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APPLICATION OF JACCARD DISTANCE MEASURE FOR IVIF MCDM PROBLEMS

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Abstract. IVIFSs have a notable advantage in coping the uncertain information and were adequately used in decision-making. In IVIFSs there exists a definite relationship between the degree of membership, non-membership, and hesitancy which is well captured by Jaccard similarity. Moreover, Jaccard similarity measure gives a very intuitive similarity between the samples and hence been implemented in decision making. Hence, in this paper a decision-making model is constructed using the Normalized Jaccard distance measure on IVIFSs. The proposed model is illustrated by taking a numerical example. Further, the problem of choosing best e-learning tool in higher education is considered as a case study. The results are compared with the existing methods and analyzed. It is observed that the proposed method is well accommodating in interpreting the results to a better extent as it provides the amount of dissimilarity along with the distance. Thus, the proposed ranking can be significantly applied to solve MCDM problems.

Key words and Phrases: IVIFS; MCDM; Interval hesitancy degree; Jaccard similarity; Normalized Jaccard Distance.

1. INTRODUCTION

IVIFSs introduced by Atanassov and Gargov [1] have substantial benefit of handling with vague and imperfect data and hence are ingeniously useful in various fields, particularly in decision-making. Ranking of fuzzy numbers is an elementary problem in fuzzy decision theory. Several ranking methodologies are proposed by many researchers in decision making in the form of accuracy functions, score functions and distance measures.

In recent past, the distance based similarity measures and entropy measures are extensively applied in fuzzy decision problems as they specify the degree of evenness while ranking. Therefore, selecting alternatives by distance-based ranking is

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an interesting and important topic of research for handling MCDM problems. Further, Jaccard similarity measure gives a very intuitive similarity amount between the samples and hence been frequently implemented in decision support systems with various domains.

Inspired by the applicability of Jaccard index in decision making, we proposed a distance measure entitled Normalized Jaccard distance for IVIFSs and presented a model using this distance measure to solve decision making problems. The distance measure developed considers the interval hesitancy degree along with membership functions and non-membership functions and it gives a very intuitive similarity amount between the samples.

During the Covid-19 pandemic, rise of e-learning in education system is increased. As a result, education scenario has changed dramatically, with the distinctive rise of e-learning platforms, whereby teaching is undertaken remotely and on digital platforms. Research suggests that online learning has been shown to increase retention of information, and take less time. Therefore, selecting a best platform for online-learning has become a significant problem in education sector. Hence, the problem of choosing best e-learning tool in higher education is studied using the proposed model.

The study is summarized as: brief Literature Review is presented in section 2. Basics of IVIFSs are presented in section 3. In section 4, the Normalized Jaccard distance measure between IVIFSs is defined and a model for solving MCDM problem is presented. In section 5 the proposed model is illustrated through a numerical example from literature and a case study of choosing best e-learning tool in higher education is solved. In section 6, a detailed comparative analysis of proposed method and existing methods is given. The conclusion is drawn in section 7.

2. Literature Review

Decision making is a cognitive process and fuzzy sets are extensively used in decision-making. Over decades, various decision making methods [16, 21] were developed under various fuzzy sets. Belbag et al. [3] combined fuzzy sets theory with two different MCDM methods to eliminate the vagueness of linguistic factors that stem from the uncertain and imprecise assessment of decision-makers to overcome the problem of facility location selection. Nihan et al. [10] used the methods of fuzzy set theory, linguistic value, Fuzzy TOPSIS and Fuzzy VIKOR to consolidate decision-makers assessments about criteria weightings and described the application of Fuzzy TOPSIS and Fuzzy VIKOR for solving location selection problem of textile factory. Bolturk Eda et al. [4] developed COmbinative Distance-based Assessment (CODAS) method for solving MCDM problems, by taking the hesitancy of decision makers into consideration based on both Euclidean and Taxicab distances according to the negative-ideal point. Deveci et al. [5] investigated the degree of importance of criteria affecting the optimal site selection of offshore wind farms using Fuzzy Delphi method. Dogan et al. [7] proposed a fuzzy decision model combining analytic hierarchy process (AHP) and technique for order of preference by similarity to ideal solution (TOPSIS) techniques with intuitionistic fuzzy sets is used to solve the problem of corridor selection for locating autonomous vehicles. Deveci et al. [6] highlighted the advantages of 6 real-time traffic management methods and proposed novel extensions of MCDM methodology COmbining COmpromise SOlution (CoCoSo) with the logarithmic method and the fuzzy Power Heronian function.

The distance measures and the similarity measures indicate the degree of likeness of two sets, thus significantly used for ranking of fuzzy sets in decision making. A variety of distance measures and similarity measures of IVIFSs/ IV-IFNs were defined in literature [16], these measures are the various combinations and generalizations of the weighted Hamming distance, the weighted Euclidean distance, and the weighted Hausdorff distance. Zhang et al. [18] presented a method for evaluating similarity measures of IVIFSs on the basis of Hausdorff metric and applied to solve pattern recognition problem. Ye [17] proposed a cosine similarity measure and a weighted cosine similarity measure for IVIFSs based on the extension of the cosine similarity measure to solve MCDM problems. Nayagam et al. [9] proposed a distance based similarity measure and showed the application through TOPSIS method in decision making. Therefore, to rank and select alternatives through measuring the amount of similarity/ dissimilarity in decision making problems is a significant topic of research.

3. Mathematical Preliminaries

In this section, we recall the notion and operations of IVIFSs.

Definition 3.1. ***Interval-Valued Intuitionistic fuzzy Set[1]***
Let E be a fixed subset of a universal set X, then

$$\tilde{\mathrm{A}} = \{< x_i, \mu_{\tilde{\mathrm{A}}}(x_i), \nu_{\tilde{\mathrm{A}}}(x_i) >, x_i \in E\}$$

where $\mu_{\tilde{\mathrm{A}}} : X \to S[0,1]$ and $\nu_{\tilde{\mathrm{A}}} : X \to S[0,10]$ are the membership and nonmembership intervals of the element x_i to the set $\tilde{\mathrm{A}}$. with $0 \leq sup\mu_{\tilde{\mathrm{A}}}(x_i) + sup\nu_{\tilde{\mathrm{A}}} \leq 1$; and S[0,1] be closed subintervals of [0,1].
And $\mu^{\mathrm{L}}_{\tilde{\mathrm{A}}}(x_i)$; $\mu^{\mathrm{U}}_{\tilde{\mathrm{A}}}(x_i)$; $\nu^{\mathrm{L}}_{\tilde{\mathrm{A}}}(x_i)$ and $\nu^{\mathrm{U}}_{\tilde{\mathrm{A}}}(x_i)$ are lower and upper boundaries of the intervals $\mu_{\tilde{\mathrm{A}}}$ and $\nu_{\tilde{\mathrm{A}}}$.
We can denote by

$$\tilde{\mathrm{A}} = \{< x_i, [\mu^{\mathrm{L}}_{\tilde{\mathrm{A}}}(x_i), \mu^{\mathrm{U}}_{\tilde{\mathrm{A}}}(x_i)], [\nu^{\mathrm{L}}_{\tilde{\mathrm{A}}}(x_i), \nu^{\mathrm{U}}_{\tilde{\mathrm{A}}}(x_i)] >, x_i \in E\}$$

The interval hesitancy degree of any IVIFS $\tilde{A}$ *is defined as [1]*

$$\pi_{\tilde{A}}(x_i) = [1 - \mu^{U}_{\tilde{A}}(x_i) - \nu^{U}_{\tilde{A}}(x_i), 1 - \mu^{L}_{\tilde{A}}(x_i) - \nu^{L}_{\tilde{A}}(x_i)]$$

Atanassov and Gargov (1989) further gave some basic relations and operational laws of IVIFSs.

Definition 3.2. ***Operations of IVIFSs [2]*** *Let* $\tilde{A}$ *and* $\tilde{B}$ *, be any two IVIFSs on E then*

(i) $\tilde{A} \subseteq \tilde{B}$ *iff* $\mu^{L}_{\tilde{A}}(x_i) \leq \mu^{L}_{\tilde{B}}(x_i)), \mu^{U}_{\tilde{A}}(x_i) \leq \mu^{U}_{\tilde{B}}(x_i)), \nu^{L}_{\tilde{A}}(x_i) \geq \nu^{L}_{\tilde{B}}(x_i)) and \nu^{U}_{\tilde{A}}(x_i) \geq \nu^{U}_{\tilde{B}}(x_i))])\}$

(ii) $\tilde{A} = \tilde{B}$ *iff* $\tilde{A} \subseteq \tilde{B}$ *and* $\tilde{B} \subseteq \tilde{A}$

(iii) $\tilde{A} \bigcap \tilde{B} = \{< x_i, [min(\mu^{L}_{\tilde{A}}(x_i), \mu^{L}_{\tilde{B}}(x_i)), min(\mu^{U}_{\tilde{A}}(x_i), \mu^{U}_{\tilde{B}}(x_i))], [max(\nu^{L}_{\tilde{A}}(x_i), \nu^{L}_{\tilde{B}}(x_i)), max(\nu^{U}_{\tilde{A}}(x_i), \nu^{U}_{\tilde{B}}(x_i))] >, x_i \in E\}$

(iv) $\tilde{A} \bigcup \tilde{B} = \{< x_i, [max(\mu^{L}_{\tilde{A}}(x_i), \mu^{L}_{\tilde{B}}(x_i)), max(\mu^{U}_{\tilde{A}}(x_i), \mu^{U}_{\tilde{B}}(x_i))], [min(\nu^{L}_{\tilde{A}}(x_i), \nu^{L}_{\tilde{B}}(x_i)), min(\nu^{U}_{\tilde{A}}(x_i), \nu^{U}_{\tilde{B}}(x_i))] >, x_i \in E\}$

The Interval Valued Intuitionistic Fuzzy Number (IVIFN) is denoted by $\tilde{\gamma} = ([s,t],[u,v])$ where $[s,t] \subseteq [0,1], [u,v] \subseteq [0,1] and t + v \leq 1$[16].

Definition 3.3. ***Arithmetic Operations of IVIFNs [16]***

Let $\tilde{\gamma}_1 = ([s_1,t_1],[u_1,v_1])$ *and* $\tilde{\gamma}_2 = ([s_2,t_2],[u_2,v_2])$ *be two IVIFNs, then some arithmetic operations of* $\tilde{\gamma}_1$ *and* $\tilde{\gamma}_2$ *are defined as follows:*

(i) $\tilde{\gamma}_1 + \tilde{\gamma}_2 = ([s_1 + s_2 - s_1.s_2, t_1 + t_2 - t_1.t_2],[u_1.u_2, v_1.v_2])$;
(ii) $\tilde{\gamma}_1 \times \tilde{\gamma}_2 = ([s_1.s_2, t_1.t_2],[u_1 + u_2 - u_1.u_2, v_1 + v_2 - v_1.v_2])$;
(iii) $r.\tilde{\gamma}_1 = ([1-(1-s_1)^r, 1-(1-t_1)^r],[{u_1}^r, {v_1}^r]), r > 0$;
(iv) $\tilde{\gamma}_1^r = ([s^r, t^r],[1-(1-u_1)^r, 1-(1-v_1)^r]), r > 0.$

Definition 3.4. ***Jaccard distance[13]***

Jaccard distance measures the dissimilarity between the sets. The distance between $\tilde{A}, \tilde{B}$ *is given by* $d_J(\tilde{A}, \tilde{B}) = 1 - S_J(\tilde{A}, \tilde{B})$.

where $S_J(\tilde{A}, \tilde{B}) = \frac{|\tilde{A} \bigcap \tilde{B}|}{|\tilde{A} \bigcup \tilde{B}|}$ *is Jaccard similarity co-efficient.*

Definition 3.5. ***Jaccard distance on IVIFSs[15]***

Let $\tilde{A}, \tilde{B}$ *be any two IVIFSs on E, then the Jaccard distance between* $\tilde{A}, \tilde{B}$ *is*

$d_J(\tilde{A}_1, \tilde{A}_2) = 1 -$
$\Big\{ \mid [min(\mu^{L}_{\tilde{A}_1}(x_i), \mu^{L}_{\tilde{A}_2}(x_i)), min(\mu^{U}_{\tilde{A}_1}(x_i), \mu^{U}_{\tilde{A}_2}(x_i))], [max(\nu^{L}_{\tilde{A}_1}(x_i), \nu^{L}_{\tilde{A}_2}(x_i)),$

$max(\nu^{\mathrm{U}}_{\tilde{\mathrm{A}}_1}(x_i), \nu^{\mathrm{U}}_{\tilde{\mathrm{A}}_2}(x_i))], [max(\pi^{\mathrm{L}}_{\tilde{\mathrm{A}}_1}(x_i), \pi^{\mathrm{L}}_{\tilde{\mathrm{A}}_2}(x_i)), min(\pi^{\mathrm{U}}_{\tilde{\mathrm{A}}_1}(x_i), \pi^{\mathrm{U}}_{\tilde{\mathrm{A}}_2}(x_i))] \mid \Big\} \Big/$

$\Big\{ \mid [max(\mu^{\mathrm{L}}_{\tilde{\mathrm{A}}_1}(x_i), \mu^{\mathrm{L}}_{\tilde{\mathrm{A}}_2}(x_i)), max(\mu^{\mathrm{U}}_{\tilde{\mathrm{A}}_1}(x_i), \mu^{\mathrm{U}}_{\tilde{\mathrm{A}}_2}(x_i))], [min(\nu^{\mathrm{L}}_{\tilde{\mathrm{A}}_1}(x_i), \nu^{\mathrm{L}}_{\tilde{\mathrm{A}}_2}(x_i)),$
$min(\nu^{\mathrm{U}}_{\tilde{\mathrm{A}}_1}(x_i), \nu^{\mathrm{U}}_{\tilde{\mathrm{A}}_2}(x_i))], [min(\pi^{\mathrm{L}}_{\tilde{\mathrm{A}}_1}(x_i), \pi^{\mathrm{L}}_{\tilde{\mathrm{A}}_2}(x_i)), max(\pi^{\mathrm{U}}_{\tilde{\mathrm{A}}_1}(x_i), \pi^{\mathrm{U}}_{\tilde{\mathrm{A}}_2}(x_i))] \mid \Big\}$

4. Proposed methodology for sovling IVIF MCDM problems

The procedure for solving IVIF MCDM problems using a distance measure is discussed in this section.

4.1. **Proposed distance on IVIFSs:** Here, a distance measure is proposed on IVIFSs and named as Normalized Jaccard Distance (NJD) measure. The NJD gives the distance between IVIFSs, which has been scaled to have unit norm.

Definition 4.1. ***Normalized Jaccard distance***
For IVIFSs $\tilde{\mathrm{A}}, \tilde{\mathrm{B}}$ on E, the Normalized Jaccard distance is defined as

$d_J(\tilde{\mathrm{A}}_1, \tilde{\mathrm{A}}_2) = 1 - \frac{1}{n}\sum_{i=1}^{n}$
$\Big\{ \mid [min(\mu^{\mathrm{L}}_{\tilde{\mathrm{A}}_1}(x_i), \mu^{\mathrm{L}}_{\tilde{\mathrm{A}}_2}(x_i)), min(\mu^{\mathrm{U}}_{\tilde{\mathrm{A}}_1}(x_i), \mu^{\mathrm{U}}_{\tilde{\mathrm{A}}_2}(x_i))], [max(\nu^{\mathrm{L}}_{\tilde{\mathrm{A}}_1}(x_i), \nu^{\mathrm{L}}_{\tilde{\mathrm{A}}_2}(x_i)),$
$max(\nu^{\mathrm{U}}_{\tilde{\mathrm{A}}_1}(x_i), \nu^{\mathrm{U}}_{\tilde{\mathrm{A}}_2}(x_i))], [max(\pi^{\mathrm{L}}_{\tilde{\mathrm{A}}_1}(x_i), \pi^{\mathrm{L}}_{\tilde{\mathrm{A}}_2}(x_i)), min(\pi^{\mathrm{U}}_{\tilde{\mathrm{A}}_1}(x_i), \pi^{\mathrm{U}}_{\tilde{\mathrm{A}}_2}(x_i))] \mid \Big\} \Big/$

$\Big\{ \mid [max(\mu^{\mathrm{L}}_{\tilde{\mathrm{A}}_1}(x_i), \mu^{\mathrm{L}}_{\tilde{\mathrm{A}}_2}(x_i)), max(\mu^{\mathrm{U}}_{\tilde{\mathrm{A}}_1}(x_i), \mu^{\mathrm{U}}_{\tilde{\mathrm{A}}_2}(x_i))], [min(\nu^{\mathrm{L}}_{\tilde{\mathrm{A}}_1}(x_i), \nu^{\mathrm{L}}_{\tilde{\mathrm{A}}_2}(x_i)),$
$min(\nu^{\mathrm{U}}_{\tilde{\mathrm{A}}_1}(x_i), \nu^{\mathrm{U}}_{\tilde{\mathrm{A}}_2}(x_i))], [min(\pi^{\mathrm{L}}_{\tilde{\mathrm{A}}_1}(x_i), \pi^{\mathrm{L}}_{\tilde{\mathrm{A}}_2}(x_i)), max(\pi^{\mathrm{U}}_{\tilde{\mathrm{A}}_1}(x_i), \pi^{\mathrm{U}}_{\tilde{\mathrm{A}}_2}(x_i))] \mid \Big\}$

Proposition 4.2. $d_{N_J}(\tilde{\mathrm{A}}, \tilde{\mathrm{B}})$ *satisfies the axioms:*

(A1) $0 \le d_{N_J}(\tilde{\mathrm{A}}, \tilde{\mathrm{B}}) \le 1$
(A2) $d_{N_J}(\tilde{\mathrm{A}}, \tilde{\mathrm{B}}) = 0$ *iff* $\tilde{\mathrm{A}}_1 = \tilde{\mathrm{B}}$
(A3) $d_{N_J}(\tilde{\mathrm{A}}, \tilde{\mathrm{B}}) = d_{N_J}(\tilde{\mathrm{B}}, \tilde{\mathrm{A}})$
(A4) $d_{N_J}(\tilde{\mathrm{A}}, \tilde{\mathrm{B}}) = 0, d_{N_J}(\tilde{\mathrm{A}}, \tilde{\mathrm{C}}) = 0$ *then* $d_{N_J}(\tilde{\mathrm{B}}, \tilde{\mathrm{C}}) = 0$ *for any IVIFSs* $\tilde{\mathrm{A}}, \tilde{\mathrm{B}}, \tilde{\mathrm{C}}$ *on X.*

Proof. (A1): Since, $min(\mu^L_{\tilde{A}}(x_i), \mu^L_{\tilde{B}}(x_i)) \le max(\mu^L_{\tilde{A}}(x_i), \mu^L_{\tilde{B}}(x_i))$,
$min(\mu^U_{\tilde{A}}(x_i), \mu^U_{\tilde{B}}(x_i)) \le max(\mu^U_{\tilde{A}}(x_i), \mu^U_{\tilde{B}}(x_i))$,
$max(\nu^L_{\tilde{A}}(x_i), \nu^L_{\tilde{B}}(x_i)) \ge min(\nu^L_{\tilde{A}}(x_i), \nu^L_{\tilde{B}}(x_i))$,
$max(\nu^U_{\tilde{A}}(x_i), \nu^U_{\tilde{B}}(x_i)) \ge min(\nu^U_{\tilde{A}}(x_i), \nu^U_{\tilde{B}}(x_i))$,
By Definition 2.2 (i), $\tilde{A} \bigcap \tilde{B} \subseteq \tilde{A} \bigcup \tilde{B}$, which implies $\mid \tilde{A} \bigcap \tilde{B} \mid \le \mid \tilde{A} \bigcup \tilde{B} \mid$. Therefore, $\frac{|\tilde{A} \bigcap \tilde{B}|}{|\tilde{A} \bigcup \tilde{B}|} \le 1$ Also it is always true that $\frac{|\tilde{A} \bigcap \tilde{B}|}{|\tilde{A} \bigcup \tilde{B}|} \ge 0$. Thus, $0 \le d_{N_J}(\tilde{A}, \tilde{B}) \le 1$

(A2): Let $\tilde{A} = \tilde{B}$ then $\tilde{A} \bigcap \tilde{B} = \tilde{A} = \tilde{B}$ and $\tilde{A} \bigcup \tilde{B} = \tilde{A} = \tilde{B}$

$\Rightarrow \mid \tilde{A} \bigcap \tilde{B} \mid = \mid \tilde{A} \bigcup \tilde{B} \mid \Rightarrow d_{N_J}(\tilde{A}, \tilde{B}) = 0.$

(A3): Since $min(\mu_{\tilde{A}}^L(x_i), \mu_{\tilde{B}}^L(x_i)) = min(\mu_{\tilde{B}}^L(x_i), \mu_{\tilde{A}}^L(x_i))$
$min(\mu_{\tilde{A}}^U(x_i), \mu_{\tilde{B}}^U(x_i)) = min(\mu_{\tilde{B}}^U(x_i), \mu_{\tilde{A}}^U(x_i))$
$max(\nu_{\tilde{A}}^L(x_i), \nu_{\tilde{B}}^L(x_i)) = max(\nu_{\tilde{B}}^L(x_i), \nu_{\tilde{A}}^L(x_i))$
$max(\nu_{\tilde{A}}^U(x_i), \nu_{\tilde{B}}^U(x_i)) = max(\nu_{\tilde{B}}^U(x_i), \nu_{\tilde{A}}^U(x_i)).$

Therefore, $\tilde{A} \bigcap \tilde{B} = \tilde{B} \bigcap \tilde{A}$

$\Rightarrow \mid \tilde{A} \bigcap \tilde{B} \mid = \mid \tilde{B} \bigcap \tilde{A} \mid$ ... (1)
$\Rightarrow$ *Similarly*, $\mid \tilde{A} \bigcup \tilde{B} \mid = \mid \tilde{B} \bigcup \tilde{A} \mid$ ... (2)

Hence, from equations (1) and (2) $\frac{|\tilde{A} \bigcap \tilde{B}|}{|\tilde{A} \bigcup \tilde{B}|} = \frac{|\tilde{B} \bigcap \tilde{A}|}{|\tilde{B} \bigcup \tilde{A}|}$
$\Rightarrow d_{N_J}(\tilde{A}, \tilde{B}) = d_{N_J}(\tilde{B}, \tilde{A}).$

(A4):
Now consider $d_{N_J}(\tilde{A}, \tilde{B}) = 0$, $d_{N_J}(\tilde{A}, \tilde{C}) = 0$

$\Rightarrow \mu_{\tilde{A}}^L(x_i) = \mu_{\tilde{B}}^L(x_i), \mu_{\tilde{A}}^U(x_i) = \mu_{\tilde{B}}^U(x_i), \nu_{\tilde{A}}^L(x_i) = \nu_{\tilde{B}}^L(x_i)$ and $\nu_{\tilde{A}}^U(x_i) = \nu_{\tilde{B}}^U(x_i)$ ... (3)

also $\mu_{\tilde{A}}^L(x_i) = \mu_{\tilde{C}}^L(x_i), \mu_{\tilde{A}}^U(x_i) = \mu_{\tilde{C}}^U(x_i), \nu_{\tilde{A}}^L(x_i) = \nu_{\tilde{C}}^L(x_i)$ and $\nu_{\tilde{A}}^U(x_i) = \nu_{\tilde{C}}^U(x_i)$ ... (4)

From equations (3) and (4)
$\mu_{\tilde{B}}^L(x_i) = \mu_{\tilde{C}}^L(x_i), \mu_{\tilde{B}}^U(x_i) = \mu_{\tilde{C}}^U(x_i), \nu_{\tilde{B}}^L(x_i) = \nu_{\tilde{C}}^L(x_i)$ and $\nu_{\tilde{B}}^U(x_i) = \nu_{\tilde{C}}^U(x_i)$

$\Rightarrow d_{N_J}(\tilde{B}, \tilde{C}) = 0$

Thus, Normalized Jaccard distance on IVIFS satisfies the properties of distance measure.

□

Definition 4.3. ***Ranking***
The ranking for $\tilde{\text{A}}and\tilde{\text{B}}$ *is given as*

(i) $\tilde{\text{A}} > \tilde{\text{B}}$ *if* $d_{N_J}(\tilde{\text{A}}, \tilde{I}) < d_{N_J}(\tilde{\text{B}}, \tilde{I})$

(ii) $\tilde{\text{A}} < \tilde{\text{B}}$ *if* $d_{N_J}(\tilde{\text{A}}, \tilde{I}) > d_{N_J}(\tilde{\text{B}}, \tilde{I})$

(iii) $\tilde{\text{A}} = \tilde{\text{B}}$ *if* $d_{N_J}(\tilde{\text{A}}, \tilde{I}) = d_{N_J}(\tilde{\text{B}}, \tilde{I})$

where $\tilde{I} = < x_i, [1, 1], [0, 0] >$ *is the ideal IVIFS.*

4.2. **A procedure to MCDM:.** In this section, steps for solving a decision-making problem are given.

Consider a finite set of m criteria $C = \{C_1, C_2, ..., C_m\}$ for 'n' alternatives $A = \{A_1, A_2, ..., A_n\}$.The expert delivers the performance values with respect to each alternative in the form of IVIFSs. Let $\tilde{p_{ij}}$ represents the performance of the alternative $\tilde{A}_i, i = 1, 2, ..., n$ and criteria $\tilde{C}_j, j = 1, 2, ..., m$.

The below steps are followed for selecting best alternative.

Step 1: Let $\tilde{A}_0$ be the ideal choice of best alternative i.e., $\tilde{A}_0$ has the performance
$\{< x_i, [1, 1], [0, 0] >\}$ in each criteria $\tilde{C}_j$.

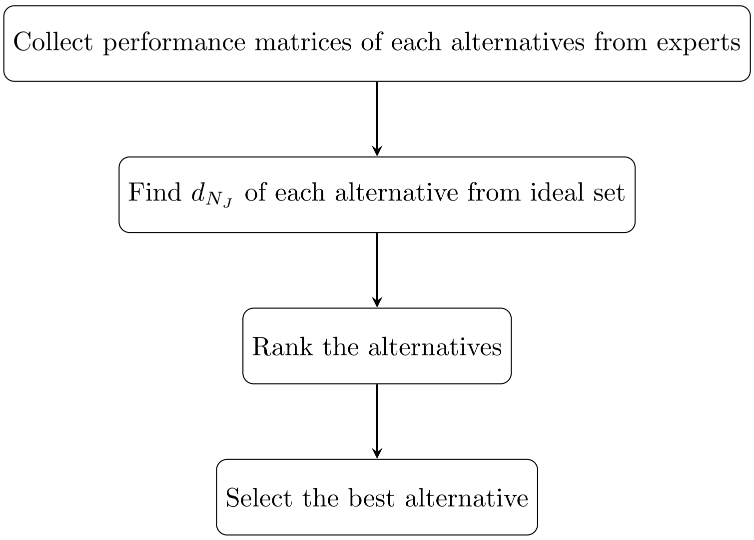
Step 2: Find d_{N_J} of each alternative $\tilde{A}_i$ to $\tilde{A}_0$ by definition 4.1.

Step 3: Rank the alternatives using definition 4.3.

Step 4: Choose the best alternative by means of the ranking order.

The flowchart of the proposed methodology is given below:

Flowchart of proposed IVIF MCDM method



5. Numerical Examples

To validate the proposed method, a real world problem from the literature [11] is taken and discussed.

Problem 5.1
The problem is about purchasing a house. There are six houses under consideration, $h_1, h_2, h_3, h_4, h_5, h_6$ and the criteria for selection are beautiful (e_1), large (e_2)", cheap (e_3)", modern (e_4)", and green surroundings (e_5)". The assessment for each house with respect each criterion is given in Table 1.

TABLE 1. IVIFSs over U

	e_1	e_2	e_3	e_4	e_5
h_1	([0.7,0.8], [0.1,0.2])	([0.82,0.84], [0.05, 0.15])	([0.52,0.72], [0.18, 0.25])	([0.55,0.6], [0.3,0.35])	([0.7,0.8], [0.1,0.2])
h_2	([0.85,0.9], [0.05, 0.1])	([0.7,0.74], [0.17,0.25])	([0.7,0.75], [0.1,0.23])	([0.7,0.75], [0.15, 0.25])	([0.75,0.9], [0.05, 0.1])
h_3	([0.5,0.7], [0.2, 0.3])	([0.86,0.9], [0.04, 0.1])	([0.6,0.7], [0.2,0.28])	([0.2,0.3], [0.5,0.6])	([0.65,0.8], [0.15, 0.2])
h_4	([0.4,0.6], [0.3,0.4])	([0.52,0.64], [0.23, 0.35])	([0.72,0.78], [0.11, 0.21])	([0.3,0.5], [0.4,0.5])	([0.8,0.9], [0.05, 0.1])
h_5	([0.6,0.8], [0.15,0.2])	([0.3,0.35], [0.5, 0.65])	([0.58,0.68], [0.18, 0.3])	([0.68,0.77], [0.1,0.2])	([0.72,0.85], [0.1, 0.15])
h_6	([0.3,0.5], [0.3,045])	([0.5,0.84], [0.25, 0.3])	([0.33,0.43], [0.5, 0.55])	([0.62,0.65], [0.15,0.35])	([0.84,0.93], [0.04, 0.07])

Step 1: Let the ideal IVIFS for best alternative be h_0 which is represented in Table 2.

TABLE 2. IVIFSs h_0

	e_1	e_2	e_3	e_4	e_5
h_0	([1,1], [0,0])	([1,1], [0,0])	([1,1], [0,0])	([1,1], [0,0])	([1,1], [0,0])

Step 2: The Normalised Jaccard distance d_{N_J} of each alternative h_i to h_0 is given in Table 3.

TABLE 3. Distance between h_i from the ideal house h_0

	h_1	h_2	h_3	h_4	h_5	h_6
h_0	0.238	0.183	0.311	0.318	0.305	0.351

Step3: The ranking order of the alternatives is $h_2 > h_1 > h_5 > h_3 > h_4 > h_6$.

Step4: The best alternative by the ranking order is h_2.

The result is compared with the existing methods [8, 11, 12]. The comparison reveals that the best house to purchase is h_2.
Further a decision making problem of choosing best e-learning tool in higher education is considered as a case study.

Case Study The education sector has a lot of challenges during this Covid pandemic. As a result, rise of e-learning in education system is increased. The teaching is undertaken remotely and on digital platforms. And the panic situation about the future of the students has overcome to a certain percentage, by a proficient use of resources available for e-learning.

The alternatives for e-learning tools are T_1, T_2, T_3 and T_4. And to analyze the best one, the following features are considered as criteria: f_1- Web Features,f_2-Tech Support, f_3- Mobile accessibility, f_4- Price, f_5- Integration with Learning Management System and f_6- User accountability.

The linguistic terms and the corresponding IVIF values are given in Table 4. The performances of alternatives are obtained as linguistic variables from expert (Table 5). Then the IVIF decision matrix is formed by changing the linguistic variables (see Table 6).

TABLE 4. Linguistic terms and values for IVIF Decision matrix

Linguistic terms	IVIF values
Efficient (E)	([0.8,0.9],[0,0.1])
Good (G)	([0.7,0.8],[0.1,0.2])
Average (A)	([0.5,0.6],[0.3,0.4])
Poor (P)	([0.3,0.4],[0.5,0.6])
Low (L)	([0.1,0.2],[0.7,0.8])

TABLE 5. Decision matrix in the form of linguistic terms

	f_1	f_2	f_3	f_4	f_5	f_6
T_1	E	G	G	A	G	E
T_2	E	A	E	A	E	G
T_3	G	G	E	G	G	E
T_4	A	A	G	E	G	A

TABLE 6. IVIF Decision matrix

	f_1	f_2	f_3	f_4	f_5	f_6
T_1	([0.8,0.9], [0,0.1])	([0.7,0.8], [0.1,0.2])	([0.7,0.8], [0.1,0.2])	([0.5,0.6], [0.3,0.4])	([0.7,0.8], [0.1,0.2])	([0.8,0.9], [0,0.1])
T_2	([0.8,0.9], [0,0.1])	([0.5,0.6], [0.3,0.4])	([0.8,0.9], [0,0.1])	([0.5,0.6], [0.3,0.4])	([0.8,0.9], [0,0.1])	([0.7,0.8], [0.1,0.2])
T_3	([0.7,0.8], [0.1,0.2])	([0.7,0.8], [0.1,0.2])	([0.8,0.9], [0,0.1])	([0.7,0.8], [0.1,0.2])	([0.7,0.8], [0.1,0.2])	([0.8,0.9] , [0,0.1])
T_4	([0.5,0.6], [0.3,0.4])	([0.5,0.6], [0.3,0.4])	([0.7,0.8], [0.1,0.2])	([0.8,0.9] , [0,0.1])	([0.7,0.8], [0.1,0.2])	([0.5,0.6], [0.3,0.4])

Step 1: Let the ideal IVIFS for best alternative be T_0 and represented in Table 7.

TABLE 7. IVIF set T_0

	f_1	f_2	f_3	f_4	f_5	f_6
T_0	([1,1],[0,0])	([1,1],[0,0])	([1,1],[0,0])	([1,1],[0,0])	([1,1],[0,0])	([1,1],[0,0])

Step 2: The Normalised Jaccard distance d_{N_J} of each alternative T_i to T_0 is given in Table 8.

TABLE 8. Distance of T_i from the ideal house T_0

	T_1	T_2	T_3	T_4
T_0	0.19	0.21	0.16	0.28

Step3: The ranking order of the alternatives is $T_3 > T_1 > T_2 > T_4$.

Step4: The best alternative is T_3.

The result of the problem is coinciding with the results of the approaches [8, 12].

6. COMPARATIVE ANALYSIS

The proposed method is compared with two different categories of MCDM methods: distance based approaches [8, 12] and fuzzy choice based MCDM approach. It is observed that the most preferred house is h_2 in all the methods. But, the order preference of other alternatives is altered. From [11], it is noted that the ordering is $h_2 > h_1 \sim h_5 > h_3 \sim h_4 > h_6$. Compared with the result of [11], the proposed method provides more flexible and rational results. As not only the best alternative but also the second best alternative is also important while taking decisions, it is concluded that the proposed method is more effective in finding the order of the alternatives when compared to [11]. Further, the result is compared with the existing methods [8, 12]. The comparison disclosed that the result obtained by proposed method is coinciding with the results of these methods.

Moreover, an MCDM problem of selecting the best online learning tool for higher education is considered as a case study and solved using the proposed method. The best alternative obtained is T_3. To validate the authenticity of the result, the problem is solved using the distance based approaches [8, 12]. The results of proposed method are promisingly coinciding with [8, 12]. Comparing with the existing MCDM methods, the presented decision making model with the proposed distance measure is compact. Also, it gives an innate degree of dissimilarity of each alternative from the best alternative.

7. Conclusion

The IVIFSs are proven to be effective in expressing the uncertain information effectively and Jaccard similarity measure gives a very intuitive similarity amount between the samples. Hence a normalized Jaccard distance measure for IVIFSs is proposed in this paper. This measure not only takes interval hesitancy degree into account while ranking but provides the amount of the dissimilarity of given IVIFS to the ideal set. An IVIF decision making model by means of this distance is proposed and its efficacy is studied by solving a numerical example taken from literature. The comparison analysis with existing methods disclosed that the result obtained by proposed method is coinciding with the results of existing methods. Further it is observed that the proposed method is helpful in understanding the results to a better extent as it provides the amount of dissimilarity of alternatives from the best alternative along with the distance. Thus the proposed method is applied in choosing the best e-learning tool in higher education. In further studies, the proposed measure can be integrated in the decision making methods such as TOPSIS, ELECTRE and PROMETHEE etc. to solve IVIF MCDM problems and a weighed normalized Jaccard distance measure can be defined to solve the MCDM problems when weights are assigned to criteria/decision makers.

REFERENCES

[1] Atanassov, KT. and Gargov, "Interval valued Intuitionistic fuzzy sets", *Fuzzy Sets Syst*, **31** (1989), 343-349.

[2] Atanassov, K.T, "New operations defined over the intuitionistic fuzzy sets", *Fuzzy Sets Syst*, **61 (2)**, (1994), 137-142.

[3] Belbag, S. Deveci, Muhammet and Uluda, Ahmet, "Comparison of two fuzzy multi criteria decision methods for potential airport location selection", *ICORES 2013 - Proceedings of*

the 2nd International Conference on Operations Research and Enterprise Systems. (2013), 270-276.

[4] Bolturk Eda, Kahraman and Cengiz , "Interval-valued Intuitionistic fuzzy CODAS method and its Application to Wave energy facility location selection problem", *J. Intell. Fuzzy Syst.*, **35 (4)** (2018), 4865-4877.

[5] Deveci M, zcan E, John R, Covrig CF, Pamucar D, "A study on offshore wind farm siting criteria using a novel interval-valued fuzzy-rough based Delphi method". *Environ. Manage.*, **270 (1)** (2020), 110916. doi: 10.1016/j.jenvman.

[6] Deveci,M., Dragan, P. and Ilgin, G., "Fuzzy Power Heronian function based CoCoSo method for the advantage prioritization of autonomous vehicles in real-time traffic management", *Sustainable Cities and Society*, **6**9, (2021), 102846, ISSN 2210-6707, https://doi.org/10.1016/j.scs.2021.102846.

[7] Dogan, O., Deveci, M., Cantez, F. et al., "A Corridor selection for locating autonomous vehicles using an interval-valued intuitionistic fuzzy AHP and TOPSIS method",*Soft Comput.*, **24** (2020), 89378953.

[8] Feng, Qinrong and Guo, Xiao. , "Uncertainty Measures of Interval-valued Intuitionistic Fuzzy Soft Sets and their Applications in Decision Making", *Intell. Data Anal.*, **21 (1)** (2017), 77-95.

[9] Nayagam V L G, Dhanasekharan, P. and Jeevraj, S., "Similarity Measure on Incomplete Imprecise Interval information and its Applications", *Neural Comput. Appl.*, **32** (2020), 3749-3761.

[10] Nihan, C. D., Deveci and Gizem, E., Comparative analysis of fuzzy multi-criteria decision making for location selection of Textile plant in Turkey, *Proceedings of International Academic Conferences from International Institute of Social and Economic Sciences*, No 400652.

[11] H.W. Qin, X.Q. Ma, T. Herawan and J.M. Zain, "An Adjustable approach to Interval-Valued Intuitionistic Fuzzy Soft Sets based Decision making", *In: Lecture Notes in Computer Science, Springer-Verlag*, **6592** (2011), 80-89.

[12] Liu, Y. and Jiang, W., "A New Distance Measure of Interval-Valued Intuitionistic fuzzy sets and its Application in Decision making", *Soft Comput.*, **24** (2020), 6987-7003.

[13] Michael Levandowsky and David, W. "Distance between Sets", *Nature* **134** (1971), 34 -35.

[14] Sireesha, V. and Hima bindu, K., "Decision making with Unknown criteria weight information in the Frame work of Interval Valued Trapezoidal Intuitionistic Fuzzy Sets", *Internat. J. Innov. Tech. Explo. Eng.*, **9 (1)** (2019), 4424-4429.

[15] V Anusha and V Sireesha, "Ranking Interval Valued Intuitionistic Fuzzy Sets by a New Distance Measure", *Adv. Math., Sci. J.*, **10 (3)** (2021), 1249-1258.

[16] Xu, Z. and Chen, "An Overview of Distance and Similarity measures of Intuitionistic Fuzzy Sets", *Internatl J. Uncertainty, Fuzziness and Knowledge-Based Systems*, **16** (2008), 529-555.

[17] Ye, J., "Cosine similarity measures for intuitionistic fuzzy sets and their applications", *Math. Comput. Model.*, **53 (1-2)** (2011), 91-97.

[18] Zhang, Q.S., Yao, H.X. and Zhang, Z.H., "Some Similarity Measures of Interval-Valued Intuitionistic Fuzzy Sets and Application to Pattern Recognition", *App. Mech. Mater.*, **4447**, 38883892.