ANOTHER PROOF OF THE INSOLUBILITY OF
FERMAT’S CUBIC EQUATION IN EISENSTEIN’S RING

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Abstract. We present an other proof of the well known insolubility of Fermat’s
equation $x^3 + y^3 = z^3$ in Eisenstein’s ring $\mathbb{Z}[\omega]$ when $\omega^3 = 1$, $\omega \neq 1$, $xyz \neq 0$.
Assuming the existence of a nontrivial solution $(a_1 + b_1 \omega, a_2 + b_2 \omega, a_3 + b_3 \omega)$ the
proof exploits the algebraic properties, (degree, kind of roots, coefficients’ relations),
of the polynomial $f(x) = (a_1 + b_1 x)^3 + (a_2 + b_2 x)^3 - (a_3 + b_3 x)^3$. In the course
of action, the well known algebraic structure of the group of rational points of the
elliptic curve $y^2 = x^3 + 16$ provides the final result.

Key words: Fermat’s cubic equation, Eisenstein’s ring, elliptic curves.

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