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COMMON-EDGE SIGNED GRAPH OF A SIGNED GRAPH

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Abstract. A *Smarandachely k-signed graph* (*Smarandachely k-marked graph*) is an ordered pair $S = (G, \sigma)$ $(S = (G, \mu))$ where $G = (V, E)$ is a graph called *underlying graph of* S and $\sigma : E \to (\overline{e}_1, \overline{e}_2, ..., \overline{e}_k)$ $(\mu : V \to (\overline{e}_1, \overline{e}_2, ..., \overline{e}_k))$ is a function, where each $\overline{e}_i \in \{+, -\}$. Particularly, a Smarandachely 2-signed graph or Smarandachely 2-marked graph is abbreviated a *signed graph* or a *marked graph*. The *common-edge graph* of a graph $G = (V, E)$ is a graph $C_E(G) = (V_E, E_E)$, where $V_E = \{A \subseteq V; |A| = 3$, and A is a connected set$\}$ and two vertices in V_E are adjacent if they have an edge of G in common. Analogously, one can define the *common-edge signed graph* of a signed graph $S = (G, \sigma)$ as a signed graph $C_E(S) = (C_E(G), \sigma')$, where $C_E(G)$ is the underlying graph of $C_E(S)$, where for any edge (e_1e_2, e_2e_3) in $C_E(S)$, $\sigma'(e_1e_2, e_2e_3) = \sigma(e_1e_2)\sigma(e_2e_3)$. It is shown that for any signed graph S, its common-edge signed graph $C_E(S)$ is balanced. Further, we characterize signed graphs S for which $S \sim C_E(S)$, $S \sim L(S)$, $S \sim J(S)$, $C_E(S) \sim L(S)$ and $C_E(S) \sim J(S)$, where $L(S)$ and $J(S)$ denotes line signed graph and jump signed graph of S respectively.

Key words and Phrases: Smarandachely k-signed graphs, Smarandachely k-marked graphs, balance, switching, common-edge signed graph, line signed graph, jump signed graph.

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Abstrak. Sebuah *graf bertanda-k Smarandachely* (*Smarandachely k-marked graph*) adalah sebuah pasangan terurut $S = (G, \sigma)$ $(S = (G, \mu))$ dimana $G = (V, E)$ adalah *graf pokok (underlying graph) dari* S dan $\sigma : E \to (\overline{e}_1, \overline{e}_2, ..., \overline{e}_k)$ $(\mu : V \to (\overline{e}_1, \overline{e}_2, ..., \overline{e}_k))$ adalah sebuah fungsi, dimana tiap $\overline{e}_i \in \{+, -\}$. Kemudian, sebuah graf bertanda-k Smarandachely disingkat dengan sebuah graf bertanda *marked graph*. *Graf sekutu-sisi* dari sebuah graf $G = (V, E)$ adalah sebuah graf $C_E(G) = (V_E, E_E)$, dimana $V_E = \{A \subseteq V; |A| = 3$, dan A adalah sebuah himpunan terhubung$\}$ dan dua titik di V_E bertetangga jika mereka mempunyai sebuah sisi sekutu di G. Secara analog, kita dapat mendefinisikan *graf bertanda sekutu-sisi* dari sebuah graf bertanda $S = (G, \sigma)$ sebagai sebuah graf bertanda $C_E(S) = (C_E(G), \sigma')$, dimana $C_E(G)$ adalah graf pokok dari $C_E(S)$, dimana untuk suatu sisi (e_1e_2, e_2e_3) di $C_E(S)$, $\sigma'(e_1e_2, e_2e_3) = \sigma(e_1e_2)\sigma(e_2e_3)$. Pada paper ini, akan ditunjukkan bahwa untuk setiap graf bertanda S, graf bertanda sekutu-sisi $C_E(S)$ adalah seimbang. Lebih jauh, kami mengkarakterisasi graf bertanda S untuk $S \sim C_E(S)$, $S \sim L(S)$, $S \sim J(S)$, $C_E(S) \sim L(S)$ dan $C_E(S) \sim J(S)$, dimana $L(S)$ dan $J(S)$ masing-masing menyatakan graf bertanda garis dan graf bertanda lompat dari S.

Kata kunci: Graf bertanda-k Smarandachely, seimbang, pertukaran, graf bertanda sekutu-sisi, graf bertanda garis, graf bertanda lompat.

1. Introduction

Unless mentioned or defined otherwise, for all terminology and notion in graph theory the reader is to refer to [7]. We consider only finite, simple graphs free from self-loops.

A *Smarandachely k-signed graph* (*Smarandachely k-marked graph*) is an ordered pair $S = (G, \sigma)$ $(S = (G, \mu))$ where $G = (V, E)$ is a graph called *underlying graph of* S and $\sigma : E \to (\overline{e}_1, \overline{e}_2, ..., \overline{e}_k)$ $(\mu : V \to (\overline{e}_1, \overline{e}_2, ..., \overline{e}_k))$ is a function, where each $\overline{e}_i \in \{+, -\}$. Particularly, a Smarandachely 2-signed graph or Smarandachely 2-marked graph is called abbreviated a *signed graph* or a *marked graph*. A *signed graph* is an ordered pair $S = (G, \sigma)$, where $G = (V, E)$ is a graph called *underlying graph of* S and $\sigma : E \to \{+, -\}$ is a function. A signed graph $S = (G, \sigma)$ is *balanced* if every cycle in S has an even number of negative edges (See [8]). Equivalently, a signed graph is balanced if product of signs of the edges on every cycle of S is positive.

A *marking* of S is a function $\mu : V(G) \to \{+, -\}$; A signed graph S together with a marking μ is denoted by S_μ.

The following characterization of balanced signed graphs is well known.

Proposition 1.1. (**E. Sampathkumar** [10]) *A signed graph $S = (G, \sigma)$ is balanced if and only if there exists a marking μ of its vertices such that each edge uv in S satisfies $\sigma(uv) = \mu(u)\mu(v)$.*

The idea of switching a signed graph was introduced by Abelson and Rosenberg [1] in connection with structural analysis of marking μ of a signed graph S. Switching S with respect to a marking μ is the operation of changing the sign of every edge of S to its opposite whenever its end vertices are of opposite signs. The signed graph obtained in this way is denoted by $\mathcal{S}_\mu(S)$ and is called *μ-switched signed graph* or just *switched signed graph*. Two signed graphs $S_1 = (G, \sigma)$ and $S_2 = (G', \sigma')$ are said to be *isomorphic*, written as $S_1 \cong S_2$ if there exists a graph isomorphism $f : G \to G'$ (that is a bijection $f : V(G) \to V(G')$ such that if uv is an edge in G then $f(u)f(v)$ is an edge in G') such that for any edge $e \in G$, $\sigma(e) = \sigma'(f(e))$. Further a signed graph $S_1 = (G, \sigma)$ *switches* to a signed graph $S_2 = (G', \sigma')$ (or that S_1 and S_2 are *switching equivalent*) written $S_1 \sim S_2$, whenever there exists a marking μ of S_1 such that $\mathcal{S}_\mu(S_1) \cong S_2$. Note that $S_1 \sim S_2$ implies that $G \cong G'$, since the definition of switching does not involve change of adjacencies in the underlying graphs of the respective signed graphs.

Two signed graphs $S_1 = (G, \sigma)$ and $S_2 = (G', \sigma')$ are said to be *weakly isomorphic* (see [17]) or *cycle isomorphic* (see [18]) if there exists an isomorphism $\phi : G \to G'$ such that the sign of every cycle Z in S_1 equals to the sign of $\phi(Z)$ in S_2. The following result is well known (See [18]):

Proposition 1.2. (**T. Zaslavasky** [18]) *Two signed graphs S_1 and S_2 with the same underlying graph are switching equivalent if and only if they are cycle isomorphic.*

2. Common-edge Signed Graph of a Signed Graph

In [4], the authors define *path graphs* $P_k(G)$ of a given graph $G = (V, E)$ for any positive integer k as follows: $P_k(G)$ has for its vertex set the set $\mathcal{P}_k(G)$ of all distinct paths in G having k vertices, and two vertices in $\mathcal{P}_k(G)$ are adjacent if they represent two paths $P, Q \in \mathcal{P}_k(G)$ whose union forms either a path P_{k+1} or a cycle C_k in G.

Much earlier, the same observation as above on the formation of a line graph $L(G)$ of a given graph G, Kulli [9] had defined the *common-edge graph* $C_E(G)$ of G as the *intersection graph* of the family $\mathcal{P}_3(G)$ of 2-paths (i.e., paths of length two) each member of which is treated as a set of edges of corresponding 2-path; as shown by him, it is not difficult to see that $C_E(G) \cong L^2(G)$, for any isolate-free graph G, where $L(G) := L^1(G)$ and $L^t(G)$ denotes the t^{th} *iterated line graph* of G for any integer $t \geq 2$.

In this paper, we extend the notion of $C_E(G)$ to realm of signed graphs: Given a signed graph $S = (G, \sigma)$ its *common-edge signed graph* $C_E(S) = (C_E(G), \sigma')$ is that signed graph whose underlying graph is $C_E(G)$, the common-edge graph of G, where for any edge (e_1e_2, e_2e_3) in $C_E(S)$, $\sigma'(e_1e_2, e_2e_3) = \sigma(e_1e_2)\sigma(e_2e_3)$. This differs from the common-edge signed graph defined in [15].

Further a signed graph is a common-edge signed graph if there exists a signed graph S' such that $S \cong C_E(S')$.

Proposition 2.1. *For any signed graph $S = (G, \sigma)$, its common-edge signed graph $C_E(S)$ is balanced.*

Proof. Let $\sigma^{'}$ denote the signing of $C_E(S)$ and let the signing σ of S be treated as a marking of the vertices of $C_E(S)$. Then by definition of $C_E(S)$ we see that $\sigma'(e_1e_2, e_2e_3) = \sigma(e_1e_2)\sigma(e_2e_3)$, for every edge (e_1e_2, e_2e_3) of $C_E(S)$ and hence, by Proposition 1.1, the result follows. □

For any signed graph $S = (G, \sigma)$, its common edge signed graph is balanced. However the converse need not be true. The following result gives a sufficient condition for a signed graph to be a common-edge signed graphs.

Theorem 2.2. *A connected signed graph $S = (G, \sigma)$ is a common-edge signed graph if there exists a consistent marking μ of vertices of S such that for any edge uv, $\sigma(uv) = \mu(u)\mu(v)$ and its underlying graph G is a common-edge graph. Conversely if S is a common edge signed graph, then S is balanced.*

Proof. Suppose that there exists a consistent marking μ of vertices of S such that for any edge uv, $\sigma(uv) = \mu(u)\mu(v)$ and G is a common-edge graph. Then there exists a graph H such that $C_E(H) \cong G$. Now consider the signed graph $S' = (L(H), \sigma')$, where for any edge $e = (uv, vw)$ in $L(H)$, $\sigma'(e)$ is the marking of the corresponding vertex uvw in $C_E(H) = G$. Then S' is balanced since the edges in any cycle C of S' which corresponds to a cycle in S and the marking μ is a consistent marking. Thus S' is a line signed graph. That is there exists a signed graph S'' such that $S'' \cong L(S')$. Then clearly $C_E(S) \cong S''$.

Conversely, suppose that $S = (G, \sigma)$ is a common edge signed graph. That is there exists a signed graph $S' = (G', \sigma')$ such that $C_E(S) \cong S'$. Consider $L(S') = (L(G'), \sigma'')$ where $\sigma''(uv, vw) = \sigma(uv)\sigma(vw)$. Now consider the marking $\mu : V(G) \to \{+, -\}$ defined by $\mu(uvw) = \sigma''(uv, vw)$. Then by definition for any edge$e = (uvw, vwx)$ in S, where $uv, vw, wx \in E(G')$, $\sigma(e) = \sigma'(uv)\sigma'(wx) = \sigma'(uv)\sigma'(vw)\sigma'(vw)\sigma'(wx) = \sigma''(uv, vw)\sigma''(vw, wx) = \mu(uvw)\mu(vwx)$. Hence by Proposition 1.1, S is balanced. □

For any positive integer k, the k^{th} iterated common-edge signed graph, $C_E^k(S)$ of S is defined as follows:

$$C_E^0(S) = S,\ C_E^k(S) = C_E(C_E^{k-1}(S))$$

Corollary 2.3. *For any signed graph $S = (G, \sigma)$ and any positive integer k, $C_E^k(S)$ is balanced.*

In [15], the author characterized those graphs that are isomorphic to their corresponding common-edge graphs.

Proposition 2.4. (**D. Sinha** [15]) *For a simple connected graph $G = (V, E)$, $G \cong C_E(G)$ if and only if G is a cycle.*

We now characterize those signed graphs that are switching equivalent to their common-edge signed graphs.

Proposition 2.5. *For any signed graph $S = (G, \sigma)$, $S \sim C_E(S)$ if and only if S is a balanced signed graph which is 2-regular.*

Proof. Suppose $S \sim C_E(S)$. This implies, $G \cong C_E(G)$ and hence by Proposition 2.4, we see that the graph G is 2-regular. Now, if S is any signed graph with underlying graph as 2-regular, Proposition 2.1 implies that $C_E(S)$ is balanced and hence if S is unbalanced and its common-edge signed graph $C_E(S)$ being balanced can not be switching equivalent to S in accordance with Proposition 1.2. Therefore, S must be balanced.

Conversely, suppose that S balanced 2-regular signed graph. Then, since $C_E(S)$ is balanced as per Proposition 2.1 and since $G \cong C_E(G)$ by Proposition 2.4, the result follows from Proposition 1.2 again. □

Corollary 2.6. *For any signed graph $S = (G, \sigma)$ and for any positive integer k, $S \sim C_E^k(S)$ if and only if S is a balanced signed graph which is 2-regular.*

3. Line Signed Graphs

The *line graph* $L(G)$ of graph G has the edges of G as the vertices and two vertices of $L(G)$ are adjacent if the corresponding edges of G are adjacent. The *line signed graph* of a signed graph $S = (G, \sigma)$ is a signed graph $L(S) = (L(G), \sigma')$, where for any edge ee' in $L(S)$, $\sigma'(ee') = \sigma(e)\sigma(e')$. This concept was introduced by M. K. Gill [6] (See also E. Sampathkumar et al. [12, 13]).

Proposition 3.1. (**M. Acharya** [2]) *For any signed graph $S = (G, \sigma)$, its line signed graph $L(S)$ is balanced.*

For any positive integer k, the k^{th} iterated line signed graph, $L^k(S)$ of S is defined as follows:

$$L^0(S) = S,\ L^k(S) = L(L^{k-1}(S))$$

Corollary 3.2. (**P. Siva Kota Reddy & M. S. Subramanya** [16]) *For any signed graph $S = (G, \sigma)$ and for any positive integer k, $L^k(S)$ is balanced.*

We now characterize those signed graphs that are switching equivalent to their line signed graphs.

Proposition 3.3. *For any signed graph $S = (G, \sigma)$, $S \sim L(S)$ if and only if S is a balanced signed graph which is 2-regular.*

Proof. Suppose $S \sim L(S)$. This implies, $G \cong L(G)$ and hence G is 2-regular. Now, if S is any signed graph with underlying graph as 2-regular, Proposition 3.1 implies that $L(S)$ is balanced and hence if S is unbalanced and its line signed graph $L(S)$ being balanced can not be switching equivalent to S in accordance with Proposition 1.2. Therefore, S must be balanced.

Conversely, suppose that S is balanced 2-regular signed graph. Then, since $L(S)$ is balanced as per Proposition 3.1 and since $G \cong L(G)$, the result follows from Proposition 1.2 again. □

Corollary 3.4. *For any signed graph $S = (G, \sigma)$ and for any positive integer k, $S \sim L^k(S)$ if and only if S is a balanced signed graph which is 2-regular.*

Proposition 3.5. (**D. Sinha** [15])
For a connected graph $G = (V, E)$, $L(G) \cong C_E(G)$ if and only if G is cycle or $K_{1,3}$.

Theorem 3.6. *For any graph G, $C_E(G) \cong L^k(G)$ for some $k \geq 3$, if and only if G is either a cycle or $K_{1,3}$.*

Proof. Suppose that $C_E(G) \cong L^k(G)$ for some $k \geq 3$. Since $C_E(G) \cong L^2(G)$, we observe that $L^k(G) = L^{k-2}(L^2(G)) = L^{k-2}(C_E(G))$ and so $C_E(G) \cong L^{k-2}(C_E(G))$. Hence, by Proposition 3.5, $C_E(G)$ must be a cycle. But for any graph G, $L(G)$ is a cycle if and only if G is either cycle or $K_{1,3}$. Since $K_{1,3}$ is a forbidden to line graph and $L(G)$ is a line graph, $G \neq K_{1,3}$. Hence $L(G)$ must be a cycle. Finally $L(G)$ is a cycle if and only if G is either a cycle or $K_{1,3}$.

Conversely, if G is a cycle C_r , of length r, $r \geq 3$ then for any $k \geq 2$, $L^k(G)$ is a cycle and if $G = K_{1,3}$ then for any $k \geq 2$, $L^k(G) = C_3$. Since $C_E(G) = L^2(G)$, $C_E(G) = L^k(G)$ for any $k \geq 3$. This completes the proof. □

We now characterize those line signed graphs that are switching equivalent to their common-edge signed graphs.

Proposition 3.7. *For any signed graph $S = (G, \sigma)$, $L(S) \sim C_E(S)$ if and only if G is a cycle or $K_{1,3}$.*

Proof. Suppose $L(S) \sim C_E(S)$. This implies, $L(G) \cong C_E(G)$ and hence by Proposition 3.5, we see that the graph G must be isomorphic to either 2-regular or $K_{1,3}$.

Conversely, suppose that G is a cycle or $K_{1,3}$. Then $L(G) \cong C_E(G)$ by Proposition 3.5. Now, if S any signed graph on any of these graphs, By Propositions 2.1 and 3.1, $C_E(S)$ and $L(S)$ are balanced and hence, the result follows from Proposition 1.2. □

Corollary 3.8. *For any signed graph $S = (G, \sigma)$ and for any integers $k \geq 3$, $C_E(S) \sim L^k(S)$ if and only if G is 2-regular.*

4. Jump Signed Graphs

The *jump graph* $J(G)$ of a graph $G = (V, E)$ is $\overline{L(G)}$, the complement of the line graph $L(G)$ of G (See [5] and [7]).The *jump signed graph* of a signed graph $S = (G, \sigma)$ is a signed graph $J(S) = (J(G), \sigma')$, where for any edge ee' in $J(S)$, $\sigma'(ee') = \sigma(e)\sigma(e')$. This concept was introduced by M. Acharya and D. Sinha [3] (See also E. Sampathkumar et al. [11]).

Proposition 4.1. (**M. Acharya and D.Sinha** [3])
For any sigraph $S = (G, \sigma)$, its jump sigraph $J(S)$ is balanced.

For any positive integer k, the k^{th} iterated jump signed graph, $J^k(S)$ of S is defined as follows:

$$J^0(S) = S,\ J^k(S) = J(J^{k-1}(S))$$

Corollary 4.2. *For any signed graph $S = (G, \sigma)$ and for any positive integer k, $J^k(S)$ is balanced.*

In the case of graphs the following result is due to Simic [14] (see also [5]) where $H \circ K$ denotes the *corona* of graphs H and K [7].

Proposition 4.3. (**S. K. Simic** [14])
The jump graph $J(G)$ of a graph G is isomorphic with G if and only if G is either C_5 or $K_3 \circ K_1$.

Lemma 4.4. (Kulli [9]**)**
For a graph $G = (V, E)$ with n vertices and m edges, the number of vertices in $L^2(S)$ is $\sum_{u \in V} \binom{deg(v)}{2}$

Lemma 4.5. (D. Sinha [15]**)**
For any simple connected graph $G = (V, E)$ on $n \geq 2$ vertices,

$$|E(G)| = \sum_{v \in V} \binom{deg(v)}{2}$$

if and only if G is a cycle or a 3-spider.

Proposition 4.6. *For a connected graph $G = (V, E)$, $J(G) \cong C_E(G)$ if and only if G is C_5.*

Proof. Suppose that $J(G) \cong C_E(G)$. Then the number of vertices in $J(G)$ must be equal to the number of vertices in $C_E(G)$. By Lemma 4.4, the number of vertices in $C_E(G)$ is $\sum_{u \in V} \binom{deg(v)}{2}$. Now, since both $J(G)$ and $L(G)$ have same number of

vertices whence by Lemma 4.5, G must either be a cycle or a 3-spider.
We note that $L^2(G) \cong C_E(G)$ and $J(G) = \overline{L(G)}$. Hence $\overline{J(L(G))} \cong L(G)$. By Proposition 4.3, it follows that $L(G)$ is either C_5 or K_3oK_1. Now, $L(G) \neq K_{1,3}$, since $K_{1,3}$ is not a line graph. Hence $G \cong C_5$. The converse is obvious.

□

We now characterize those jump signed graphs that are switching equivalent to their common-edge signed graphs.

Proposition 4.7. *For any signed graph $S = (G, \sigma)$, $J(S) \sim C_E(S)$ if and only if $G \cong C_5$.*

Proof. Suppose $J(S) \sim C_E(S)$. This implies, $J(G) \cong C_E(G)$ and hence by Proposition 4.6, we see that $G \cong C_5$.

Conversely, suppose $G \cong C_5$. Then $J(G) \cong C_E(G)$ by Proposition 4.6. Now, if S is a signed graph with underlying graph as C_5, by Propositions 2.1 and 4.1, $C_E(S)$ and $J(S)$ are balanced and hence, the result follows from Proposition 1.2. □

The following result is a stronger form of the above result.

Theorem 4.8. *A connected graph satisfies $J(S) \cong C_E(S)$ if and only if G is C_5.*

Proof. Clearly $C_E(C_5) \cong J(C_5)$. Consider the map $f : V(C_E(G)) \rightarrow V(J(G))$ defined by $f(u_1u_2u_3, u_2u_3u_4) = (u_1u_2, u_3u_4)$ is an isomorphism. Let σ be any signing C_5 . Let $e = (v_1v_2v_3, v_2v_3v_4)$ be an edge in $C_E(C_5)$. Then sign of the edge e in $C_E(G)$ is the $\sigma(u_1u_2)\sigma(u_3u_4)$ which is the sign of the edge (u_1u_2, u_3u_4) in $J(C_5)$. Hence the map f is also a signed graph isomorphism between $J(S)$ and $C_E(S)$. □

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