

DYNAMIC PRICING POLICY USING PRESERVATION TECHNOLOGY WITH STOCK AND PRICE SENSITIVE DEMAND

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Abstract. This paper aims to develop a dynamic pricing policy for deteriorating items with price and stock dependent demand. In declining market demand of items decreases with respect to time and also after a duration items get outdated. In this situation it needs a pricing policy to sale the items before end season. The proposed dynamic pricing policy is applicable for a limited period to clease the stock. Policy decision regarding the selling price could aggressively attracts the costumers. Objectives are to maximize the profit/revenue, pricing strategy and economic order level for such a stock dependent and price sensitive items. We are giving numerical example and simulation to illustrate the proposed model.

Key words and Phrases: Inventory, stock and price sensitive dependent demand, preservation technology, optimal profit.

1. INTRODUCTION

Inventory management that maintains an adequate balance between demand and supply is an essential need for any business. The nature of items plays an important role in inventory management. The items, for this purpose, can be affected by and at the same time can affect several logistical and market forces like production, demand, transportation, price, etc. Further, the life of items and the nature of deterioration of item characteristics also play a very important role. Some inventory items are perishable over long duration of time, while some perish over small time interval. In addition, inventory problem is also affected by the inflation rate, shortage of items, order size, price fluctuations, discount strategies

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and marketing plans. In the present e-shopping environment, customers have many options to compare the similar products on the basis of merits, price, and according to place of order. In this process one can save expenditure on shopping and time both. The price reduction with e-shopping is a tool to generate the excess demand of item. In super market, presence of larger quantity of item attracts the customer to buy more items; this phenomenon reveals that stock dependent demand also can be a tool to generate the demand. Pricing strategy is a tool that attracts customer to buy more. Within (1955) was first to integrate the concept of inventory theory to the concept of price theory and simultaneously determined the price prospect and order quantity of a retailer. In a situation when all other assumptions of the EOQ model are valid, he took demand as a price sensitive parameter. You (2005) picked up the problem of joint determination of order size and optimal price for the perishable inventory system subjected to condition that when demand is exponentially decreasing and price sensitive. It is with the assumptions that a decision maker gets an optimal level to adjust price before entering into next time interval to influence demand and revenue both. Taleizadeh *et al.* (2015) revisited the idea of replenishment frequency after each replenishment cycle and production rate in vendor managed inventory system with deterioration. Internal process of dengue virus transmission in the human body studied by Nuraini *et al.* (2007). They developed a mathematical model and analyzed the dynamic of dengue virus and supported by a numerical example.

Son and Ikuta (2007) dealt with a problem of selecting profitable orders to accept out of sequentially arriving customers in a custom production company where only one customer is allowed to be held. The delay cost would be paid for every order not completed up to its due date. Inventory cost plays a vital role in inventory management. Sarkar and Sarkar (2013a) expanded an inventory model for deteriorating items with stock-dependent demand. This is applicable in which time varying backlogging rate and time varying deterioration rate. They determined the optimal cycle length of each product such that the expected total cost including holding, shortage, ordering, deterioration and opportunity cost. Further, they provided the necessary and sufficient conditions for existence and uniqueness of the optimal solution some of the literature are giving in references like Sarkar and Sarkar (2013b, c), Sarkar and Goswami(2012), Tjandra *et al.* (2015), Sarkar *et al.* (2011), Masjkur, *et al.* (2018) Taleizadeh *et al.*(2015), Khedlekar and Shukla (2013).

Expenditure sources like ordering cost, safety, lead time and numbers of lots are the integral parts of decision making. An integrated inventory model focusing on these issues has been discussed by Lo (2007). Skouri and Papachristos (2002) also attempted the temporary price reductions to resellers when manufacturers, distributors, wholesalers are offer trade promotion. Khedlekar *et al.* (2013) computed the time periods before and after disrupt a production system. Further useful contributions on pricing theory, are due to Chen and Chen (2007).

The EOQ model designed by Hou and Lin (2006) reflects how a demand pattern which is price and stock dependent affects the discount in cash. You (2007)

investigated a perishable inventory problem wherein a firm purchases and prices a service or a perishable item before a sales season and sells it through a reservation system during the sales season. Taking the cancellation phenomenon and the effect of waiting time on demand into account, he developed two continuous time models to respectively solve a problem with a time independent refund. A thought from the literature survey appears rarely considered stock dependent with price sensitive demand in research contributions. In view of above literature, we have motivated to design a dynamic model by considering price sensitive and stock dependent demand. We assumed the market is in decline nature and that's why we are reducing the selling price and generate the excess demand to increase the revenue. We have divided the sale session into n equal parts and find out the replenishment size and number of price settings.

2. ASSUMPTIONS AND NOTATIONS

Suppose in an inventory system where a company purchases the items before starting of a sales season and sales it to the customers in time horizon L which is divided into n equal parts i.e. $T = L/n$. The inventory managers decides to change the selling price n times in n equal intervals $[0, T], [T, 2T] \cdots [(j-1)T, jT] \cdots [(n-1)T, nT]$ and the respective prices are $p_1, p_2, \cdots p_j \cdots p_n$ in decreasing order. Suppose $\alpha > 0$ is the intersection of the demand curve (initial demand) of the item and if η is parameter of contribution of stock in demand which is proportional to it and β is parameter contribution of price in demand. If at time t on hand inventory is $I_j(t)$ and the demand of the item is $d_j(p, t) = \alpha - \beta p_j + \eta I_j(t)$.

If sold quantity of items are $Q(j)$ for period the index j . If there is some cost occurs to change the selling price once like advertisers and displaying the change in price in super market say which is A , and c is unit purchasing cost. The holding cost of inventory is h unit per unit time. It is assumed that demand increases accordingly reduction in selling price and proposition to on hand inventory $I_j(t)$. The following notations are used to formulate the proposed model.

- $\lambda(\alpha)$ preservation technology function ($\lambda(\alpha) = \lambda_0 e^{-\delta\alpha}$)
- L sale season,
- Q order quantity,
- η parameter of stock dependent demand $\eta > 0$,
- n number of permissible price settings,
- T length of period for one price setting, $T = L/n$,
- j period j refers the time interval $[(j-1)T, jT]$,
- c unit purchasing cost,
- A price setting cost per price change,
- p_j j^{th} selling price of product,
- $d_j(p, t)$ demand of product in j^{th} interval $[(j-i)t, jt]$.

3. THE MATHEMATICAL MODEL

The rate of decay in inventory due to demand $(\alpha - \beta p_j + \eta I_j(t) + \lambda(\alpha) I_j(t))$ in the interval $[0, T]$, satisfied the following differential equation.

$$\frac{d}{dt} I_j(t) = -(\alpha - \beta p_j + \eta I_j(t) + \lambda(\alpha) I_j(t)) \tag{1}$$

where $0 \leq t \leq T$ and boundary condition $I_j(0) = I_{j-1}(T)$

On hand inventory denoted by $I_j(t)$ at time t expressed as following equation

$$I_j(t) = -q_j e^{(\eta + \lambda(\alpha))t} - \frac{\alpha - \beta p_j}{\eta} (1 - e^{(\eta + \lambda(\alpha))t})$$

Sold items $q_j(t)$ at j^{th} period is

$$q_j(t) = -x^{-1} q_{j-1} - \frac{\alpha - \beta p_j - 1}{\eta + \lambda(\alpha)} (1 - x^{-1}), \text{ where } x = e^{-(\eta + \lambda(\alpha))T}$$

If $q_1 = Q$, then above equation leads to

$$q_j(t) = x^{-(j-1)} Q - \frac{\alpha}{\eta + \lambda(\alpha)} (1 - x^{-(j-1)}) + \beta (1 - x^{-1}) \sum_{i=2}^j (x^{-(j-i)}) p_{(i-1)} \tag{2}$$

Sale quantity of stock in interval $[(j - 1)T, jT]$ is just difference of q_j and q_{j+1} , where $j = 1, 2, 3, \dots, n$

$$\delta q_j = q_j - q_{j+1}$$

$$\begin{aligned} \delta q_j &= x^j (x - 1) (\alpha + (\eta + \lambda(\alpha)) Q) - \frac{\beta (1 - x^{-1}) p_j}{\eta + \lambda(\alpha)} \\ &+ \frac{\beta}{\eta + \lambda(\alpha)} (1 - x^{-1})^2 \left(\sum_{i=1}^{j-1} x^{-(j-i-1)} (1 - x^{-1}) p_{i-1} \right) \end{aligned} \tag{3}$$

$$\begin{aligned} I_j(t) &= e^{(\eta + \lambda(\alpha))t} \left(x^{-(j-1)} Q - \frac{\alpha (1 - x^{-(j-1)})}{(\eta + \lambda(\alpha))} + \frac{(1 - x^{-1}) \beta (\sum_{i=2}^j x^{-(j-i)} p_{(i-1)})}{\eta} \right) \\ &- \frac{(\alpha - \beta p_j)}{\eta} (1 - e^{(\eta + \lambda(\alpha))t}) \end{aligned} \tag{4}$$

Boundary condition $I_{j-1}(t) = 0$, leads to

$$Q(n) = \frac{1}{(\eta + \lambda(\alpha))} \left(\alpha (x^n - 1) - \beta (1 - x^{-1}) \sum_{i=1}^n p_i + \beta (1 - x^{-1}) \sum_{i=2}^n x^{-(i-1)} p_{i-1} \right) \tag{5}$$

Revenue $R(n)$ when price changes n times over time L is

$$R(n) = (x - 1) \left(\frac{\alpha}{(\eta + \lambda(\alpha))} \right) \sum_{j=1}^n p_j + \frac{\beta (1 - x^{-1})^2}{(\eta + \lambda(\alpha))} \left(\sum_{j=2}^n \sum_{i=1}^{j-1} p_i p_j \right) - \frac{\beta}{\eta + \lambda(\alpha)} \sum_{i=1}^n p_i^2 \tag{6}$$

Holding cost $H(n)$ over time L is

$$H(n) = \sum_{j=1}^n \int_0^T hI_j(t)dt$$

Profit function $F(n, \mathbf{p})$ is the difference of Revenue and total incurred cost

$$F(n, \mathbf{p}) = R(n) - H(n) - cQ(n) - nA \quad (7)$$

4. SPECIAL CASES

If $n = 1$

If firm is not permit to change the selling price in entire season. It means the business is running with only one price in sale season $[0, L]$. That is $n = 1$ and $\mathbf{p}=(p_1)$. on putting $n = 1$ in equation (3.7) leads to

$$\begin{aligned} \frac{\partial}{\partial p_1} F(1, \mathbf{p}) &= -\frac{2\beta}{\eta + \lambda(\alpha)} p_1 + \frac{x-1}{(\eta + \lambda(\alpha))^2} ((\alpha + c\beta)(\eta + \lambda(\alpha)) + h\beta) \\ &\quad - \frac{hT\beta}{\eta + \lambda(\alpha)} = 0, \end{aligned} \quad (8)$$

$$\frac{\partial^2}{\partial p_1^2} F(1, \mathbf{p}) = \frac{-2\beta}{\eta} \leq 0, \quad p_1 = \frac{\alpha(\eta + \lambda(\alpha)) + c\beta\eta + h}{(\eta + \lambda(\alpha))} + \frac{hT}{2(1-x)}$$

and equation (3.4) leads to

$$Q(1) = \frac{\beta Th}{2(\eta)} + \frac{(x-1)(\alpha\eta - c(\eta + \lambda(\alpha))\beta - h)}{2(\eta + \lambda(\alpha))^2} \quad (9)$$

If $n = 2$

If firm decide to change the selling price once over the entire season L . It means the business is running with two prices p_1 and p_2 in intervals $[0, T]$ and in $[T, 2T]$, where $T = L/2$. That is $n = 2$ and $\mathbf{p}=(p_1, p_2)$, thus on putting $n = 2$ the equation (3.7) leads to

$$\frac{\partial}{\partial p_1} F(2, \mathbf{p}) = 0, \quad \frac{\partial}{\partial p_2} F(2, \mathbf{p}) = 0$$

Above two equations evaluates two prices p_1 and p_2 in decereasing order. To obtain optimization conditions we have find followings

$$A_{11} = \frac{\partial^2}{\partial p_1^2} F(2, \mathbf{p}), \quad A_{12} = \frac{\partial^2}{\partial p_1 \partial p_2} F(2, \mathbf{p})$$

$$A_{21} = \frac{\partial^2}{\partial p_2 \partial p_1} F(2, \mathbf{p}), \quad A_{22} = \frac{\partial^2}{\partial p_2^2} F(2, \mathbf{p})$$

and Hessian matrix H_{22} is

$$H_{22} = \begin{pmatrix} \frac{\partial^2 F(2, \mathbf{p})}{\partial p_1 \partial p_1} & \frac{\partial^2 F(2, \mathbf{p})}{\partial p_1 \partial p_2} \\ \frac{\partial^2 F(2, \mathbf{p})}{\partial p_1 \partial p_2} & \frac{\partial^2 F(2, \mathbf{p})}{\partial p_2 \partial p_2} \end{pmatrix}.$$

and quantity $Q(2)$ is

$$Q(2) = \frac{\beta Th}{2(\eta + \lambda(\alpha))} + \frac{2(x - 1)(\beta h + c\beta(\eta + \lambda(\alpha)) - \alpha\eta)}{2(\eta + \lambda(\alpha))^2} + \frac{hT\beta}{(\eta + \lambda(\alpha))} \quad (10)$$

If $n = 3$

If firm decide to change the price twice that is in this case $n = 3$, and three prices p_1, p_2, p_3 are applied in the intervals $[0, T], [T, 2T]$ and $[2T, 3T]$ respectively, where $T = L/3$ and $\mathbf{p} = (p_1, p_2, p_3)$. Equation (3.7) leads for $n = 3$,

$$\frac{\partial}{\partial p_1} F(3, \mathbf{p}) = 0, \frac{\partial}{\partial p_2} F(3, \mathbf{p}) = 0, \frac{\partial}{\partial p_3} F(3, \mathbf{p}) = 0$$

Above three equations evaluates three prices p_1, p_2 and p_3 . To obtain optimization conditions we have to find followings.

$$A_{11} = \frac{\partial^3}{\partial p_1^3} F(3, \mathbf{p}), A_{12} = \frac{\partial^3}{\partial p_1 \partial p_2^2} F(3, \mathbf{p}), A_{13} = \frac{\partial^3}{\partial p_1 \partial p_3^2} F(3, \mathbf{p}),$$

$$A_{21} = \frac{\partial^3}{\partial p_1^2 \partial p_2} F(3, \mathbf{p}), A_{22} = \frac{\partial^3}{\partial p_2^3} F(3, \mathbf{p}), A_{23} = \frac{\partial^3}{\partial p_2 \partial p_3^2} F(3, \mathbf{p}),$$

$$A_{31} = \frac{\partial^3}{\partial p_1^3} F(3, \mathbf{p}), A_{32} = \frac{\partial^3}{\partial p_1 \partial p_1 \partial p_2} F(3, \mathbf{p}), A_{33} = \frac{\partial^3}{\partial p_3^3} F(3, \mathbf{p}).$$

and to find rest third derivatives of $F(3, \mathbf{p})$, and $Q(3)$ could be found by equation (3.5) and equation (3.7) . Thus the Hessian matrix H_{33} is,

$$H_{33} = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}.$$

5. NUMERICAL EXAMPLE

Suppose the parameters are $\alpha = 900, L = 90, \eta = 0, \beta = 0.2, h = 0.005$. If charge for one price change is $A = 1000$, then on applying model output for $n = 1$ the selling price p_1 is 55.12 in interval $[0, 120]$, and order quantity $Q(1) = 1698$ and respective profit of the system $F_1(1, p) = 7285$. If firm has to decide that there will be two selling prices in the time intervals $(0, 45]$ and $(45, 90]$, then selling price $p_1 = 58.43$, and selling price $p_2 = 51.80$, quantity will be $Q(2) = 2034$, and respective profit will be $F_2(2, p) = 7882$. If firm has to decide to take three selling prices $p_1 = 59.72, p_2 = 55.00, p_3 = 50.85$. The order quantity $Q(3) = 2113$, respective profit $F_3 = 6688$. In this example we observed that the optimum profit for $n = 2$ is maximum than $n = 1$ and $n = 3$. Hence the optimum value exists at $n = 2$.

6. SENSITIVE ANALYSIS

Table 1: Sensitivity analysis parameter(β)in demand

β	n	p_1	p_2	p_3	Q	F(n, \mathbf{P})
0.2	1	287.60	-	-	2278	598511
0.2	2	311.00	264.22	-	3751	607160
0.2	3	319.00	287.82	257.50	3763	607750
0.4	1	175.10	-	-	2269	12190
0.4	2	182.25	167.96	-	2304	12991
0.4	3	184.66	175.32	1666.7	2311	607750
0.6	1	137.60	-	-	845	9497
0.6	2	139.34	138.87	-	859	8637
0.6	3	139.92	137.82	136.4	861	5050

Discussion

To simulate the model we use the data set as per second date in table 1, and set the selling price once, twice and thrice simultaneously. For $n = 1, 2, 3$, therefore no change, one change and twice change in selling price over sale season L . The selling price p_1, p_2 and p_3 are in $[0, T], (T, 2T], (2T, 3T]$ optimal quantity is $Q(3)$ and profit is F_3 are given in table 1. For $\beta=0.02$, the profits for $n = 2$ is higher than $n = 1$ and $n = 3$. Optimal profit and consumed quantity are very sensitive on parameter β which are increasing in terms of β increasing, higher initial demand is flexible to put higher selling price initially. However profit and quantity are higher at $n = 3$ and $p_1 > p_2 > p_3$ in decreasing order to sell the entire stock. For rest value of $\beta=0.2, 0.4$ and 0.6 the profits are increases number of price setting n increases. So it is still continue to investigate to find optimal price settings and respective profit.

Table 2: Sensitivity analysis for parameter purchasing cost (c)

c	n	p_1	p_2	p_3	Q	F(n, \mathbf{P})
350	1	456.30	-	-	6533	693025
350	2	506.30	406.23	-	7280	771298
350	3	524.45	456.27	388.20	7439	787148
400	1	481.27	-	-	4996	404804
400	2	519.53	443.01	-	5566	450149
400	3	533.40	481.27	429.20	5688	787148
450	1	506.27	-	-	3458	193458
450	2	532.75	479.79	-	3853	214659
450	3	542.35	506.27	470.30	3938	218158
500	1	531.27	-	-	1921	58986
500	2	545.97	516.57	-	2140	64829
500	3	551.30	531.27	511.30	64971	64971

Discussion

From table 2, we observed that the p_1, p_2, p_3 are increases as well as unit purchasing cost (c) increases this is a strength of the proposed model. This characteristic could be explained as follow. First to note that if unit purchasing cost increases the profit

reduced accordingly, so to coop up this amount of revenue, proposed model set the higher selling prices p_1, p_2, p_3 increases accordingly. For $c = 350$, the profit is gradually increases for $n = 1$, $n = 2$ and $n = 3$, and this phenomenon followed for $c = 400$, $c = 450$, and $c = 500$.

7. CONCLUSION

In this paper we have developed a dynamic pricing policy and effect of parameters analyzed by computing the selling prices and respective output. It is advised to management keep Longer length of sale season (L) that will provides more number of price settings because of proposed model is highly sensitive on this parameter (L). Unit purchasing cost (c) negatively affects the business, because of that model provides higher selling prices and revenues accordingly. If β increases means some higher amounts deduces from α , and thus demand decreases therefore profit become less, to coop up this amount of losses, model sets higher selling price accordingly to uplift the profit. This is the strength of the proposed model. Sufficient conditions for optimality as concaveness shown in the model and analyzed the model parameters. On substituting $\eta = 0$, the model converted for only price sensitive demand, and on putting $\beta = 0$, it converted into only stock dependent model. Theoretical and analytical evidence indicates the existence of an optimal number of changes in the value of selling prices for achieving an optimal profit in any business setup.

One can reformulated the model with variable holding cost and as well as variable deterioration. Also One can divide the sale season in unequal length of time intervals and determine the respective selling prices.

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