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An Economic Order Quantity Model for Flawed Units With Quality Screening and Time Dependent Backlogging

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Abstract. This study introduces an Economic Order Quantity (EOQ) model that addresses inventory systems dealing with imperfect quality items, incorporating both quality screening processes and time-dependent backlogging. Recognizing that a proportion of received items may be defective, the model integrates a screening mechanism to identify and separate flawed units before they reach customers. Additionally, the model considers a backlogging scenario where unmet demand is partially backordered, with the backlogging rate being a function of the waiting time until the next replenishment. The objective is to determine the optimal order quantity and backordering level that minimize the total cost, which includes ordering, holding, screening, backordering and shortage costs. Analytical solutions are derived and numerical examples are provided to illustrate the model's applicability. Sensitivity analyses are conducted to examine the impact of key parameters on the optimal solution.

Key words and Phrases: Flawed units, quality screening, time dependent backlogging and partially backordered.

1. INTRODUCTION

Economic Order Quantity (EOQ) models often assume that all items received are of perfect quality and that any shortages are either completely backordered or result in lost sales. However, in real-world scenarios, a certain proportion of items may be defective, and customers willingness to wait for backordered items can vary over time. To address these practical considerations, researchers have developed EOQ models that incorporate imperfect quality items, inspection processes and time-dependent backlogging. Incorporating imperfect quality into EOQ models acknowledges that not all items received are usable, necessitating inspection or screening processes to identify and separate defective units. Additionally, customer

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behaviour regarding backorders is not static. The longer customers have to wait for a backordered item, the less likely they are to complete the purchase, leading to time-dependent backlogging. This dynamic necessitates models that can account for the decreasing rate of backorders over time.

Industries such as electronics manufacturing, textiles and apparel, pharmaceuticals and chemicals, automotive components, and food processing are leveraging advanced models to enhance quality control, rework strategies, and inventory management. In electronics manufacturing, particularly in printed circuit board assembly, these models help manage inspections and rework processes to reduce production defects. The textile and apparel sector benefits by addressing fabric defects and stitching errors, optimizing rework decisions, and managing inventory to meet changing demand. Pharmaceutical and chemical industries utilize these models to oversee quality control in batch productions, reprocess defective items, and make informed decisions considering product perishability. Automotive component manufacturers apply these strategies to handle imperfections in parts, incorporate rework plans, and manage customer expectations during shortages. In food processing, the focus is on dealing with spoilage and quality variations, implementing screening and rework where feasible, and managing time-sensitive backorders due to product perishability. Collectively, these industries are adopting innovative approaches to enhance operational efficiency and meet quality standards.

This study aims to develop an EOQ model that simultaneously considers the presence of defective items requiring quality screening and the time-dependent nature of backlogging. By integrating these factors, the model seeks to provide a more realistic and practical approach to inventory management, minimizing total costs associated with ordering, holding, screening and shortages.

Mandeep Mittal and Chandra K. Jaggi [1] developed the EOQ model for decaying and poor-quality products. To prevent having imperfect products in the lot, Rezaei and Salimi [2] created an economic order quantity model per little worth items. This strategy seeks to maximize the amount that a consumer is ready to wage a dealer. Jaber et al.[3] introduced an entropic economic order quantity model for products of inadequate value. The above-mentioned research projects make the assumption that the flawed items are removed from inventory also offered for sale at a reduced cost after the screening period.

The inventory model established by Salameh and Jaber [4] was modified by Jaber et al. [5] by allowing defective goods to be repaired locally or sold on a subordinate marketplace. Paul et al. [6] studied the effect of the percentage of defective units on the collection procedure. Modak et al. [7] established a model that limits the optimum just in time cushion while accounting for protective conservation and the likelihood of damaged goods. Taleizadeh et al. [8] deliberate the repurchase process, screening procedure, and ordering and price decisions within a supply chain context for low-quality goods. An EOQ inventory model was integrated by Rezaei [9] with sampling screening plans for defective products. For faulty items, Alamri et al. [10] provided an effective inventory controller system. They essentially developed an economic order quantity model for goods of varying value by means of variable demand, imperfect products, a scrutiny procedure, and corrosion. Khan et al. [11] explored the impact of a seller accomplished inventory strategy in conjunction through a package ordinary agreement in a stock restraint over a solo supplier also one client. In this situation, the trader shows each manufacture lot in various lots to the customer's storeroom.

Lin [12] investigated the implications of several carbon due regimes taking place the operation of an inventory model through mixed quality goods, where buyer has influence over the seller. Rad et al. [13] recently identified manufacturing and distribution methods for defective items, when demand is driven by together retailing price also promotion.

The proposed Economic Order Quantity (EOQ) model, which incorporates flawed units with quality screening and time-dependent backlogging, offers significant advancements over the classical EOQ model and its traditional extensions.

TABLE 1. Comparison of Classical and Proposed EOQ Models

Feature	**Classical EOQ Model**	**Proposed EOQ Model**
Product Quality Assumption	Assumes all items are of perfect quality.	Accounts for a certain percentage of defective items in each lot, recognizing the reality of imperfect production processes.
Inspection Process	No inspection; all items are deemed acceptable.	Incorporates a quality screening process to identify and separate defective items, acknowledging potential inspection errors.
Rework of Defective Items	Not considered; defective items are not addressed.	Includes the possibility of reworking defective items to meet quality standards, reducing waste and improving resource utilization.

Feature	Classical EOQ Model	Proposed EOQ Model
Backlogging Approach	Assumes either complete backlogging or no backlogging during stockouts.	Introduces time-dependent partial backlogging, where the rate of backordering decreases as the waiting time increases, reflecting customer behaviour more accurately.
Demand Pattern	Assumes constant and continuous demand over time.	Accommodates time-varying demand patterns, including scenarios with seasonal fluctuations or deteriorating items.
Cost Considerations	Focuses on minimizing ordering and holding costs.	Expands cost analysis to include inspection costs, rework expenses, backordering penalties and potential lost sales due to defective items or stockouts.
Inventory Strategy	Simplistic approach with fixed order quantities and reorder points.	Develops a more dynamic inventory strategy that optimizes order quantities and cycle times by considering multiple real-world factors, leading to more efficient inventory management.

In summary, the proposed EOQ model offers a more nuanced and practical approach to inventory management by integrating factors often encountered in real-world operations but overlooked in classical models. This leads to more efficient inventory control, cost savings and improved customer satisfaction.
Implementing an EOQ model that accounts for flawed units, quality screening and time-dependent backlogging enables industries to make informed decisions regarding inventory management, quality control and customer service. By integrating these factors, businesses can enhance operational efficiency, reduce costs and improve overall supply chain performance.

The problem addressed in this study involves an Economic Order Quantity (EOQ) model that considers the presence of flawed units in an inventory system. In real-world production and procurement, not all received units are perfect; some may have defects that require screening before use. This model incorporates an quality screening process, which ensures that defective items are identified and handled appropriately. Additionally, time-dependent backlogging is considered, meaning that unmet demand can be partially fulfilled later, with the likelihood of backlogging decreasing over time. The objective is to determine the optimal order quantity that minimizes total costs, balancing procurement, holding and backlogging expenses while accounting for the presence of flawed items and the efficiency of the screening process.

2. ASSUMPTIONS AND NOTATIONS

2.1. **Assumptions.**

(1) On-hand inventory deteriorates by constant proportion $\theta(0 \leq \theta < 1)$.
(2) Demand rate stands predictable and persistent.
(3) Lead time stays predictable and persistent.
(4) The replacement is immediate.
(5) The screening process and demand occur simultaneously but the demand rate (D) is lower than the screening rate (μ), $D < \mu$.
(6) Flawed things occur in a specific batch size (Q).
(7) β: Percentage of defective item
(8) Shortages are permissible and during periods of stockouts, the backlogging rate varies and is depending upon the length of the wait for the next restocking. Specifically, the backorder rate is defined as $B(t) = \frac{1}{1+\delta(T-t)}$, where δ represents the backlogging parameter with a range of 0 to 1 and the parameter $(T-t)$ denotes the waiting time, where $t_2 \leq t \leq T$.

2.2. **Notations.**

(1) $I_1(t)$: Inventory level at time t, $0 \leq t \leq t_2$.
(2) $I_2(t)$: Inventory level at time t, $t_2 \leq t \leq T$.
(3) $Q^{'}$: Total order quantity
(4) Q: Initial inventory level
(5) T : Duration of the inventory cycle.
(6) t_2 : Period during which there is no lack of inventories.
(7) C_0 : Ordering price
(8) C_p : Purchasing price
(9) C_h : Holding price
(10) C_d : Deterioration price

(11) C_2 : Shortage price used for backlogged things
(12) C_3 : Lost sales price
(13) C_T : Total price
(14) δ : Backlogging parameter
(15) BI : Backlog inventory level
(16) S : Salvage price per defective unit, $S < C_p$
(17) α : Screening price

3. MAIN RESULTS

In this inventory system, Q items with a purchasing price C_p and an ordering price C_0 are used at the beginning of the period. There are β percent defective products in each lot.

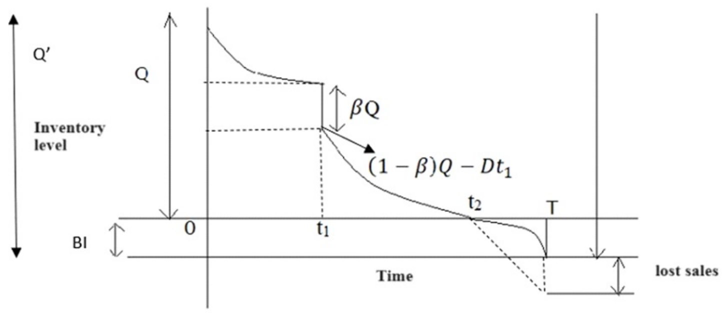

FIGURE 1. Graphic illustration of the structure

From 0 to t_1, the screening method applies to all received quantities by the screening rate μ, provided that the demand rate (D) is lower than the screening rate. The commodities that the screening procedure finds to be of perfect value satisfy demand, which emerges concurrently with the screening procedure (Dt_1). By time t_1, a group of defective items (βQ) is provided at a discounted price (s per unit). By time t_1, inventory level is $(1-\beta)Q - Dt_1$. By time t_2, demand and partial deterioration lead to the inventory level becoming zero.
Let $I_1(t)$ is an inventory level at time $t(0 \leq t \leq t_2)$, throughout the temporal period $[0, t_2]$, the differential equation is

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = -D, \qquad 0 \leq t \leq t_2 \tag{1}$$

with boundary conditions $t = 0, I_1(t) = Q$ and $t = t_1, I_1(t_1) = (1-\beta)Q - Dt_1$.

By solving (1), using boundary conditions $I_1(t) = Q$ when $t = 0$, we get the following solution

$$I_1(t) = Qe^{-\theta t} + \frac{D}{\theta}[e^{-\theta t} - 1], \qquad 0 \le t \le t_1 \tag{2}$$

Using boundary condition $I_1(t_1) = (1-\beta)Q - Dt_1$ when $t = t_1$, we get the following solution

$$I_1(t) = e^{\theta(t_1-t)}[(1-\beta)Q - Dt_1] + \frac{D}{\theta}[e^{\theta(t_1-t)} - 1], \quad t_1 \le t \le t_2 \tag{3}$$

and

$$Q = \frac{1}{1 - e^{\theta t_1}(1-\beta)}\Big[\frac{D}{\theta}(e^{\theta t_1} - 1) - Dt_1 e^{\theta t_1}\Big]. \tag{4}$$

Throughout the interval $[t_2, T]$, a scarcity happened, also demand was somewhat backlogged. Let $I_2(t)$ is an inventory level at time $t(t_2 \le t \le T)$. The differential equations is

$$\frac{dI_2(t)}{dt} = \frac{-D}{1 + \delta(T - t)}, \qquad t_2 \le t \le T \tag{5}$$

with boundary conditions $t = t_2, I_2(t) = 0$.
The result of (5) stands

$$I_2(t) = D(t_2 - t)[1 - \delta T + \frac{\delta}{2}(t_2 + t)]. \tag{6}$$

The maximum backordered inventory BI is reached at $t = T$.

$$BI = -I_2(t) = -D(t_2 - T)[1 + \frac{\delta}{2}(t_2 - T)]. \tag{7}$$

Thus the total order quantity(Q') throughout entire period $[0, T]$ is

$$Q' = Q + BI = \frac{1}{1 - e^{\theta t_1}(1-\beta)}\Big[\frac{D}{\theta}(e^{\theta t_1} - 1) - Dt_1 e^{\theta t_1}\Big] - D(t_2 - T)[1 + \frac{\delta}{2}(t_2 - T)]. \tag{8}$$

The Ordering Price is represented by C_0, which is a constant value.
Ordering Price $= C_0$
The Purchasing Price (PC) is calculated as the product of the unit purchasing price (C_p) and the total order quantity (Q').
Purchasing Price $(PC) = C_p Q'$

$$PC = C_p\Big\{\frac{1}{1 - e^{\theta t_1}(1-\beta)}\Big[\frac{D}{\theta}(e^{\theta t_1} - 1) - Dt_1 e^{\theta t_1}\Big] - D(t_2 - T)[1 + \frac{\delta}{2}(t_2 - T)]\Big\}. \tag{9}$$

The Screening Price is calculated as the product of the screening cost (α) and the order quantity (Q).
Screening Price $= \alpha Q$
The Salvage Value is determined by multiplying the salvage price (S) by the number of defective items (βQ).

Salvage value $= S\beta Q$
The Holding Price (HC) is calculated as the product of the holding cost rate (C_h) and the sum of the integrals of the inventory level function ($I_1(t)$) over the time intervals $[0, t_1]$,and $[t_1, t_2]$.
Holding Price $(HC) = C_h\Big[\int_0^{t_1} I_1(t)dt + \int_{t_1}^{t_2} I_1(t)dt\Big]$

$$HC = C_h\Big\{\frac{Q}{\theta}(1-e^{-\theta t_1}) - \frac{D}{\theta^2}[\theta t_1 + e^{-\theta t_1} - 1] + \frac{1}{\theta}[(1-\alpha)Q - Dt_1][1-e^{\theta(t_1-t_2)}] - \frac{D}{\theta^2}[e^{\theta(t_1-t_2)} + (t_1-t_2)\theta - 1]\Big\}. \tag{10}$$

Deterioration price $(DC) = C_d\Big\{Q - \int_0^t D(t)dt + \int_{t_1}^{t_2} D(t)dt - \beta Q\Big\}$

$$DC = C_d\Bigg\{(1-\beta)\Bigg[\frac{1}{1-e^{\theta t_1}(1-\beta)}\Big[\frac{D}{\theta}(e^{\theta t_1}-1) - Dt_1e^{\theta t_1}\Big]\Bigg] - Dt_2\Bigg\}. \tag{11}$$

Shortage Price $= -C_2\int_{t_2}^{T} I_2(t)dt$

$$= C_2D\Big[\frac{\delta}{3}(T^3 - t_2^3) - \frac{1}{2}(T^2 + t_2^2) - \delta t_2T(T-t_2) + Tt_2\Big] \tag{12}$$

Lost sales price $= C_3\int_{t_2}^{T}\Big[1 - \frac{1}{1+\delta(T-t)}\Big]Ddt$

$$= C_3\frac{\delta}{2}D(T-t_2)^2. \tag{13}$$

Total Price per cycle= Ordering price + Holding price + Purchase price + Deterioration price + Shortage price + Lost sales price + Screening price − Salvage value

$$\begin{aligned} C_T = C_0 &+ C_h\Big\{\frac{Q}{\theta}(1-e^{-\theta t_1}) - \frac{D}{\theta^2}[\theta t_1 + e^{-\theta t_1} - 1] + \frac{1}{\theta}[(1-\alpha)Q - Dt_1][1-e^{\theta(t_1-t_2)}] \\ &- \frac{D}{\theta^2}[e^{\theta(t_1-t_2)} + (t_1-t_2)\theta - 1]\Big\} + C_p\Big\{\frac{1}{1-e^{\theta t_1}(1-\beta)}\Big[\frac{D}{\theta}(e^{\theta t_1}-1) - Dt_1e^{\theta t_1}\Big] \\ &- D(t_2-T)[1 + \frac{\delta}{2}(t_2-T)]\Big\} + C_d\Bigg\{(1-\beta)\Bigg[\frac{1}{1-e^{\theta t_1}(1-\beta)}\Big[\frac{D}{\theta}(e^{\theta t_1}-1) - Dt_1e^{\theta t_1}\Big]\Bigg] \\ &- Dt_2\Bigg\} + C_2D\Big[\frac{\delta}{3}(T^3-t_2^3) - \frac{1}{2}(T^2+t_2^2) - \delta t_2T(T-t_2) + Tt_2\Big] \\ &+ C_3\frac{\delta}{2}D(T-t_2)^2 + \alpha Q - S\beta Q. \end{aligned} \tag{14}$$

Our purpose is to minimize the total price.

The necessary condition is $\frac{\partial C_T}{\partial t_2} = 0$ and $\frac{\partial^2 C_T}{\partial t_2^2} > 0$ for all $t_2 > 0$ We get

$$\frac{\partial C_T}{\partial t_2} = C_h\Big[e^{\theta(t_1-t_2)}[(1-\alpha)Q - Dt_1] - \frac{D}{\theta}e^{\theta(t_1-t_2)} + \frac{D}{\theta}\Big] + C_p[-D - D\delta t_2 + D\delta T]$$
$$-C_d D + C_2 D[2T\delta t_2 - t_2 - \delta t_2^2 - \delta T^2 + T] - C_3\delta D[T - t_2] = 0. \quad (15)$$

and

$$\frac{\partial^2 C_T}{\partial t_2^2} = C_h[Q\alpha\theta - \theta Q + D\theta t_1 - D] - C_p D\delta + 2C_2 DT\delta - 2C_2 D\delta t_2 + C_3\delta D > 0. \quad (16)$$

4. NUMERICAL EXAMPLES

The following numerical examples illustrate the application of a given mathematical model using specific input values. By substituting these values into the model, we compute key results, including t_2, Q' and C_T. These examples demonstrate how variations in input parameters influence the final outcomes, providing insights into the model's behaviour and its practical implications.

4.1. **Numerical Example 1:**

Let us Consider the following input values $[\theta, t_1, T, \alpha, \beta, \delta, D, S, C_0, C_d, C_h, C_p, C_2, C_3]$ $=[0.2, 1.5, 3, 0.2, 0.1, 0.0001, 30, 1, 2, 0.005, 1, 10, 5, 1]$. Then we get $t_2 = 2.6890, Q' = 47.7936$ and $C_T = 832.1161$.
In this scenario, with a moderate deterioration rate and a relatively low defective rate, the system achieves a balance between ordering and holding costs. The non-shortage period (t_2) indicates the duration within the cycle where inventory is available to meet demand without shortages. The optimal order quantity ensures that inventory levels are sufficient to cover demand while minimizing costs associated with ordering, holding, and shortages.

4.2. **Numerical Example 2:**

Let us Consider the following input values $[\theta, t_1, T, \alpha, \beta, \delta, D, S, C_0, C_d, C_h, C_p, C_2, C_3]$ $=[0.4, 2, 4, 0.1, 0.2, 0.0005, 50, 2, 3, 0.01, 5, 20, 10, 2]$. Then we get $t_2 = 3.6192, Q' = 107.9138$ and $C_T = 4031.5$.
In this case, the higher deterioration rate and increased demand necessitate a larger optimal order quantity to meet customer needs and compensate for inventory losses due to deterioration. The extended non-shortage period reflects the need to maintain inventory availability over a longer cycle. Consequently, the total cost is significantly higher, driven by increased holding costs, higher penalty costs for shortages and more substantial screening and defective item costs.

5. SENSITIVE ANALYSIS

Sensitivity analysis examines how changes in input parameters affect the output of a mathematical model. By varying one or more parameters while keeping others constant, we can identify which factors have the most significant impact on the results. This analysis helps in understanding the robustness of the model, optimizing decision-making and assessing the reliability of predictions. In the given numerical examples, sensitivity analysis can be performed to observe how alterations in parameters like θ, D and T influence key outcomes such as t_2, Q' and C_T, providing valuable insights into system behaviour.

Based on Table 2 and Figure 2, the analysis demonstrates that as the deterioration rate increases:

(1) The non-shortage period (t_2) decreases.
(2) The optimal order quantity ($Q^{'}$) increases.
(3) The total cost (C_T) decreases.

These findings highlight the importance of accounting for deterioration rates in inventory management to optimize ordering strategies and minimize costs.

TABLE 2. Variation in deterioration rate (θ)

θ	t_2	$Q^{'}$	C_T
0.18	2.8825	44.2454	891.1516
0.19	2.7806	46.0746	857.3326
0.2	2.6890	47.7936	832.1161
0.21	2.6062	49.4048	813.4958
0.22	2.5310	50.9128	799.9815

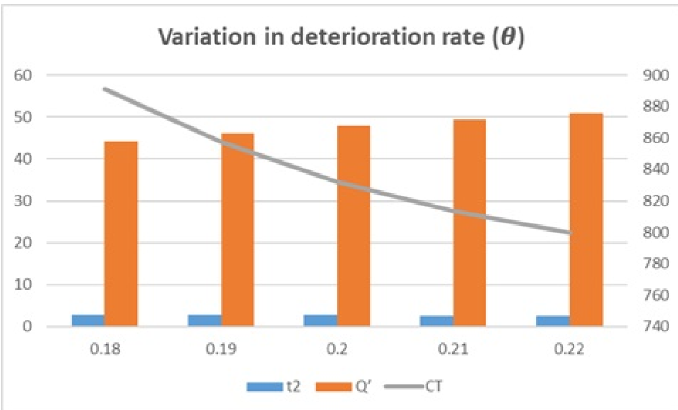

FIGURE 2. Variation in deterioration rate (θ)

Based on Table 3 and Figure 3, the sensitivity analysis demonstrates that while the non-shortage period (t_2) remains unchanged, both the optimal order quantity ($Q^{'}$) and total cost (C_T) escalate with increasing demand rates. This underscores the importance of adjusting order quantities in response to demand fluctuations to maintain cost-effective inventory management.

TABLE 3. Variation in the demand rate (D)

D	t_2	$Q^{'}$	C_T
10	2.6890	15.9312	278.7054
20	2.6890	31.8624	555.4107
30	2.6890	47.7936	832.1161
40	2.6890	63.7248	1108.8
50	2.6890	79.6561	1385.5

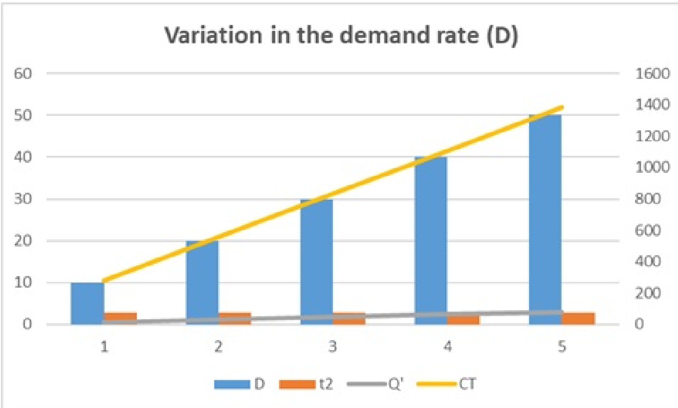

FIGURE 3. Variation in the demand rate (D)

Based on Table 4 and Figure 4, the analysis demonstrates that increasing the cycle length (T) leads to:

(1) Longer non-shortage periods (t_2)
(2) Higher optimal order quantities ($Q^{'}$) and
(3) Elevated total costs (C_T)

These findings underscore the trade-off between ordering frequency and inventory holding costs. While longer cycles reduce ordering frequency, they necessitate larger order quantities and incur higher holding costs, culminating in increased total costs. Therefore, determining the optimal cycle length is crucial for balancing these factors to minimize overall inventory costs.

Table 4. Variation in length of the cycle (T)

T	t_2	Q'	C_T
2.6	2.3503	45.9547	722.8645
2.8	2.5196	46.8757	777.7408
3.0	2.6890	47.7936	832.1161
3.2	2.8583	48.7146	886.0289
3.4	3.0276	49.6356	939.4917

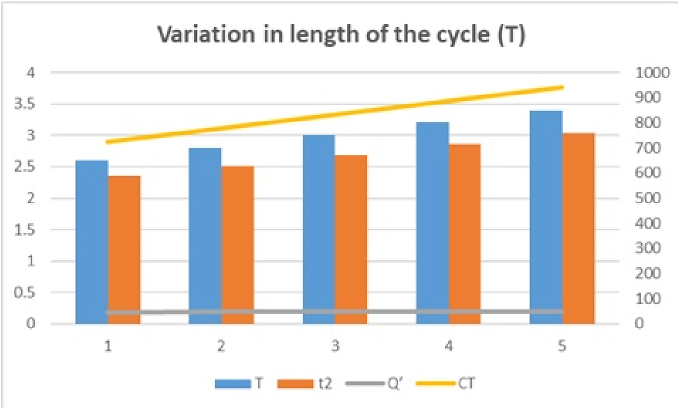

Figure 4. Variation in length of the cycle (T)

6. CONCLUDING REMARKS

This study presents an enhanced Economic Order Quantity (EOQ) model that integrates the complexities of imperfect quality items, quality screening processes, and time-dependent backlogging. By acknowledging that a portion of inventory may be defective and that customer willingness to wait for backordered items diminishes over time, the model offers a more realistic framework for inventory management. The incorporation of quality screening allows for the identification and handling of flawed units before they reach customers, thereby improving overall product quality and customer satisfaction. Additionally, the model's consideration of time-dependent backlogging reflects the dynamic nature of customer behaviour, where the likelihood of backordering decreases as the waiting time increases. Sensitivity analyses conducted within the study reveal that variations in parameters such as deterioration rate, demand rate and cycle length significantly impact the optimal order quantity and total cost. These insights enable inventory managers to make informed decisions by understanding how changes in operational factors affect inventory performance. The proposed EOQ model provides a comprehensive tool for managing inventory systems that deal with imperfect quality items and fluctuating customer demand. By integrating quality screening and time-dependent backlogging into the EOQ framework, businesses can achieve more efficient inventory control, reduce costs and enhance customer satisfaction. This approach can also be expanded to allow for permitted payment delays.

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