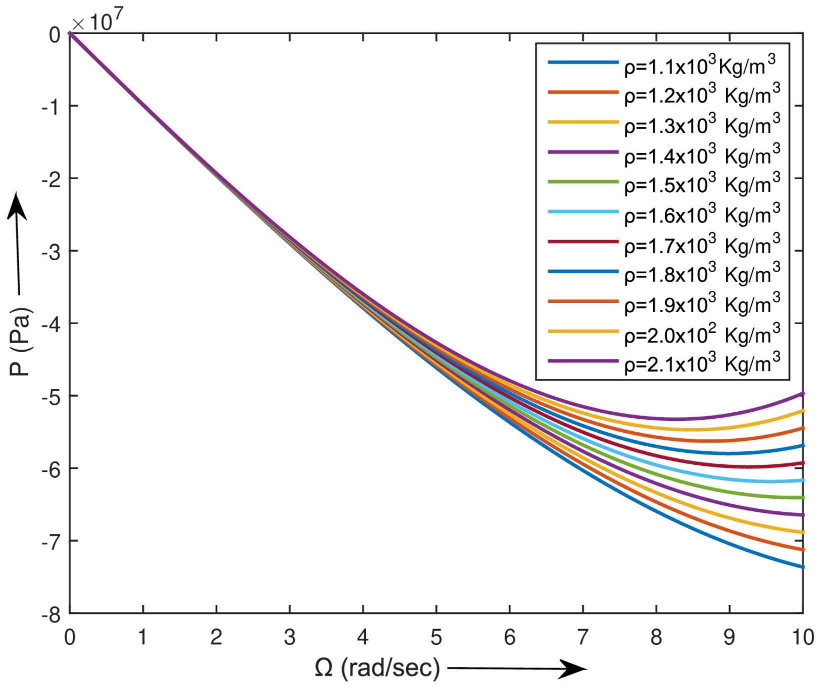


FIGURE 11. Pressure for variable density for $\frac{\eta}{k} = 5 \times 10^6$

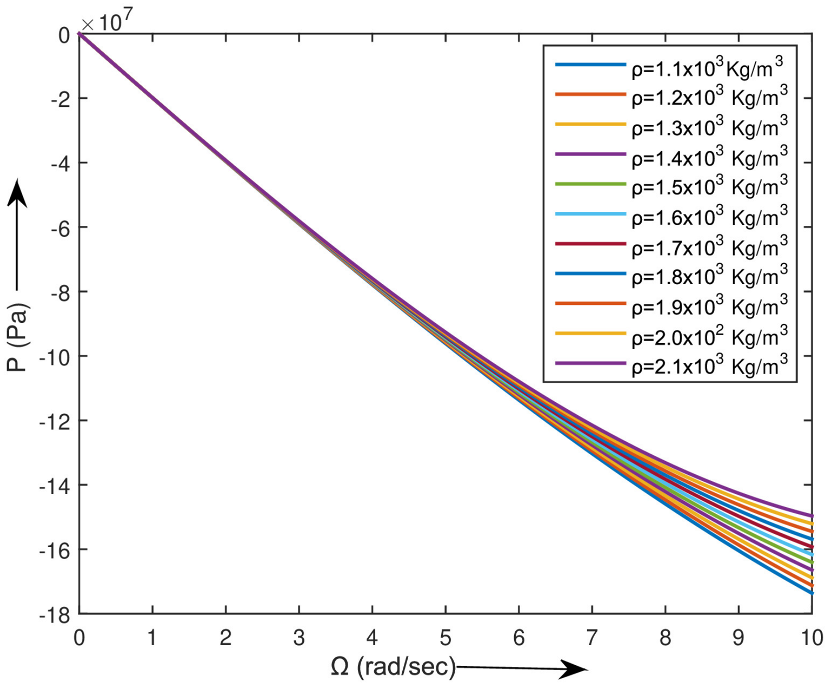


FIGURE 12. Pressure for variable density for $\frac{\eta}{k} = 10 \times 10^6$.

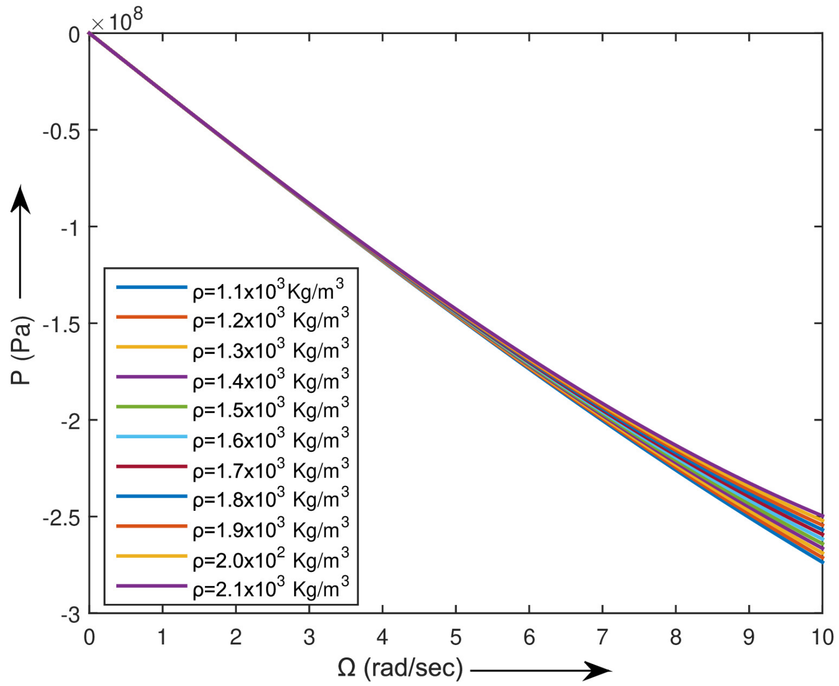


Figure 13. Pressure for variable density for $\frac{\eta}{k} = 15 \times 10^6$

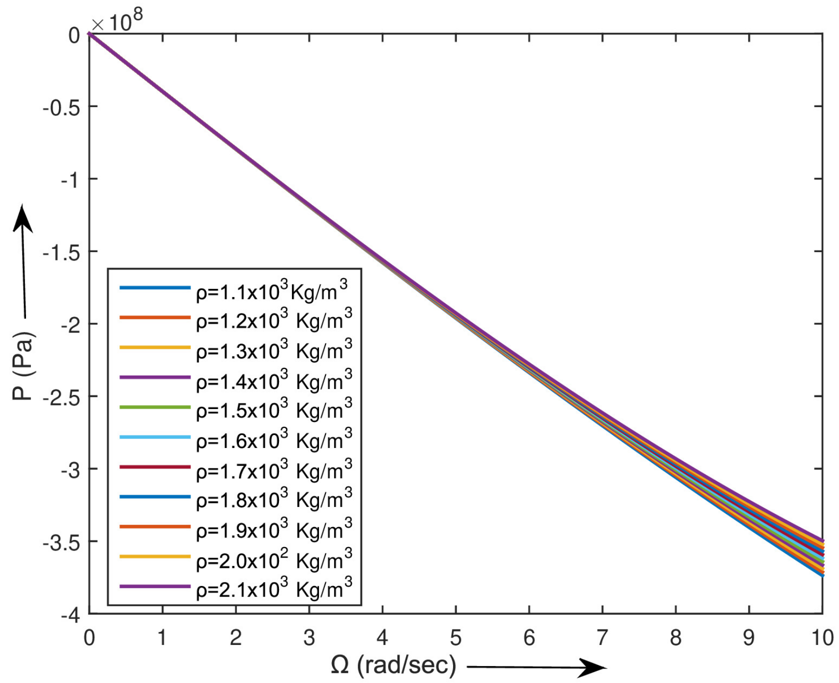


Figure 14. Pressure for variable density for $\frac{\eta}{k} = 20 \times 10^6$

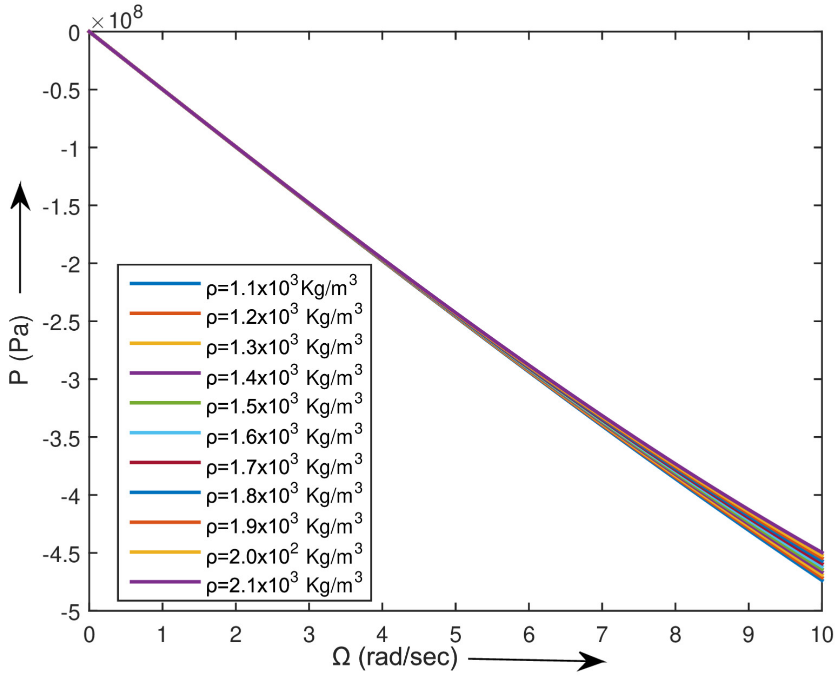


FIGURE 15. Pressure for variable density for $\frac{\eta}{k} = 25 \times 10^6$

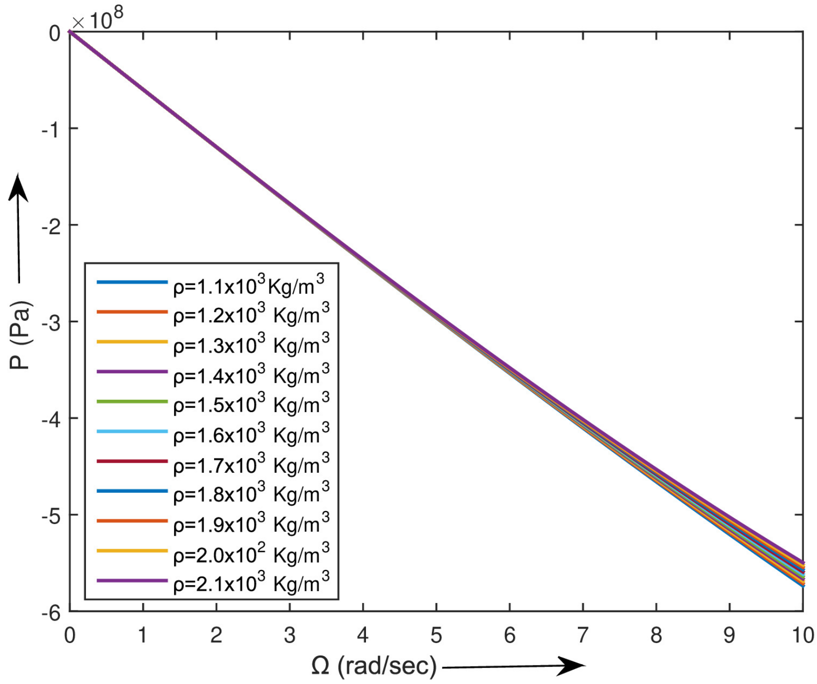


FIGURE 16. Pressure for variable density for $\frac{\eta}{k} = 30 \times 10^6$

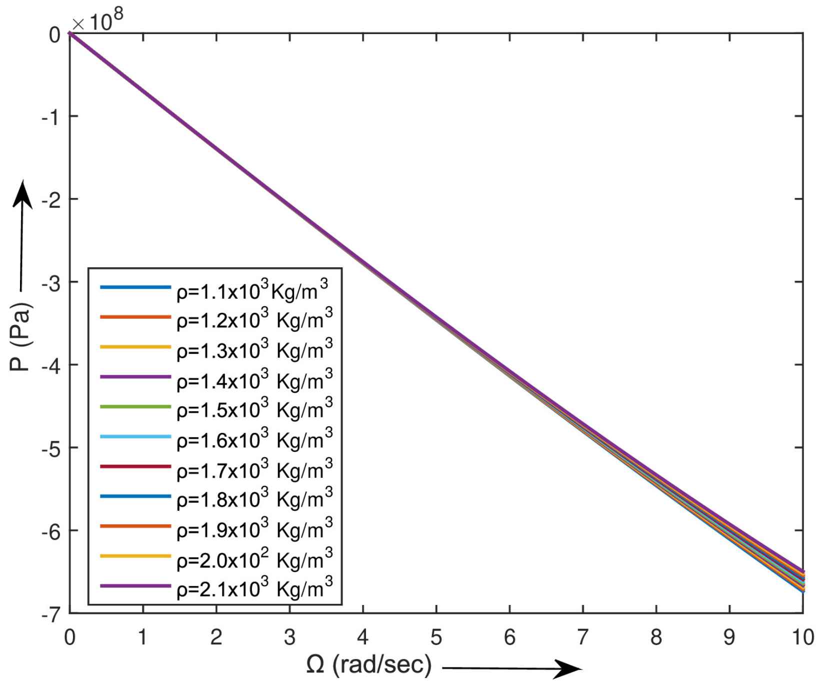


FIGURE 17. Pressure for variable density for $\frac{\eta}{k} = 35 \times 10^6$

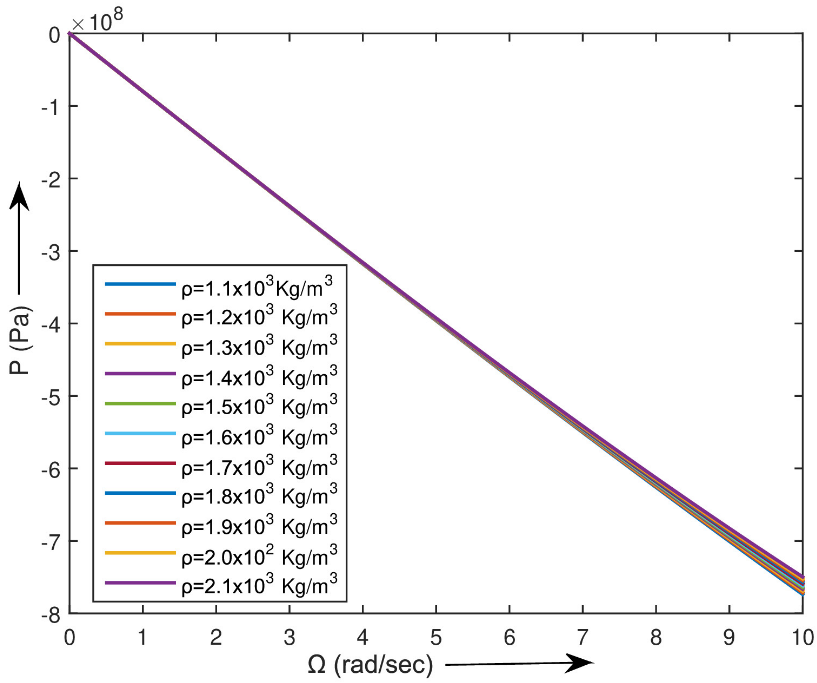


FIGURE 18. Pressure for variable density for $\frac{\eta}{k} = 40 \times 10^6$

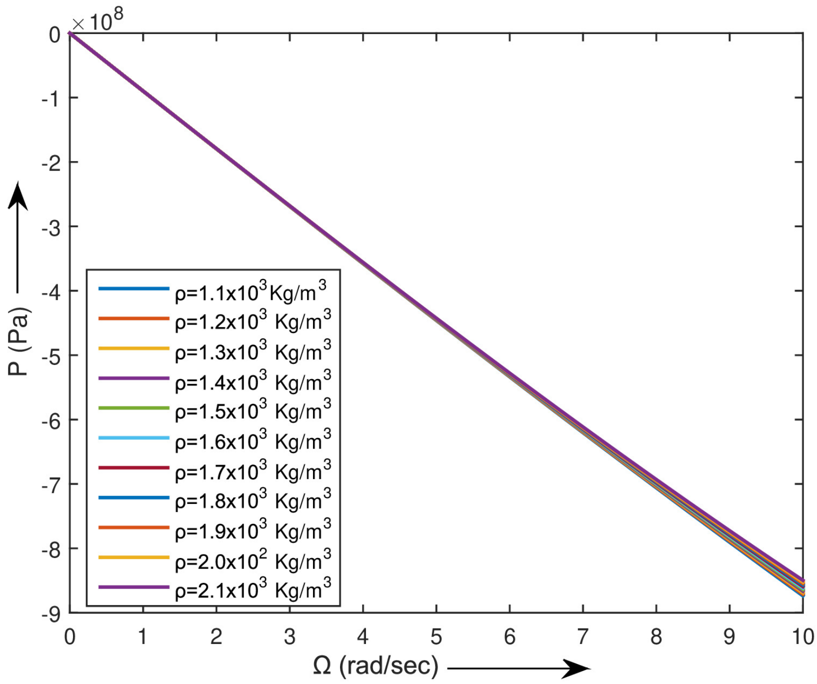


FIGURE 19. Pressure for variable density for $\frac{\eta}{k} = 45 \times 10^6$

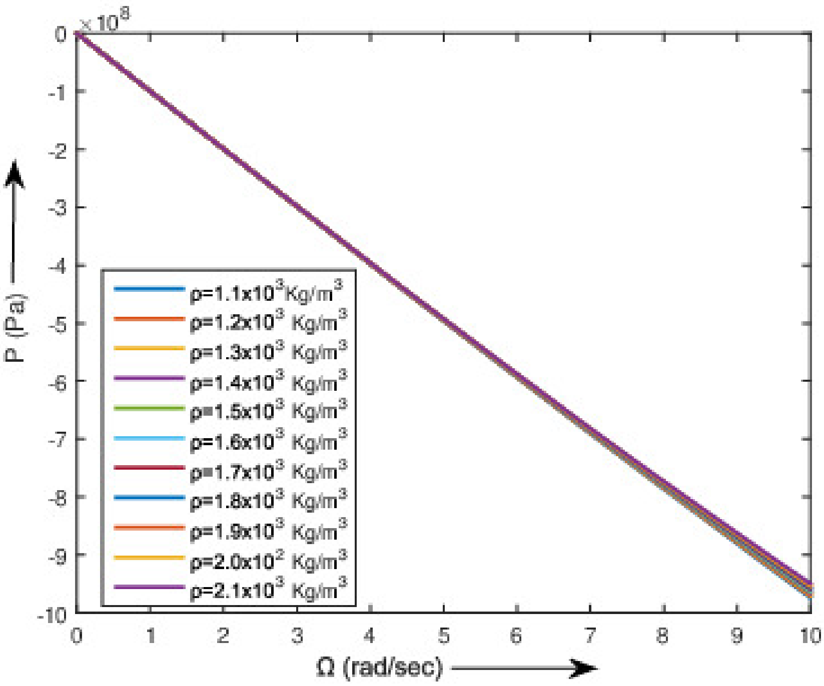


FIGURE 20. Pressure for variable density for $\frac{\eta}{k} = 50 \times 10^6$

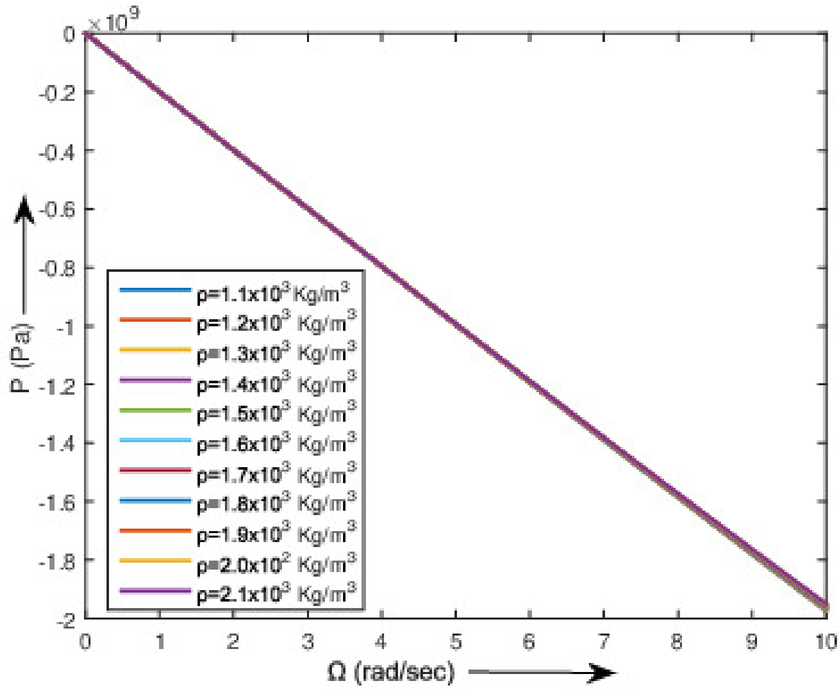


FIGURE 21. Pressure for variable density for $\frac{\eta}{k} = 100 \times 10^6$

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