J. Indones. Math. Soc.
Vol. 28, No. 01 (2022), pp. 75–83.

HEMI-SLANT SUBMANIFOLD OF $(LCS)_n$-MANIFOLD

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Abstract. In this paper, we analyse briefly some properties of hemi-slant submanifold of $(LCS)_n$-manifold. Here we discuss about some necessary and sufficient conditions for distributions to be integrable and obtain some results in this direction. We also study the geometry of leaves of hemi-slant submanifold of $(LCS)_n$-manifold. At last, we give an example of a hemi-slant submanifold of an $(LCS)_n$-manifold.

Key words and Phrases: $(LCS)_n$-manifold, hemi-slant submanifold, integrablity, leaves of distribution.

1. INTRODUCTION

An n-dimensional Lorentzian manifold $\tilde{M}$ is a smooth connected paracompact Hausdorff manifold with a Lorentzian metric $\tilde{g}$, that is $\tilde{M}$ admits a smooth symmetric tensor field $\tilde{g}$ of type (0,2) such that for each point. the tensor $\tilde{g}_p$: $T_p\tilde{M} \times T_p\tilde{M} \to \mathbb{R}$ is a non-degenerate inner-product of signature (-,+,...,+), $T_p\tilde{M}$ denotes the tangent vector space of $\tilde{M}$ at p and $\mathbb{R}$ is the real no. space. A non-zero vector $X_p \in T_p\tilde{M}$ is known to be spacelike, null or lightlike, or timelike according as $\tilde{g}_p(X_p, X_p) > 0, = 0$ or < 0 respectively.

If $\tilde{M}$ is a differentiable manifold of dimension n, and there exists a (ϕ, ξ, η) structure satisfying

$$\phi^2 = I + \eta \otimes \xi, \eta(\xi) = -1, \phi(\xi) = 0, \eta \circ \phi = 0,$$

then M is called an almost paracontact manifold.

In an almost paracontact structure $(\phi, \xi, \eta, \tilde{g})$,

$$\tilde{g}(X, \phi Y) = \tilde{g}(\phi X, Y),$$

2020 Mathematics Subject Classification: 53C05, 53C15, 53C40, 53C50.
Received: 12-03-2021, accepted: 17-01-2022.

$$2g(\phi X, Y) = (\bar{\nabla}_X \eta)Y + (\bar{\nabla}_Y \eta)X,$$
$$\phi^2 X = X + \eta(X)\xi, \eta \circ \phi = 0, \phi(\xi) = 0, \eta(\xi) = -1, \tag{1.1}$$

where ϕ is a tensor of type (1,1), ξ is a vector field, η is a 1-form and $\tilde{g}$ is Lorentzian metric satisfying

$$\tilde{g}(\phi X, \phi Y) = \tilde{g}(X, Y) + \eta(X)\eta(Y), \tilde{g}(X, \xi) = \eta(X) \tag{1.2}$$

for all vector fields X, Y on $\tilde{M}$.

In a Lorentzian manifold $(\tilde{M}, \tilde{g})$, a vector field P defined by $\tilde{g}(X, P) = A(X)$ for any $X \in \Gamma(T\tilde{M})$, is called con-circular if

$$(\bar{\nabla}_X A)(Y) = \alpha\{\tilde{g}(X, Y) + \omega(X)A(Y)\},$$

where α is a non-zero scalar and ω is a closed 1-form and $\bar{\nabla}$ denotes the operator of covariant differentiation of $\tilde{M}$ with respect to $\tilde{g}$.

Let $\tilde{M}$ admits a unit timelike concircular vector field ξ, called the structure vector field of the manifold, then $\tilde{g}(\xi, \xi) = -1$, since ξ is a unit concircular vector field, it follows that $\exists$ a non-zero 1-form η such that $\tilde{g}(X, \xi) = \eta(X)$. The following equations hold−

$$(\bar{\nabla}_X \eta)Y = \alpha[\tilde{g}(X, Y) + \eta(X)\eta(Y)],\ \alpha \neq 0,$$
$$\bar{\nabla}_X \alpha = X\alpha = d\alpha(X) = \rho\eta(X),$$

for all vector fields X, Y on $\tilde{M}$ and α is a non-zero scalar function related to ρ, by $\rho = -(\xi\alpha)$.

Let $\phi X = \frac{1}{\alpha}\bar{\nabla}_X \xi$, from which it follows that ϕ is a symmetric (1,1) tensor and call it the structure tensor on the manifold. Thus the Lorentzian manifold $\tilde{M}$ together with unit timelike concircular vector field ξ, its associated 1-form η and a (1,1) tensor field ϕ is called a Lorentzian Concircular Structure manifold i.e. $(LCS)_n$-manifold. Specially, if $\alpha = 1$, then we obtain LP-Sasakian structure of Matsumoto [15]. In an $(LCS)_n$-manifold $(n > 2)$, the following relations hold−

$$\phi^2 = I + \eta \otimes \xi, \eta(\xi) = -1,$$

where I denotes the identity transformation of the tangent space $T\tilde{M}$,

$$\phi\xi = 0, \eta \circ \phi = 0, \tilde{g}(X, \phi Y) = \tilde{g}(\phi X, Y), rank\phi = 2n, \tag{1.3}$$

$$\tilde{g}(\phi X, \phi Y) = \tilde{g}(X, Y) + \eta(X)\eta(Y), \tilde{g}(X, \xi) = \eta(X), \tag{1.4}$$

$$\bar{R}(X, Y)\xi = (\alpha^2 - \rho)[\eta(Y)X - \eta(X)Y] \tag{1.5}$$

$\forall\ X, Y \in T\tilde{M}$.

Also $(LCS)_n$-manifold satisfies−

$$(\bar{\nabla}_X \phi)Y = \alpha[\tilde{g}(X, Y)\xi + 2\eta(X)\eta(Y)\xi + \eta(Y)X], \tag{1.6}$$

$$\bar{\nabla}_X \xi = \alpha\phi X. \tag{1.7}$$

Let M be a submanifold of $\tilde{M}$ with $(LCS)_n$-structure $(\phi, \xi, \eta, \tilde{g})$ with induced metric g and let ∇ is the induced connection on the tangent bundle TM and $\nabla^{\perp}$ is the induced connection on the normal bundle $T^{\perp}M$ of M.

The Gauss and Weingarten formulae are characterized by−

$$\tilde{\nabla}_X Y = \nabla_X Y + h(X, Y), \tag{1.8}$$

$$\tilde{\nabla}_X N = -A_N X + \nabla^{\perp}_X N, \tag{1.9}$$

$\forall\ X, Y \in TM, N \in T^{\perp}M, h$ is the 2nd fundamental form and A_N is the Weingarten mapping associated with N via

$$g(A_N X, Y) = g(h(X, Y), N). \tag{1.10}$$

The mean curvature H is given by

$$H = \frac{1}{k}\sum_{i=1}^{k} h(e_i, e_i), \tag{1.11}$$

where k is the dimension of M and $\{e_i\}_{i=1}^{k}$ is the local orthonormal frame on M.

For any $X \in \Gamma(TM)$,

$$\phi X = TX + FX, \tag{1.12}$$

where TX is the tangential component and FX is the normal component of ϕX.

Similarly, for any $V \in \Gamma(T^{\perp}M)$,

$$\phi V = tV + fV, \tag{1.13}$$

where tV, fV are the tangential component and the normal component of ϕV respectively.

The covariant derivatives of the tensor fields T, F, t, f are defined as−

$$(\nabla_X T)Y = \nabla_X TY - T\nabla_X Y, \tag{1.14}$$

$$(\nabla_X F)Y = \nabla^{\perp}_X FY - F\nabla_X Y, \tag{1.15}$$

$$(\nabla_X t)V = \nabla_X tV - t\nabla^{\perp}_X V, \tag{1.16}$$

$$(\nabla_X f)V = \nabla^{\perp}_X fV - f\nabla^{\perp}_X V \tag{1.17}$$

$\forall\ X, Y \in TM, V \in T^{\perp}M$.

A submanifold is called−

i) invariant if $\forall\ X \in \Gamma(TM), \phi X \in \Gamma(TM)$,
ii) anti-invariant if $\forall\ X \in \Gamma(TM), \phi X \in \Gamma(T^{\perp}M)$,
iii) totally umbilical if $h(X, Y) = g(X, Y)H$ (1.18)
$\forall\ X, Y \in \Gamma(TM), H$ is the mean curvature,
iv) totaly geodesic if $h(X, Y) = 0\ \forall\ X, Y \in \Gamma(TM)$,
v) minimal if $H = 0$ on M.

Let M be a Riemannian manifold isometrically immersed in an almost contact metric manifold $(\tilde{M}, \phi, \xi, \eta, g)$ and ξ be tangent to M. Then the tangent bundle TM decomposes as $TM = D \oplus < \xi >$, where D is the orthogonal distribution to ξ. Now for each non-zero vector X tangent to M at x, such that X is not proportional to ξ_x, we denote the angle between ϕX and D_x by $\theta(X)$. M is called slant submanifold if the angle $\theta(X)$ is constant, which is independent of the choice of $x \in M$ and $X \in T_xM - < \xi_x >$. The constant angle $\theta \in [0, \frac{\pi}{2}]$ is then called the slant angle of M in $\tilde{M}$. If $\theta = 0$, then the submanifold is invariant, if $\theta = \frac{\pi}{2}$, then the submanifold is anti-invariant and if $\theta \neq 0, \frac{\pi}{2}$, then the submanifold is proper slant.

According to A. Lotta [9], when M is a proper slant submanifold of $\tilde{M}$ with slant angle θ, then $\forall\ X \in \Gamma(TM)$,

$$T^2(X) = -cos^2\theta(X - \eta(X)\xi). \tag{1.19}$$

A. Carriazo [3] introduced hemi-slant submanifolds as a special case of bislant submanifolds and he called them pseudo-slant submanifolds.

A submanifold M of an $(LCS)_n$-manifold is called hemi-slant if there exist two orthogonal distributions D^θ and $D^\perp$ satisfying [5]$-$
i) $TM = D^\theta \oplus D^\perp \oplus < \xi >$,
ii) D^θ is a slant distribution with slant angle $\theta \neq \frac{\pi}{2}$,
iii) $D^\perp$ is totally real i.e., $\phi D^\perp \subseteq T^\perp M$.

A hemi-slant submanifold is called proper if $\theta \neq 0, \frac{\pi}{2}$.

CR-submanifolds and slant submanifolds are hemi-slant submanifolds with slant angle $\theta = \frac{\pi}{2}$ and $D^\theta = 0$ respectively.

In the rest of this paper, we use M as a hemi-slant submanifold of an $(LCS)_n$-manifold $\tilde{M}$. If we denote the dimensions of the distributions $D^\perp$ and D^θ by m_1, m_2 respectively, then we have$-$
i) if $m_2 = 0$, then M is anti-invariant,
ii) if $m_1 = 0, \theta = 0$, then M is invariant,
iii) if $m_1 = 0, \theta \neq 0$, then M is proper-slant with slant angle θ,
iv) if $m_1 m_2 \neq 0, \theta \in (0, \frac{\pi}{2})$, then M is proper hemi-slant.

Let M be hemi-slant submanifold of an $(LCS)_n$-manifold $\tilde{M}$, then for any $X \in TM$,

$$X = P_1X + P_2X + \eta(X)\xi, \tag{1.20}$$

where P_1, P_2 are projection maps on the distributions $D^\perp, D^\theta$ respectively. Now operating ϕ on (1.20), we get

$$\phi X = \phi P_1X + \phi P_2X + \eta(X)\phi\xi.$$

Using (1.1) and (1.12), we obtain

$$TX + FX = FP_1X + TP_2X + FP_2X.$$

On comparing, we get

$$TX = TP_2X,$$
$$FX = FP_1X + FP_2X.$$

If we denote the orthogonal complement of $\phi(TM)$ in $T^{\perp}M$ by μ, then the normal bundle $T^{\perp}M$ can be decomposed as

$$T^{\perp}M = F(D^{\perp}) \oplus F(D^{\theta}) \oplus < \mu > . \tag{1.21}$$

Since $F(D^{\perp})$ and $F(D^{\theta})$ are orthogonal distributions, $g(X, Y) = 0$ for each $X \in D^{\perp}$ and $Y \in D^{\theta}$. Hence by (1.5) and (1.12), we have

$$\forall\ Z \in D^{\perp}, W \in D^{\theta},\ g(FZ, FW) = g(\phi Z, \phi W) = g(Z, W) = 0,$$

which shows that $F(D^{\perp}), F(D^{\theta})$ are mutually perpendicular. So, (1.21) is an orthogonal direct decomposition.

There are various types of works done on hemi-slant submanifolds. H. I. Abutuqayqah worked on geometry of hemi-slant submanifolds of almost contact manifolds [1]. M. A. Khan et al. discussed about totally umbilical hemi-slant submanifolds of Kahler manifolds [2] and of cosymplectic manifolds [4], and they also discussed about a classification on totally umbilical proper slant and hemi-slant submanifolds of a nearly trans-Sasakian manifold [6]. B. Laha et al. studied totally umbilical hemi-slant submanifolds of LP-Sasakian manifold [7] and hemi-slant submanifold of Kenmotsu manifold [10]. H. M. Tastan et al. discussed about hemi-slant submanifolds of a locally product Riemannian manifold [12] and of a locally conformal Kahler manifold [13]. Another important works on hemi-slant submanifolds were done by A. Lotta in 1996 [9], by M. A. Lone et al. in 2016 [8] and by M. S. Siddesha et al. in 2018 [11]. Motivated from these works, in this paper, we analyse some properties regarding distributions and leaves of hemi-slant submanifold of $(LCS)_n$-manifold.

2. MAIN RESULTS

In this section, we discuss about some necessary and sufficient conditions for distributions to be integrable and obtain some results in this direction. We also study the geometry of leaves of hemi-slant submanifold of $(LCS)_n$-manifold.

Theorem 2.1. *Let M be a hemi-slant submanifold of an $(LCS)_n$-manifold $\tilde{M}$, then* $\forall\ Z, W \in D^{\perp},\ A_{\phi W}Z = A_{\phi Z}W - \alpha\eta(W)Z - \alpha\eta(Z)W - 2\alpha\eta(Z)\eta(W)\xi.$

Proof. On using (1.10), we have

$$g(A_{\phi W}Z, X) = g(h(Z, X), \phi W) = g(\phi h(Z, X), W) = g(\phi\tilde{\nabla}_X Z, W) - g(\phi\nabla_X Z, W)$$

$= g(\phi\tilde{\nabla}_X Z, W) = g(\tilde{\nabla}_X \phi Z, W) - g((\tilde{\nabla}_X \phi)Z, W)$.

Again using (1.6) and (1.9), we get
$g(A_{\phi W}Z, X) = g(A_{\phi Z}X + \nabla^{\perp}_X \phi Z, W) - \alpha g(g(X,Z)\xi + 2\eta(X)\eta(Z)\xi + \eta(Z)X, W)$
$= g(A_{\phi Z}X, W) - \alpha g(X,Z)\eta(W) - 2\alpha\eta(X)\eta(Z)\eta(W) - \alpha\eta(Z)g(X,W)$
$= g(h(W,X), \phi Z) - \alpha g(X,Z)\eta(W) - \alpha\eta(Z)g(X,W) - 2\alpha\eta(X)\eta(Z)\eta(W)$
$= g(A_{\phi Z}W - \alpha\eta(W)Z - \alpha\eta(Z)W - 2\alpha\eta(Z)\eta(W)\xi, X)$
$\Rightarrow A_{\phi W}Z = A_{\phi Z}W - \alpha\eta(W)Z - \alpha\eta(Z)W - 2\alpha\eta(Z)\eta(W)\xi$.

Theorem 2.2. *Let M be a hemi-slant submanifold of an $(LCS)_n$-manifold $\tilde{M}$. Then the distribution $D^\theta \oplus D^\perp$ is integrable if and only if $g([X,Y],\xi) = 0\ \forall\ X, Y \in D^\theta \oplus D^\perp$.*

Proof. For $X, Y \in D^\theta \oplus D^\perp$,
$g([X,Y],\xi) = g(\tilde{\nabla}_X Y, \xi) - g(\tilde{\nabla}_Y X, \xi)$
$= -g(\tilde{\nabla}_X \xi, Y) + g(\tilde{\nabla}_Y \xi, X)$
$= -g(\alpha\phi X, Y) + g(\alpha\phi Y, X)$
$= 0$. (by (1.4))

Since $TM = D^\theta \oplus D^\perp \oplus <\xi>$, therefore $[X,Y] \in D^\theta \oplus D^\perp$. So, $D^\theta \oplus D^\perp$ is integrable.

Conversely, let $D^\theta \oplus D^\perp$ is integrable. Then $\forall\ X, Y \in D^\theta \oplus D^\perp, [X,Y] \in D^\theta \oplus D^\perp$. As $TM = D^\theta \oplus D^\perp \oplus <\xi>$, therefore $g([X,Y],\xi) = 0$.

Theorem 2.3. *Let M be a hemi-slant submanifold of an $(LCS)_n$-manifold $\tilde{M}$. Then the anti-invariant distribution $D^\perp$ is integrable if and only if $\forall\ W \in D^\perp, W$ is a scalar multiple of ξ.*

Proof. For $Z, W \in D^\perp$, from (1.6), we have

$$(\tilde{\nabla}_Z \phi)W = \alpha[g(Z,W)\xi + 2\eta(Z)\eta(W)\xi + \eta(W)Z]. \tag{2.1}$$

After some calculations and using (1.12), (1.13), we get

$$-A_{FW}Z + \nabla^{\perp}_Z FW - T\nabla_Z W - F\nabla_Z W - th(Z,W) - fh(Z,W) = \alpha[g(Z,W)\xi + 2\eta(Z)\eta(W)\xi + \eta(W)Z]. \tag{2.2}$$

Comparing tangential components, we have

$$-A_{FW}Z - T\nabla_Z W - th(Z,W) = \alpha[g(Z,W)\xi + 2\eta(Z)\eta(W)\xi + \eta(W)Z]. \tag{2.3}$$

Interchanging Z, W, we obtain

$$-A_{FZ}W - T\nabla_W Z - th(W,Z) = \alpha[g(W,Z)\xi + 2\eta(W)\eta(Z)\xi + \eta(W)Z]. \tag{2.4}$$

Subtracting (2.3) from (2.4) and using the fact that h is symmetric, we have

$$A_{FW}Z - A_{FZ}W + T(\nabla_Z W - \nabla_W Z) = \alpha[\eta(Z)W - \eta(W)Z]. \tag{2.5}$$

From (2.5), we have

$$A_{FW}Z - A_{FZ}W + T([Z,W]) = \alpha[\eta(Z)W - \eta(W)Z]. \tag{2.6}$$

Now $D^\perp$ is integrable if and only if $[Z,W] \in D^\perp$ and as $D^\perp$ is anti-invariant, $\phi D^\perp \subseteq T^\perp M$ and so, $T[Z,W] = 0$.

Hence from (2.6), $D^\perp$ is integrable if and only if $A_{FW}Z - A_{FZ}W = \alpha[\eta(Z)W - \eta(W)Z]$.

From Theorem 2.1, we have as $TW = 0 = TZ$,
$A_{\phi W}Z - A_{\phi Z}W = -\alpha\eta(W)Z - \alpha\eta(Z)W - 2\alpha\eta(Z)\eta(W)\xi$
$\Rightarrow \alpha[\eta(Z)W - \eta(W)Z] = -\alpha\eta(W)Z - \alpha\eta(Z)W - 2\alpha\eta(Z)\eta(W)\xi$
$\Rightarrow 2\alpha\eta(Z)W + 2\alpha\eta(Z)\eta(W)\xi = 0$
$\Rightarrow \eta(Z)W + \eta(Z)\eta(W)\xi = 0$
$\Rightarrow W + \eta(W)\xi = 0$. Hence the result is proved.

Theorem 2.4. *Let M be a hemi-slant submanifold of an $(LCS)_n$-manifold $\tilde{M}$. Then the slant distribution D^θ is integrable if and only if $\forall\ X, Y \in D^\theta$,*

$$P_1(\nabla_X TY - \nabla_Y TX) = \alpha[\eta(Y)P_1X - \eta(X)P_1Y]. \tag{2.7}$$

Proof. We denote by P_1, P_2 the projections on $D^\perp, D^\theta$ respectively. $\forall\ X, Y \in D^\theta$, we have from (1.6),

$$(\tilde{\nabla}_X\phi)Y = \alpha[\tilde{g}(X,Y)\xi + 2\eta(X)\eta(Y)\xi + \eta(Y)X]. \tag{2.8}$$

On applying (1.8), (1.9), (1.12), (1.13), we have

$$(\tilde{\nabla}_X\phi)Y = \nabla_X TY + h(X,TY) - A_{FY}X + \nabla_X FY - (T\nabla_X Y + F\nabla_X Y) - (th(X,Y) + fh(X,Y)) = \alpha[g(X,Y)\xi + 2\eta(X)\eta(Y)\xi + \eta(Y)X]. \tag{2.9}$$

Comparing tangential components, we get

$$\nabla_X TY - A_{FY}X - T\nabla_X Y - th(X,Y) = \alpha[g(X,Y)\xi + 2\eta(X)\eta(Y)\xi + \eta(Y)X]. \tag{2.10}$$

Interchanging X, Y in (2.10) and subtracting the resultant from (2.10), we obtain

$$\nabla_X TY - \nabla_Y TX - A_{FY}X + A_{FX}Y - T\nabla_X Y + T\nabla_Y X = \alpha[\eta(Y)X - \eta(X)Y]. \tag{2.11}$$

Since $X, Y \in D^\theta, FX = 0 = FY$, applying P_1 to both sides of (2.11), we have

$$P_1(\nabla_X TY - \nabla_Y TX) = \alpha[\eta(Y)P_1X - \eta(X)P_1Y].$$

Theorem 2.5. *Let M be a hemi-slant submanifold of an $(LCS)_n$-manifold $\tilde{M}$. If the leaves of $D^\perp$ are totally geodesic in M, then $\forall\ X \in D^\theta$ and $Z, W \in D^\perp$,*

$$g(h(Z,X), FW) + g(th(Z,W), X) = 0. \tag{2.12}$$

Proof. From (1.6), (1.8), (1.9), we have
$\nabla_Z \phi W + h(Z, \phi W) - A_{FW} Z + \nabla^{\perp}_Z FW - \phi \nabla_Z W - \phi h(Z, W)$
$= \alpha[g(Z, W)\xi + 2\eta(W)\eta(Z)\xi + \eta(W)Z]$.

Comparing tangential components and on taking inner product with $X \in D^{\theta}$, we obtain

$$-g(A_{FW}Z, X) - g(th(Z, W), X) - g(T\nabla_Z W, X) = 0.$$

The leaves of $D^{\perp}$ are totally geodesic in M if for $Z, W \in D^{\perp}, \nabla_Z W \in D^{\perp}$. So, $T\nabla_Z W = 0$.

Thus $g(A_{FW}Z, X) + g(th(Z, W), X) = 0$.

Example. Now we give an example of a hemi-slant submanifold of an $(LCS)_n$-manifold.

Let $\tilde{M}(\mathbb{R}^9, \phi, \xi, \eta, g)$ denote the manifold $\mathbb{R}^9$ with the (LCS)-structure given by–
$\xi = 3\frac{\partial}{\partial z}, \eta = \frac{1}{3}(-dz + \sum_{i=1}^{4} b^i da^i)$,
$g = \frac{1}{9}\sum_{i=1}^{4}(da^i \otimes da^i \oplus db^i \otimes db^i) - \eta \otimes \eta$,
$\phi(\frac{\partial}{\partial z}) = 0, \phi(\frac{\partial}{\partial a^i}) = \frac{\partial}{\partial b^i}, i = 1, 2, 3, 4$, and
$\phi(\frac{\partial}{\partial b^i}) = \frac{\partial}{\partial a^i}$ for $i = 1, 2$ and $\phi(\frac{\partial}{\partial b^i}) = -\frac{\partial}{\partial a^i}$ for $i = 3, 4$,
where $(a^1, a^2, a^3, a^4, b^1, b^2, b^3, b^4, z) \in \mathbb{R}^9$.

Let us consider a 5-dimensional submanifold M of $\tilde{M}$ defined by
$(a^1, a^2, a^3, a^4, b^1, b^2, b^3, b^4, z) \mapsto (cos\alpha a^1 + sin\alpha a^2, cos\beta b^1 + sin\beta b^2, \frac{a^3 - b^3}{\sqrt{3}}, \frac{a^4 - b^4}{\sqrt{3}}, 3z)$.

Then it can be easily proved that M is a hemi-slant submanifold of $\tilde{M}$ by choosing the slant distribution $D_{\theta} =< e_1, e_2 >$ with slant angle $|\alpha - \beta|$ and the totally real distribution $D^{\perp} =< e_3, e_4 >$, where $e_1 = sin\alpha\frac{\partial}{\partial a^1} - cos\alpha\frac{\partial}{\partial a^2}$, $e_2 = sin\beta\frac{\partial}{\partial b^1} - cos\beta\frac{\partial}{\partial b^2}$, $e_3 = \frac{\partial}{\partial a^3} + \frac{\partial}{\partial b^3}$, $e_4 = \frac{\partial}{\partial a^4} + \frac{\partial}{\partial b^4}$ such that $\{e_1, e_2, e_3, e_4, \xi\}$ forms an orthogonal frame on TM so that $TM = D_{\theta} \oplus D^{\perp} \oplus < \xi >$.

Acknowledgement. The first author is the corresponding author and has been sponsored by University Grants Commission (UGC) Junior Research Fellowship, India. *UGC-Ref. No.*: 1139/(CSIR-UGC NET JUNE 2018). The authors would like to thank the referee for the valuable suggestions to improve the paper.

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