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AN OTHER PROOF OF THE INSOLUBILITY OF FERMAT'S CUBIC EQUATION IN EISENSTEIN'S RING

Elias Lampakis

Kiparissia, T.K: 24500, Greece
eliaslampakis@yahoo.gr

Abstract. We present an other proof of the well known insolubility of Fermat's equation $x^3 + y^3 = z^3$ in Eisenstein's ring $\mathbb{Z}[\omega]$ when $\omega^3 = 1$, $\omega \neq 1$, $x\,y\,z \neq 0$. Assuming the existence of a nontrivial solution $(a_1 + b_1\,\omega,\ a_2 + b_2\,\omega,\ a_3 + b_3\,\omega)$ the proof exploits the algebraic properties, (degree, kind of roots, coefficients' relations), of the polynomial $f(x) = (a_1 + b_1\,x)^3 + (a_2 + b_2\,x)^3 - (a_3 + b_3\,x)^3$. In the course of action, the well known algebraic structure of the group of rational points of the elliptic curve $y^2 = x^3 + 16$ provides the final result.

Key words: Fermat's cubic equation, Eisenstein's ring, elliptic curves.

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