J. Indones. Math. Soc.
Vol. 19, No. 1 (2013), pp. 41–47.

A NOTE ON THE EXISTENCE OF A UNIVERSAL POLYTOPE AMONG REGULAR 4-POLYTOPES

Jin Akiyama[1], Sin Hitotumatu[2], and Ikuro Sato[3]

[1]**Research Center for Math and Science Education, Tokyo University of Science, 1-3 Kagurazaka, Shinjuku, Tokyo 162-8601 Japan, ja@jin-akiyama.com**
[2]**Research Institute of Mathematical Science, University of Kyoto**
[3]**Department of Pathology, Research Institute, Miyagi Cancer Center, 47-1, Medeshima-shiote (Azanodayama), Natori-city, Miyagi 981-1293 Japan, sato-iku510@miyagi-pho.jp**

Abstract. For a polytope P, the set of all of its vertices is denoted by $V(P)$. For polytopes P and Q of the same dimension, we write $P \subset Q$ if $V(P) \subset V(Q)$. An n-polytope (n-dimensional polytope) Q is said to be universal for a family $\mathfrak{P}_n$ of all regular n-polytopes if $P \subset Q$ holds for every $P \in \mathfrak{P}_n$. The set $\mathfrak{P}_4$ consists of six regular 4-polytopes. It is stated implicitly in Coxeter (1973) by applying finite discrete groups that a regular 120-cell is universal for $\mathfrak{P}_4$. Our purpose of this note is to give a simpler proof by using only metric properties. Furthermore, we show that the corresponding property does not hold in any other dimension but 4.

Key words and Phrases: Inclusion property.

2000 Mathematics Subject Classification: 52B05, 52B45
Received: 30-08-2012, revised: 13-01-2013, accepted: 17-01-2013.