J. Indones. Math. Soc.
Special Edition (2011), pp. 59–69.

ON THE SUPER EDGE-MAGIC DEFICIENCY AND α-VALUATIONS OF GRAPHS

Rikio Ichishima[1] and Akito Oshima[2]

[11]**College of Humanities and Sciences, Nihon University, 3-25-40 Sakurajousui Setagaya-ku, Tokyo 156-8550, Japan, ichishim@chs.nihon-u.ac.jp**

Department of Mathematical Information Science, Faculty of Science, Tokyo University of Science, 1-3 Kagurazaka Shinjuku-ku, Tokyo 162-8601, Japan, akito_o@rs.kagu.tus.ac.jp

Abstract. A graph G is called super edge-magic if there exists a bijective function $f : V(G) \cup E(G) \to \{1, 2, \ldots, |V(G)| + |E(G)|\}$ such that $f(u) + f(v) + f(uv)$ is a constant for each $uv \in E(G)$ and $f(V(G)) = \{1, 2, \ldots, |V(G)|\}$. The super edge-magic deficiency, $\mu_s(G)$, of a graph G is defined as the smallest nonnegative integer n with the property that the graph $G \cup nK_1$ is super edge-magic or $+\infty$ if there exists no such integer n. In this paper, we prove that if G is a graph without isolated vertices that has an α-valuation, then $\mu_s(G) \leq |E(G)| - |V(G)| + 1$. This leads to $\mu_s(G) = |E(G)| - |V(G)| + 1$ if G has the additional property that G is not sequential. Also, we provide necessary and sufficient conditions for the disjoint union of isomorphic complete bipartite graphs to have an α-valuation. Moreover, we present several results on the super edge-magic deficiency of the same class of graphs. Based on these, we propose some open problems and a new conjecture.

Key words: Super edge-magic labeling, super edge-magic deficiency, sequential labeling, sequential number, α-valuation.

2000 Mathematics Subject Classification: Primary 05C78.
Received: 09-08-2011, revised: 09-09-2011, accepted: 04-12-2012.