J. Indones. Math. Soc.
Vol. 29, No. 03 (2023), pp. 271–288.

TOPOLOGICAL INDICES OF RELATIVE g-NONCOMMUTING GRAPH OF DIHEDRAL GROUPS

Nur Ain Supu[1], Intan Muchtadi Alamsyah[2], Erma Suwastika [3]

[1] **Master's Program in Mathematics, Institut Teknologi Bandung, Indonesia, nurainsupu23@gmail.com**
[2] **Algebra Research Group, Institut Teknologi Bandung, ntan@itb.ac.id**
[3]**Combinatorial Mathemathics Research Group, Institut Teknologi Bandung, ermasuwastika@itb.ac.id**

Abstract. Let G be a finite group, H be a subgroup of G and g be a fixed element of G. The relative g-noncommuting graph $\Gamma_{g,H,G}$ of G is defined as a graph with vertex set is G and two distinct vertices x and y are adjacent if $[x,y] \neq g$ or $[x,y] \neq g^{-1}$, where at least x or y belong to H. In this paper, we will discuss the relative g-noncommuting graph of the dihedral groups D_{2n}, in particular case when n is an odd number. We give several topological indices of the relative g-noncommuting graph of the dihedral groups D_{2n} including the first Zagreb index, Wiener index, Edge-Wiener index, Hyper-Wiener index, and Harary index.

Key words and Phrases: Relative g-noncommuting graph, Dihedral group, Topological indices.

1. INTRODUCTION

The topological index is one of the applications of graph theory and group theory in chemistry that can be used to predict the chemical and physical properties of molecular structures with numerical values. In a graph, the molecular structure of atoms is represented as vertices and the bonds between atoms as edges [7]. A graph can be constructed from a finite group. Some of the graphs are built from groups, including commuting graphs, noncommuting graphs, relative g-noncommuting graphs, and so on. A noncommuting graph is a graph where two vertices are connected if $xy \neq yx$, the vertices are all members of the group except the identity while a g-noncommuting graph is a graph where two vertices x, y are adjacent if $[x,y] \neq g$ and $[x,y] \neq g^{-1}$ where x or y belong to H and the vertices are all members of the group [15][12].

2020 Mathematics Subject Classification: 20C05, 05C07, 05C09
Received: 10-02-2023, accepted: 31-10-2023.

Research on graphs of a group has been continued in recent years including Raza and Faizi [15] and Abdollahi et al [1] discussed the noncommuting graph of a finite group. Jahandideh et al examined the topological index of noncommuting graphs of finite groups while Alimon et al [2] examined the topological index on noncommuting graphs more specifically on dihedral groups. Nasiri et al [12] and Sharma and Nath [17] studied the relative g-noncommuting graphs of finite groups.

In this paper, we will discuss further the relative g-noncommuting graphs of a group that refer more to the research of Nasiri et al [12]. In Nasiri et al.'s research, we discuss the relative g-noncommuting graphs of groups in general, while in this research specifically discuss dihedral groups. In this paper, the dihedral group D_{2n} is restricted to the case of n is odd and two types of subgroups, namely the subgroups $H = \langle a \rangle$ and $H = \{e, a^j b\}$ for $j = 0, 1, \ldots, n-1$. Furthermore, in this paper we will determine some topological indices such as the first Zagreb index, Wiener index, edge Wiener index, hyper Wiener index, and Harary index.

2. LITERATURE REVIEW

This section will provide some definitions, lemmas, proposition, and theorems that will be used in this research.

Definition 2.1. [5] *The dihedral group of order* $2n$, D_{2n} *where* $n \in \mathbf{N}$ *and* $n \geq 3$, *is the group generated by two elements* a, b *with the properties*

$$a^n = e = b^2, bab^{-1} = a^{-1}$$

Proposition 2.2. [16] *Let* H *be a subgroup of* $G \cong \langle a, b : a^n = e, b^2 = e, bab = a^{-1} \rangle$ *with* $n \geq 3$, $n \in \mathbf{N}$. *Then*

$$Z(G) = \begin{cases} \{e\} & , n \text{ is odd} \\ \{e, a^{n/2}\} & , n \text{ is even.} \end{cases}$$

Definition 2.3. [9] *Define the commutator* $[x, y]$ *of* $x, y \in G$ *as*

$$[x, y] = x^{-1}y^{-1}xy.$$

For H *a subgroup of* G *and* $g \in G$ *define*

$$K(H, G) = \{(x, y) | x \in H, y \in G \backslash H; [x, y] = g \text{ or } [x, y] = g^{-1}\}$$

$$K_H = \{x \in H : (x, y) \in K(H, G)\} \quad (1)$$

$$K_{G \backslash H} = \{y \in G \backslash H : (x, y) \in K(H, G)\} \quad (2)$$

Definition 2.4. [4] *A graph* Γ *is defined as a pair* $(V(\Gamma), E(\Gamma))$ *with* $V(\Gamma)$ *is a finite nonempty set of elements called vertices and* $E(\Gamma)$ *is a set of unordered pairs of vertices in* $V(\Gamma)$ *whose elements are named edges.*

Definition 2.5. *Let G be a finite group, H be a subgroup of G and g be a fixed element of G. The relative g-noncommuting graph $\Gamma_{g,H,G}$ of G is defined as a graph with vertex set is G and two distinct vertices x and y are adjacent if $[x,y] \neq g$ or $[x,y] \neq g^{-1}$, where at least x or y belong to H* [12].

These are some examples of relative g-noncommuting graphs of dihedral group for $n = 5$.

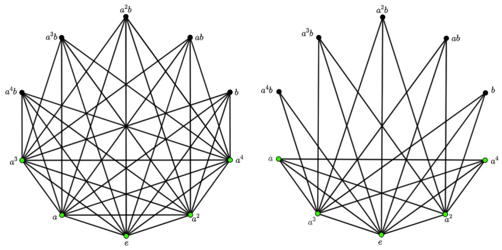

FIGURE 1. Case $H = \langle a \rangle$ with $g = b$ and $g = a^2$.

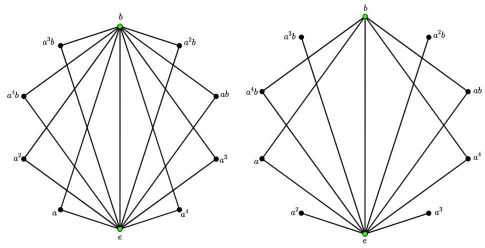

FIGURE 2. Case $H = \{e, b\}$ with $g = b$ and $g = a$.

Theorem 2.6. [12] *Let G be a non-identity element of G. Then*

(1) $\Gamma_{g,H,G}$ has no isolated vertex,
(2) The diameter of $\Gamma_{g,H,G}$ is 2.

If $x \in G$ and H is a subgroup of G, $C_H(x)$ denotes the centralizer of x in H.

Lemma 2.7. [12] *Let G be a group and H a subgroup of G. The degree of $x \in G$ as a vertex in $\Gamma_{g,H,G}$ is given as follows:*

(1) Let $x \in G \backslash H$.
- *If $g^2 = e$, then deg(x)=$|H| - \varepsilon |C_H(x)|$ where $\varepsilon = 1$ if x if conjugate to xg or xg^{-1} in H, but not both and $\varepsilon = 2$ ix x is conjugate to xg and xg^{-1} in H.*
- *If $g^2 = e$ and $g \neq e$, then deg(x)=$|H| - |C_H(x)|$ whenever xg is conjugate to x in H. For $g = e$ we have deg(x)=$|H| - |C_H(x)|$.*

- *If xg and xg^{-1} are not conjugate to x in H, then deg(x)=H.*

(2) Let $x \in H$.

- *If $g^2 \neq e$, then $deg(x) = |G| - \varepsilon|C_G(x)| - 1$ where $\varepsilon = 1$ if x is conjugate to xg or xg^{-1}, but not both and $\varepsilon = 2$ if x is conjugate to xg and xg^{-1}.*
- *If $g^2 = e$ and $g \neq e$, then $deg(x) = |G| - |C_G(x)| - 1$ whenever xg is conjugate to x. For $g = e$ we have $deg(x) = |G| - |C_G(x)|$.*
- *If xg and xg^{-1} are not conjugate to x, then $deg(x) = |G| - 1$ [12].'*

Definition 2.8. [6][13][3][14] *Given Γ a connected graph.*

(1) The first Zagreb index of Γ is the sum of all squares of the degree of each vertex of Γ, written as

$$M_1(\Gamma) = \sum_{x \in V(\Gamma)} (deg(x))^2.$$

(2) The Wiener index of Γ is the sum of the distances of every pair of unordered vertex of the graph Γ, written as

$$W(\Gamma) = \sum_{\{x,y\} \subset V(\Gamma)} d(x, y).$$

(3) The edge Wiener index of Γ is the sum of all distances between any two edges of the graph Γ, written as

$$W_e(\Gamma) = \sum_{\{e,f\} \subset E(\Gamma)} d(e, f).$$

(4) The hyper Wiener index of Γ is the sum of all distances between vertices of Γ, written as

$$WW(\Gamma) = \frac{1}{2} \left(\sum_{\{u,v\} \subset V(\Gamma)} d(u, v) + (d(u, v))^2 \right)$$

(5) The Harary index of Γ is the sum of reciprocals of distances between all pairs of vertices of a connected graph, written as

$$H(\Gamma) = \sum_{\{u,v\} \subset V(\Gamma)} \frac{1}{d(u, v)}$$

Below is an example of the topological index of the relative g-noncommuting graph of the dihedral group, for $n = 3$, $H = \{e, b\}$, and $g = b$.

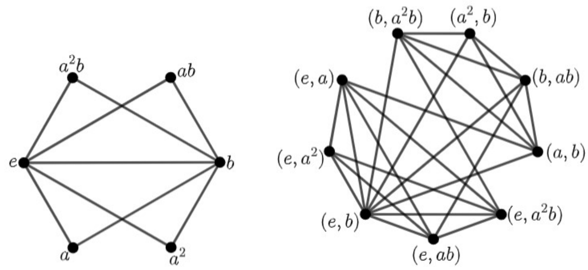

FIGURE 3. Graph $\Gamma_{g,H,G}$ and graph $L(\Gamma_{g,H,G})$, case $n = 3$, $H = \{e, b\}$, $g = b$.

Example 2.9. $M_1(\Gamma_{g,H,G}) = deg(e)^2 + deg(a)^2 + deg(a^2)^2 + deg(b)^2 + deg(ab)^2 + deg(a^2b)^2 = 5^2 + 2^2 + 2^2 + 2^2 + 2^2 + 5^2 = 66.$

Example 2.10. $W(\Gamma_{g,H,G}) = d(e, a) + d(e, a^2) + d(e, b) + d(e, ab) + d(e, a^2b) + d(a, a^2) + d(a, b) + d(a, ab) + d(a, a^2b)d(a^2, b) + d(a^2, ab) + d(a^2, a^2b) + d(b, ab) + d(b, a^2b) + d(ab, a^2b) = 1+1+1+1+1+2+1+2+2+1+2+2+1+1+2 = 21.$

Example 2.11. $WW(\Gamma_{g,H,G}) = \frac{1}{2}[d(e, a) + d(e, a^2) + d(e, b) + d(e, ab) + d(e, a^2b) + d(a, a^2) + d(a, b) + d(a, ab) + d(a, a^2b) + d(a^2, b) + d(a^2, ab) + d(a^2, a^2b) + d(b, ab) + d(b, a^2b) + d(ab, a^2b) + d(e, a)^2 + d(e, a^2)^2 + d(e, b)^2 + d(e, ab)^2 + d(e, a^2b)^2 + d(a, a^2)^2 + d(a, b)^2 + d(a, ab)^2 + d(a, a^2b)^2 + d(a^2, b)^2 + d(a^2, ab)^2 + d(a^2, a^2b)^2 + d(b, ab)^2 + d(b, a^2b)^2 + d(ab, a^2b)^2] = \frac{1}{2}[1+1+1+1+1+2+1+2+2+1+2+2+1+1+2+1^2+1^2+1^2+1^2+1^2+2^2+1^2+2^2+2^2+1^2+2^2+2^2+1^2+1^2+2^2 = 27.$

Example 2.12. $H(\Gamma_{g,H,G}) = \frac{1}{d(e,a)} + \frac{1}{d(e,a^2)} + \frac{1}{d(e,b)} + \frac{1}{d(e,ab)} + \frac{1}{d(e,a^2b)} + \frac{1}{d(a,a^2)} + \frac{1}{d(a,b)} + \frac{1}{d(a,ab)} + \frac{1}{d(a,a^2b)} + \frac{1}{d(a^2,b)} + \frac{1}{d(a^2,ab)} + \frac{1}{d(a^2,a^2b)} + \frac{1}{d(b,ab)} + \frac{1}{d(b,a^2b)} + \frac{1}{d(ab,a^2b)} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{2} + \frac{1}{1} + \frac{1}{2} + \frac{1}{2} + \frac{1}{1} + \frac{1}{2} + \frac{1}{2} + \frac{1}{1} + \frac{1}{1} + \frac{1}{2} = 12.$

From Lemma 2.6 we can calculate the topology indices using the following theorem.

Theorem 2.13. [11] *Given Γ a simple graph with $diam(\Gamma) \leq 2$.*

(1) The Wiener index of Γ is

$$W(\Gamma) = |V(\Gamma)|(|V(\Gamma)| - 1) - |E(\Gamma)|.$$

(2) Then hyper Wiener index of Γ is

$$WW(\Gamma) = \frac{3}{2}|V(\Gamma)|(|V(\Gamma)| - 1) - 2|E(\Gamma)|.$$

(3) The Harary index of Γ is

$$H(\Gamma) = \frac{1}{4}|V(\Gamma)|(|V(\Gamma)| - 1) + \frac{1}{2}|E(\Gamma)|.$$

PROOF. Note that $diam(\Gamma) \leq 2$, then the number of unordered pairs in Γ that have distance 2 is

$$\binom{|V(\Gamma)|}{2} - |E(\Gamma)|$$

hence

(1) The Wiener index of Γ is

$$\begin{aligned} W(\Gamma) &= |E(\Gamma)| + 2\left(\binom{|V(\Gamma)|}{2} - |E(\Gamma)|\right) \\ &= |V(\Gamma)|(|V(\Gamma)| - 1 - |E(\Gamma)|. \end{aligned}$$

(2) The hyper Wiener index of Γ is

$$\begin{aligned} WW(\Gamma) &= \frac{1}{2}\left[|E(\Gamma)| + 2\left(\binom{|V(\Gamma)|}{2}) - |E(\Gamma)|\right) + |E(\Gamma)| + 4\left(\binom{|V(\Gamma)|}{2} - |E(\Gamma)|\right)\right] \\ &= \frac{3}{2}|V(\Gamma)|(|V(\Gamma)| - 1) - 2|E(\Gamma)|. \end{aligned}$$

(3) The Harary index of Γ is

$$\begin{aligned} H(\Gamma) &= |E(\Gamma)| + \frac{1}{2}\left(\binom{|V(\Gamma)|}{2}) - |E(\Gamma)|\right) \\ &= \frac{1}{4}|V(\Gamma)|(|V(\Gamma)| - 1) + \frac{1}{2}|E(\Gamma)|. \end{aligned}$$

Theorem 2.14. [11] *Given Γ a connected graph. Let $L(\Gamma)$ be a simple graph with $diam(L(\Gamma)) \leq 2$. The edge Wiener index of Γ is*

$$W_e(\Gamma) = |E(\Gamma)|^2 - \frac{1}{2}M_1(\Gamma)$$

PROOF. Note that two edges e, f on Γ will be $L(\Gamma)$-neighboring if they are incident to a point in Γ. Since every point x in Γ will have as many as $deg(x)$ edges incident to x, then the number of pairs of unordered edges in Γ that are adjacent to $L(\Gamma)$ is $\sum_{x\in V(\Gamma)} \binom{deg(x)}{2}$. We obtain

$$\begin{aligned} |E(L(\Gamma))| &= \frac{1}{2}\sum_{x\in V(\Gamma)} deg(x)(deg(x) - 1) \\ &= \frac{1}{2}\sum_{x\in V(\Gamma)} deg(x)^2 - \frac{1}{2}\sum_{x\in V(\Gamma)} deg(x) \\ &= \frac{1}{2}M_1(\Gamma) - |E(\Gamma)|. \end{aligned}$$

Since $diam(L(\Gamma)) \leq 2$, by Theorem 2.13 (1) we get

$$\begin{aligned} W_e(\Gamma) &= W(L(\Gamma)) \\ &= |V(L(\Gamma))|(|V(L(\Gamma))| - 1) - |E(L(\Gamma))| \\ &= |E(\Gamma)|(|E(\Gamma)| - 1) - \frac{1}{2}M_1(\Gamma) + |E(\Gamma)| \\ &= |E(\Gamma)|^2 - \frac{1}{2}M_1(\Gamma). \end{aligned}$$

Example 2.15. *The line graph of $\Gamma_{g,H,G}$ has diameter 2 with $|E(\Gamma)| = 9$ and $M_1(\Gamma) = 66$. Therefore, $W_e(\Gamma_{g,H,G}) = 9^2 - \frac{1}{2}(66) = 48$.*

3. MAIN RESULTS

3.1. Some Properties of Relative g-noncommuting Graph of Dihedral Groups.

In this section, we will discuss the properties of the relative g-noncommuting graph of the dihedral group (D_{2n}), for the case n is an odd number.

Lemma 3.1. *Let G be D_{2n}. If $x, y \in G$, then $[x, y] = a^i$ for some $i = 0, 1, \ldots, n-1$.*

Proof. Let G be D_{2n} and $x, y \in G$.

If $x = a^j$ and $y = a^k$ for $j, k = 0, 1, \ldots, n-1$ where $j \neq k$, then

$$[x, y] = a^{-j}a^{-k}a^ja^k = e. \tag{3}$$

If $x = a^j$ and $y = a^kb$ for $j, k = 0, 1, \ldots, n-1$, then

$$[x, y] = a^{-j}a^kba^ja^kb = a^{n-2j}. \tag{4}$$

If $x = a^jb$ and $y = a^k$ for $j, k = 0, 1, \ldots, n-1$, then

$$[x, y] = a^jba^{-k}a^jba^k = a^{2k}. \tag{5}$$

If $x = a^jb$ and $y = a^kb$ for $j, k = 0, 1, 2, \ldots, n-1$ for $j \neq k$, then

$$[x, y] = a^jba^kba^jba^kb = a^{2(j-k)}. \tag{6}$$

So, $[x, y] = a^i$ for some $i = 0, 1, \ldots, n-1$.

Lemma 3.2. *Let G be D_{2n} where n is odd. Let H be a subgroup of G and $g = a^i$ for some $i = 1, 2, \ldots, n-1$.*

(1) If $H = \langle a \rangle$, then $|K_H| = 2$.
(2) If $H = \{e, a^jb\}$ for some $j = 0, 1, \ldots, n-1$, then $|K_{G\setminus H}| = 4$.

Proof.

(1) Let $H = \langle a \rangle$ and $g = a^i$. We will prove that $|K_H| = 2$.
Based on Lemma 3.1, we obtain :

- If $x = a^j$ and $y = a^k$ for some $j, k = 0, 1, \ldots, n-1$ where $j \neq k$ then from Lemma 3.1, $[x, y] = e$. Since $g \neq e$, then $[x, y] \neq g$ and $[x, y] \neq g^{-1}$.
- If $x = a^j$ and $y = a^k b$ for some $j, k = 0, 1, \ldots, n-1$ then $[x, y] = a^{n-2j}$, so that

$$\begin{aligned}[x,y] = g &\Leftrightarrow [x,y] = a^i \\ &\Leftrightarrow a^{n-2j} = a^i \\ &\Leftrightarrow n-2j = i \mod n \\ &\Leftrightarrow j = \begin{cases} \frac{n-i}{2}, & i \text{ is odd} \\ n - \frac{i}{2}, & i \text{ is even} \end{cases}\end{aligned} \qquad \begin{aligned}[x,y] = g^{-1} &\Leftrightarrow [x,y] = a^{n-i} \\ &\Leftrightarrow a^{n-2j} = a^{n-i} \\ &\Leftrightarrow 2j = i \mod n \\ &\Leftrightarrow j = \begin{cases} \frac{i}{2}, & i \text{ is even} \\ -(\frac{n-i}{2}), & i \text{ is odd} \end{cases}\end{aligned} \tag{7}$$

If i is odd, $x \in K_H \Leftrightarrow x = a^{\frac{n-i}{2}}$ or $x = a^{-(\frac{n-i}{2})}$, and if i is even, then $x \in K_H \Leftrightarrow x = a^{\frac{i}{2}}$ or $x = a^{n-\frac{i}{2}}$. Therefore $|K_H| = 2$.

(2) Let $H = \{e, a^j b\}$ for some $j = 0, 1, \ldots, n-1$. We will show that $|K_{G\backslash H}| = 4$. Based on Lemma 3.1, we obtain:
- If $x = a^j b$ and $y = a^k$ for some $j = 0, 1, \ldots, n-1$ and $k = 1, 2, \ldots, n-1$, then $[x, y] = a^{2k}$.
- If $x = a^j b$ and $y = a^k b$ for $j, k = 0, 1, \ldots, n-1$, then $[x, y] = a^{2(j-k)}$.

By using similar argument as in Equation 7, if i is odd, then $y \in K_{G\backslash H} \Leftrightarrow y \in \{a^{\frac{n-i}{2}}, a^{-(\frac{n-i}{2})}, a^{j+\frac{n-i}{2}}b, a^{j-(\frac{n-i}{2})}b\}$. If i is even, then $y \in K_{G\backslash H} \Leftrightarrow y \in \{a^{\frac{i}{2}}, a^{n-\frac{i}{2}}, a^{j+\frac{i}{2}}b, a^{j-(\frac{i}{2})}b\}$, so that $|K_{G\backslash H}| = 4$.

Lemma 3.3. *Let G be D_{2n} where n is odd.*

(1) If $j = 1, 2, \ldots, n-1$ and $k = 0, 1, \ldots, n-1$, then $a^j a^k b \neq a^k b a^j$.
(2) If $j, k = 0, 1, \ldots, n-1$ and $j \neq k$, then $a^j b a^k b \neq a^k b a^j b$.

PROOF.

(1) If $(a^j)(a^k b) = (a^k b)(a^j)$ then $a^j \in Z(G)$, but we know $Z(G) = \{e\}$. This contradicts $j \neq 0$. So, $(a^j)(a^k b) \neq (a^k b)(a^j)$
(2)
$$a^l = a^{-l} \Leftrightarrow l = n - l \mod n \Leftrightarrow 2l = 0 \mod n. \tag{8}$$
Because n is odd, then Equation 8 is not statisfied so $a^l \neq a^{-l}$.
Consequently, $a^j b a^k b = a^{j-k} \neq a^{k-j} = a^k b a^j b$ with $j \neq k$.

Lemma 3.4. *Let G be D_{2n} with n is odd and H be a subgroup of G.*

(1) If $H = \langle a \rangle$, then

$$\begin{cases} |C_H(x)| = 1 & , x \in G\backslash H \\ |C_G(x)| = n & , x \in K_H \end{cases}$$

(2) If $H = \{e, a^j b\}$ for some $j = 0, 1, \ldots, n-1$, then

$$\begin{cases} |C_H(x)| = 1 & , x \in K_{G\backslash H} \\ |C_G(x)| = 2 & , x \in H\backslash\{e\} \end{cases}$$

Proof. Let G be D_{2n} with n is odd.

(1) Let $H = \langle a \rangle$. We will prove that $|C_H(x)| = 1$. Based on Proposition 2.2, obviously that e commutes with all $x \in G$. Let $x \in G\backslash H$ and $y \in H$. Because $x = a^j b$ and $y = a^k$, then from Lemma 3.3, $xy \neq yx$. Thus, the centralizer of $x \in G\backslash H$ is e, so that $|C_H(x)| = 1$.
Next, we will prove that $|C_G(x)| = n$ with $x \in K_H$ and $y \in G$. From Lemma 3.2 we have $x = a^{\frac{n-i}{2}}$ or $x = a^{-(\frac{n-i}{2})}$ for i is odd while $x = a^{\frac{i}{2}}$ or $x = a^{n-\frac{i}{2}}$ for i is even. It is obvious that if $y = a^j$ for some $j = 1, 2, \ldots, n-1$, then $xy = yx$. From Lemma 3.3, if $y = a^j b$ for some $j = 1, 2, \ldots, n-1$, then $xy \neq yx$.
So, the centralizer of $x \in K_H$ is H, so that $|C_G(x)| = n$.

(2) Let $H = \{e, a^j b\}$ for some $j = 0, 1, \ldots, n-1$. We will prove that $|C_H(x)| = 1$. From Lemma 3.2, $x \in \{a^{\frac{n-i}{2}}, a^{-(\frac{n-i}{2})}, a^{j+\frac{n-i}{2}}b, a^{j-\frac{n-i}{2}}b\}$ for i is odd and $x \in \{a^{\frac{i}{2}}, a^{n-\frac{i}{2}}, a^{j+\frac{i}{2}}b, a^{j-\frac{i}{2}}b\}$ for i is even. From Lemma 3.3, if $x \in K_{G\backslash H}$ and $y = a^j b$ for some $j = 0, 1, \ldots, n-1$, then $xy \neq yx$. So, centralizer of $x \in K_{G\backslash H}$ is e, so that $|C_H(x)| = 1$.
Next, we will prove that $|C_G(x)| = 2$ for $x \in H\backslash\{e\}$ and $y \in G$. Note that $x = a^j b$. Since $x^2 = e$, then the only elements that commute with x are e and x. From Lemma 3.3, if $y = a^k b$ for some $j, k = 0, 1, \ldots, n-1$ where $j \neq k$, then $xy \neq yx$, while for $y = a^k$ for some $k = 1, 2, \ldots, n-1$, then $xy \neq yx$. So, the centralizer $x \in H\backslash\{e\}$ is $\{e, a^j b\}$, so that $|C_G(x)| = 2$.

Below we will discuss the lemmas related to conjugates. Before that, we will first show the relationship between conjugates and commutators as follows.

Suppose $x, y \in G$. If $[x, y] = g$ or $[x, y] = g^{-1}$, then

$$x^{-1}y^{-1}xy = g \quad \text{or} \quad x^{-1}y^{-1}xy = g^{-1}$$
$$x^y = xg \quad \text{or} \quad x^y = xg^{-1}.$$

Lemma 3.5. *Let G be D_{2n} where n is odd. Let H be a subgroup of G and $g = a^i b$ for some $i = 0, 1, \ldots, n-1$.*

(1) If $x \in G\backslash H$, then x is not conjugate to xg and xg^{-1} in H.
(2) If $x \in H$, then x is not conjugate to xg and xg^{-1}.

Proof. It is clear because $[x, y] = g$ if and only if $y^{-1}xy = xg$, and by Lemma 3.1 $[x, y] = a^i$. Therefore, if $g = a^i b$ then $x \in G\backslash H$ is not conjugate to xg and xg^{-1} in H and $x \in H$ is not conjugate to xg and xg^{-1}.

Lemma 3.6. *Let G be D_{2n} with n is odd. Let H be a subgroup of G and $g = a^i$ for some $i = 0, 1, \ldots, n-1$.*

(1) If $H = \langle a \rangle$, then $x \in G \backslash H$ is conjugate to xg and xg^{-1} in H.
(2) If $H = \{e, a^j b\}$, then $x \in H \backslash \{e\}$ is conjugate to xg and xg^{-1}.

PROOF.

(1) Let $H = \langle a \rangle$. We will prove that $x \in G \backslash H$ and $y \in K_H$, then x is conjugate to xg and xg^{-1} in H. Let $x = a^j b$ for some $j = 0, 1, \ldots, n-1$. Based on Lemma 3.2 for case i is odd take $y_1 = a^{\frac{n-i}{2}}$ and $y_2 = a^{-(\frac{n-i}{2})}$, so that

$$\begin{aligned} x^{y_1} &= a^{\frac{n-i}{2}} a^j b a^{-(\frac{n-i}{2})} \\ &= a^{j-i} b \\ &= xg \end{aligned} \quad \text{and} \quad \begin{aligned} x^{y_2} &= a^{-(\frac{n-i}{2})} a^j b a^{\frac{n-i}{2}} \\ &= a^{j+i} b \\ &= xg^{-1} \end{aligned} \tag{9}$$

For case i is even, using the similar argument as in Equation 9, we obtain x is conjugate to xg and xg^{-1}.

Thus, every $x \in G \backslash H$ is conjugate to xg and xg^{-1} in H.

(2) Let $H = \{e, a^j b\}$ for some $j = 0, 1, \ldots, n-1$. We will prove that if $x \in H \backslash \{e\}$ and $y \in K_{G \backslash H}$, then x is conjugate to xg and xg^{-1}.

Let $x = a^j b$ for some $j = 0, 1, \ldots, n-1$. Based on Lemma 3.2, for case i is odd take $y \in \{a^{\frac{n-i}{2}}, a^{-(\frac{n-i}{2})}, a^{j+\frac{n-i}{2}} b, a^{j-(\frac{n-i}{2})} b\}$ and $y \in \{a^{\frac{i}{2}}, a^{n-\frac{i}{2}}, a^{j+\frac{i}{2}} b, a^{j-(\frac{i}{2})} b\}$ for i is even. By using similar argument as in Equation 9, we obtain $x \in H \backslash \{e\}$ is conjugate to xg and xg^{-1}.

Below is a lemma on the degree of the relative g-noncommuting graph of the dihedral group D_{2n}.

Lemma 3.7. *Let G be D_{2n} where n is odd and H be a subgroup of G. If $g = a^i b$ for some $i = 0, 1, 2, \ldots, n-1$, then*

$$deg(x) = \begin{cases} 2n-1 & , x \in H \\ |H| & , x \in G \backslash H \end{cases}$$

PROOF. Note that $g^2 = e$, but from Lemma 3.5 for $x \in H$ or $x \in G \backslash H$, x is not conjugate to xg and xg^{-1}. From Lemma 2.11, then

$$deg(x) = \begin{cases} |G| - 1 = 2n-1 & , x \in H \\ |H| & , x \in G \backslash H \end{cases}$$

Lemma 3.8. *Let G be D_{2n} where n is odd, H be a subgroup of G and $g = a^i$ for some $i = 1, 2, \ldots, n-1$.*

(1) If $H = \langle a \rangle$, then

$$deg(x) = \begin{cases} 2n-1 & , x \in H \backslash K_H \\ n-1 & , x \in K_H \\ n-2 & , x \in G \backslash H \end{cases}$$

(2) If $H = \{e, a^j b\}$ for some $j = 0, 1, 2, \ldots, n-1$, then

$$deg(x) = \begin{cases} 2n-1 & , x = e \\ 2n-5 & , x \in H \backslash \{e\} \\ 2 & , x \in G \backslash (H \cup K_{G \backslash H}) \\ 1 & , x \in K_{G \backslash H} \end{cases}$$

PROOF.

(1) Note that $H = \langle a \rangle$ and $g^2 \neq e$. Based on Lemma 3.2 there are two elements of H that are not connected to $G \backslash H$. Based on Equation 2, for $x \in K_H$ it is obviously conjugate to xg or xg^{-1} while $x \in H \backslash K_H$ is not conjugate to xg and xg^{-1}. Notice that for $x \in G \backslash H$. From Lemma 3.6, $x \in G \backslash H$ is conjugate to xg and xg^{-1}. So, from Lemma 2.11 and Lemma 3.4, we obtain

$$deg(x) = \begin{cases} 2n-1 & , x \in H \backslash K_H \\ 2n-n-1 = n-1 & , x \in K_H \\ n-2 & , x \in G \backslash H. \end{cases}$$

(2) Note that $H = \{e, a^j b\}$ for some $j = 0, 1, \ldots, n-1$. Based on Equation 4 and Equation 5, for $x = e$ and $y \in G$, $[x, y] \neq g$ and $[x, y] \neq g^{-1}$. So, x is not conjugate to xg and xg^{-1}. For $x \in K_{G \backslash H}$, based on Equation 3, x is conjugate to xg or xg^{-1} in H. From Lemma 3.6, $x \in H \backslash \{e\}$ is conjugate to xg and xg^{-1}. Otherwise, $x \in G \backslash (H \cup K_G \backslash H)$ is not conjugate to xg and xg^{-1}. So, from Lemma 2.11 and Lemma 3.4, we obtain

$$deg(x) = \begin{cases} 2n-1 & , x = e \\ 2n-2.2-1 = 2n-5 & , x \in H \backslash \{e\} \\ 2 & , x \in G \backslash (H \cup K_{G \backslash H}) \\ 2-1 = 1 & , x \in K_{G \backslash H}. \end{cases}$$

Furthermore, some lemmas on the number of edges of the relative g-noncommuting graph of the dihedral group is given below.

Lemma 3.9. *Let G be D_{2n} where n is odd and H a subgroup of G. Suppose $g = a^i b$ for some $i = 0, 1, 2, \ldots, n-1$*

(1) If $H = \langle a \rangle$, then the number of edges of $\Gamma_{g,H,G}$ is

$$|E(\Gamma_{g,H,G})| = \frac{3n^2 - n}{2}.$$

(2) If $H = \{e, a^j b\}$ for some $j = 0, 1, \ldots, n-1$, then the number of edges of $\Gamma_{g,H,G}$ is

$$|E(\Gamma_{g,H,G})| = 4n - 3.$$

PROOF. Let G be D_{2n} where n is odd and H a subgroup of G. Suppose $g = a^i b$ for some $i = 0, 1, \ldots, n-1$.

(1) If $H = \langle a \rangle$, then from Lemma 3.7,

$$2|E(\Gamma_{g,H,G})| = \sum deg(x) = (2n-n)n + n(2n-1)$$
$$|E(\Gamma_{g,H,G})| = \frac{3n^2 - n}{2}.$$

(2) If $H = \{e, a^j b\}$ for some $j = 0, 1, \ldots, n-1$, then from Lemma 3.7,

$$2|E(\Gamma_{g,H,G})| = \sum deg(x) = (2n-2)2 + 2(2n-1)$$
$$|E(\Gamma_{g,H,G})| = \frac{8n-6}{2} = 4n - 3.$$

Lemma 3.10. *Let G be D_{2n} where n is odd. Let H be a subgroup of G and $g = a^i$ for some $i = 1, 2, \ldots, n-1$.*

(1) If $H = \langle a \rangle$, then the number of edges of $\Gamma_{g,H,G}$ is

$$|E(\Gamma_{g,H,G})| = \frac{3n^2 - 5n}{2}$$

(2) If $H = \{e, a^j b\}$ for some $j = 0, 1, 2, \ldots, n-1$, then the number of edges of $\Gamma_{g,H,G}$ is

$$|E(\Gamma_{g,H,G})| = 4n - 7$$

Proof. Let G be D_{2n} where n is odd. Let H be a subgroup of G and $g = a^i$ for some $i = 0, 1, \ldots, n-1$.

(1) If $H = \langle a \rangle$, then from Lemma 3.8,

$$2|E(\Gamma_{g,H,G})| = \sum deg(x) = (n-2)(2n-1) + 2(n-1) + (2n-n)(n-2)$$
$$|E(\Gamma_{g,H,G})| = \frac{3n^2 - 5n}{2}.$$

(2) If $H = \{e, a^j b\}$ for some $j = 0, 1, \ldots, n-1$, then from Lemma 3.8,

$$2|E(\Gamma_{g,H,G})| = \sum deg(x) = 2n - 1 + 2n - 5 + (2n-6)2 + 4(1)$$
$$|E(\Gamma_{g,H,G})| = \frac{8n-14}{2} = 4n - 7.$$

3.2. Topological Indices of Relative g-noncommuting Graph of Dihedral Groups.

In this section, we will discuss some topological indices of the relative g-noncommuting graph of a group D_{2n}, including the first Zagreb index, Wiener index, hyper Wiener index, Wiener edge index, and Harary index.

3.2.1. *Case $H = \langle a \rangle$.*

Theorem 3.11. *Given $\Gamma_{g,H,G}$ the relative g-noncommuting graph of group D_{2n} with n is odd. If $H = \langle a \rangle$ and $g = a^i b$ for some $i = 0, 1, \ldots, n-1$, then*

- *the first Zagreb Index of $\Gamma_{g,H,G}$ is*
$$M_{1(\Gamma_{g,H,G})} = 5n^3 - 4n^2 + n.$$
- *the Wiener Index of $\Gamma_{g,H,G}$ is*
$$W(\Gamma_{g,H,G}) = \frac{1}{2}(5n^2 - 3n).$$
- *the edge Wiener Index of $\Gamma_{g,H,G}$ is*
$$W_e(\Gamma_{g,H,G}) = \frac{1}{4}(9n^4 - 16n^3 + 9n^2 - 2n).$$
- *the hyper Wiener Index of $\Gamma_{g,H,G}$ is*
$$WW(\Gamma_{g,H,G}) = 3n^2 - 2n.$$
- *the Harary Index of $\Gamma_{g,H,G}$ is*
$$H(\Gamma_{g,H,G}) = \frac{1}{4}(7n^2 - 3n).$$

Proof.

- From Definition 2.8 (1) and Lemma 3.7, we obtain
$$\begin{aligned} M_1(\Gamma_{g,H,G}) &= \sum (deg(x))^2 \\ &= \sum_{x \in G \setminus H} n^2 + \sum_{x \in H} (2n-1)^2 \\ &= 5n^3 - 4n^2 + n. \end{aligned} \tag{10}$$
- From Theorem 2.13 (1) and Lemma 3.7, we obtain
$$\begin{aligned} W(\Gamma_{g,H,G}) &= |V(\Gamma_{g,H,G})|(|V(\Gamma_{g,H,G})| - 1) - |E(\Gamma_{g,H,G})| \\ &= 2n(2n-1) - \left(\frac{3n^2 - n}{2}\right) = \frac{1}{2}(5n^2 - 3n). \end{aligned}$$
- From Theorem 2.14, Lemma 3.7, and Equation 10, we obtain
$$\begin{aligned} W_e(\Gamma_{g,H,G}) &= |E(\Gamma_{g,H,g})|^2 - \frac{1}{2} M_1(\Gamma_{g,H,G}) \\ &= \frac{1}{4}(9n^4 - 16n^3 + 9n^2 - 2n). \end{aligned}$$
- From Theorem 2.13 (2), and Lemma 3.7, we obtain
$$\begin{aligned} WW(\Gamma_{g,H,G}) &= \frac{3}{2}|V(\Gamma_{g,H,G})|(|V(\Gamma_{g,H,G})| - 1) - 2|E(\Gamma_{g,H,G})| \\ &= 3n^2 - 2n. \end{aligned}$$

- From Theorem 2.13 (3), and Lemma 3.7, we obtain

$$\begin{aligned}H(\Gamma_{g,H,G}) &= \frac{1}{4}|V(\Gamma_{g,H,G})|(|V(\Gamma_{g,H,G})|-1)+\frac{1}{2}|E(\Gamma_{g,H,G})|\\ &= \frac{1}{4}(7n^2-3n).\end{aligned}$$

Theorem 3.12. *Given $\Gamma_{g,H,G}$ the relative g-noncommuting graph of group D_{2n} with n is odd. If $H=\langle a\rangle$ and $g=a^i$ for some $i=1,2,\ldots,n-1$, then*

- *the first Zaegreb index of $\Gamma_{g,H,G}$ is*
$$M_1(\Gamma_{g,H,G})=5n^3-14n^2+9n.$$
- *the Wiener index of $\Gamma_{g,H,G}$ is*
$$W(\Gamma_{g,H,G})=\frac{1}{2}(5n^2+n).$$
- *the edge Wiener index of $\Gamma_{g,H,G}$ is*
$$W_e(\Gamma_{g,H,G})=\frac{1}{4}(9n^4-40n^3+53n^2-18n).$$
- *the hyper Wiener index of $\Gamma_{g,H,G}$ is*
$$WW(\Gamma_{g,H,G})=3n^2+2n.$$
- *the Harary index of $\Gamma_{g,H,G}$ is*
$$H(\Gamma_{g,H,G})=\frac{1}{4}(7n^2-7n).$$

PROOF.

- From Definition 2.8 (1) and Lemma 3.8, we obtain

$$\begin{aligned}M_1(\Gamma_{g,H,G}) &= \sum deg(x)^2\\ &= \sum_{x\in G\setminus H} deg(x)^2+\sum_{x\in H\setminus K_H} deg(x)^2+\sum_{x\in K_H} deg(x)^2\\ &= 5n^3-14n^2+9n. \qquad (11)\end{aligned}$$

- From Theorem 2.13 (1), and Lemma 3.8, we obtain

$$\begin{aligned}W(\Gamma_{g,H,G}) &= |V(\Gamma_{g,H,G})|(|V(\Gamma_{g,H,G})|-1)-|E(\Gamma_{g,H,G})|\\ &= \frac{1}{2}\left(5n^2+n\right).\end{aligned}$$

- From Theorem 2.14, Lemma 3.8, and Equation 11, we obtain

$$\begin{aligned}W_e(\Gamma_{g,H,G}) &= |E(\Gamma_{g,H,g})|^2-\frac{1}{2}M_1(\Gamma_{g,H,G})\\ &= \frac{1}{4}(9n^4-40n^3+53n^2-18n).\end{aligned}$$

- From Theorem 2.13 (2), and Lemma 3.8, we obtain

$$\begin{aligned} WW(\Gamma_{g,H,G}) &= \frac{3}{2}|V(\Gamma_{g,H,G})|(|V(\Gamma_{g,H,G})|-1) - 2|E(\Gamma_{g,H,G})| \\ &= 3n^2 + 2n. \end{aligned}$$

- From Theorem 2.13 (3), and Lemma 3.8, we obtain

$$\begin{aligned} H(\Gamma_{g,H,G}) &= \frac{1}{4}|V(\Gamma_{g,H,G})|(|V(\Gamma_{g,H,G})|-1) + \frac{1}{2}|E(\Gamma_{g,H,G})| \\ &= \frac{1}{4}(7n^2 - 7n). \end{aligned}$$

3.2.2. *Case $H = \{e, a^j b\}$.*

Theorem 3.13. *Given $\Gamma_{g,H,G}$ the relative g-noncommuting graph of group D_{2n} with n is odd. If $H = \{e, a^j b\}$ and $g = a^i b$ for some $i, j = 0, 1, \ldots, n-1$, then*

- *the first Zagreb index of $\Gamma_{g,H,G}$ is*

$$M_1(\Gamma_{g,H,G}) = 8n^2 - 6.$$

- *the Wiener index of $\Gamma_{g,H,G}$ is*

$$W(\Gamma_{g,H,G}) = 4n^2 - 6n + 3.$$

- *the edge Wiener index of $\Gamma_{g,H,G}$ is*

$$W_e(\Gamma_{g,H,G}) = 12n^2 - 24n + 12.$$

- *the hyper Wiener index of $\Gamma_{g,H,G}$ is*

$$WW(\Gamma_{g,H,G}) = 6n^2 - 11n + 6.$$

- *the Harary index of $\Gamma_{g,H,G}$ is*

$$H(\Gamma_{g,H,G}) = \frac{1}{2}(2n^2 + 3n - 3).$$

PROOF.

- From Definition 2.8 (1) and Lemma 3.7, we obtain

$$\begin{aligned} M_1(\Gamma_{g,H,G}) &= \sum (deg(x))^2 \\ &= \sum_{x \in G \setminus H} 2^2 + \sum_{x \in H} (2n-1)^2 \\ &= (2n-2)2^2 + 2(2n-1)^2 = 8n^2 - 6. \end{aligned} \tag{12}$$

- From Theorem 2.13 (1), and Lemma 3.7, we obtain

$$\begin{aligned} W(\Gamma_{g,H,G}) &= |V(\Gamma_{g,H,G})|(|V(\Gamma_{g,H,G})|-1) - |E(\Gamma_{g,H,G})| \\ &= 4n^2 - 6n + 3. \end{aligned}$$

- From Theorem 2.14, Lemma 3.7, and Equation 12, we obtain

$$\begin{aligned} W_e(\Gamma_{g,H,G}) &= |E(\Gamma_{g,H,g})|^2 - \frac{1}{2}M_1(\Gamma_{g,H,G}) \\ &= 12n^2 - 24n + 12. \end{aligned}$$

- From Theorem 2.13 (2), and Lemma 3.7, we obtain

$$\begin{aligned} WW(\Gamma_{g,H,G}) &= \frac{3}{2}|V(\Gamma_{g,H,G})|(|V(\Gamma_{g,H,G})| - 1) - 2|E(\Gamma_{g,H,G})| \\ &= 6n^2 - 11n + 6. \end{aligned}$$

- From Theorem 2.13 (3), and Lemma 3.7, we obtain

$$\begin{aligned} H(\Gamma_{g,H,G}) &= \frac{1}{4}|V(\Gamma_{g,H,G})|(|V(\Gamma_{g,H,G})| - 1) + \frac{1}{2}|E(\Gamma_{g,H,G})| \\ &= \frac{1}{2}(2n^2 + 3n - 3). \end{aligned}$$

Theorem 3.14. *Given $\Gamma_{g,H,G}$ the relative g-noncommuting graph of group D_{2n} with n is odd. If $H = \{e, a^j b\}$ and $g = a^i$ for some $i = 1, 2, \ldots, n-1$ and $j = 0, 1, \ldots, n-1$, then*

- *the first Zagreb index of $\Gamma_{g,H,G}$ is*

$$M_1(\Gamma_{g,H,G}) = 8n^2 - 16n + 6.$$

- *the Wiener index of $\Gamma_{g,H,G}$ is*

$$W(\Gamma_{g,H,G}) = 4n^2 - 6n + 7.$$

- *the edge Wiener index of $\Gamma_{g,H,G}$ is*

$$W_e(\Gamma_{g,H,G}) = 12n^2 - 48n + 46.$$

- *the hyper Wiener index of $\Gamma_{g,H,G}$ is*

$$WW(\Gamma_{g,H,G}) = 6n^2 - 11n + 14.$$

- *the Harary index of $\Gamma_{g,H,G}$ is*

$$H(\Gamma_{g,H,G}) = \frac{1}{2}(2n^2 + 3n - 7).$$

Proof.

- From Defnition 2.8 (1) and Lemma 3.8, we obtain

$$\begin{aligned} M_1(\Gamma_{g,H,G}) &= \sum (deg(x))^2 \\ &= \sum_{x=e}(2n-1)^2 + \sum_{x\in H\setminus\{e\}}(2n-5)^2 + \sum_{x\in G\setminus(H\cup K_{G\setminus H})} 2^2 + \sum_{x\in K_{G\setminus H}} 1^2 \\ &= 8n^2 - 16n + 6. \qquad (13) \end{aligned}$$

- From Theorem 2.13 (1), and Lemma 3.8, we obtain

$$\begin{aligned} W(\Gamma_{g,H,G}) &= |V(\Gamma_{g,H,G})|(|V(\Gamma_{g,H,G})|-1)-|E(\Gamma_{g,H,G})| \\ &= 4n^2-6n+7. \end{aligned}$$

- From Theorem 2.14, Lemma 3.8, and Equation 13, we obtain

$$\begin{aligned} W_e(\Gamma_{g,H,G}) &= |E(\Gamma_{g,H,g})|^2-\frac{1}{2}M_1(\Gamma_{g,H,G}) \\ &= 12n^2-48n+46. \end{aligned}$$

- From Theorem 2.13 (2), and Lemma 3.8, we obtain

$$\begin{aligned} WW(\Gamma_{g,H,G}) &= \frac{3}{2}|V(\Gamma_{g,H,G})|(|V(\Gamma_{g,H,G})|-1)-2|E(\Gamma_{g,H,G})| \\ &= 6n^2-11n+14 \end{aligned}$$

- From Theorem 2.13 (3), and Lemma 3.8, we obtain

$$\begin{aligned} H(\Gamma_{g,H,G}) &= \frac{1}{4}|V(\Gamma_{g,H,G})|(|V(\Gamma_{g,H,G})|-1)+\frac{1}{2}|E(\Gamma_{g,H,G})| \\ &= \frac{1}{2}(2n^2+3n-7). \end{aligned}$$

Conclusion. In this research, several topological indices are obtained, namely the first Zagreb index, Wiener index, edge Wiener index, hyper Wiener index, and Harary index. In further research, we will discuss the vertex degree of the relative g-noncommuting graph and the topological index for another case, namely the case of n even.

REFERENCES

[1] Abdollahi,A., Akbari.S. and Maimani,H.R., " Noncommuting Graph of a Group", *Journal of Algebra.*, 298(2006), 468-492.

[2] Alimon, N.I., Sarmin, N.H. and Erfanian, A., "Topological Indices of Noncommuting Graph of Dihedral Groups" , *Malaysian Journal of Fundamental and Applied Sciences.*, (2018), 473-476.

[3] Dankelman,P., Gutman,I., Mukwembi, S. and Swart, H.C., " The Edge Wiener Index of a Graph"., *Discrete Mathematics.*, 309(2009), 3452-3457.

[4] Diestel, R., *Graph Theory*, Springer, 2017.

[5] Dummit, D.S. and Foote, R.M., *Abstract Algebra*, John Wiley and Sons, 2004.

[6] Ghorbani, M. and Hosseinzadeh, M.A., "A New Version of Zagreb Indices", *Filomat.*, 26(1)(2012), 93-100.

[7] Gutman, I. and Polansky, O., *Mathematical Consepts in Organic Chemistry*, Springer-Verlag, 1986.

[8] Hildebrandt, T.H., "Linear Continuous Functionals on the Space (BV) with Weak Topologies", *Proc. Amer. Math. Soc..*, **17** (1966), 658-664.

[9] Issacs, Martin., *Algebra A Graduate Course*, American Mathematical Society, 2009.

[10] Jahandideh, M., Sarmin, N.H. and Omer, S.M.S., " The Topological Indices of Noncommuting Graph of a Finite Group", *International Journal of Pure and Applied Mathemathics.*, 105(1)(2015), 27-38.

[11] Maulana, F., Alamsyah, I.M. and Suwastika, E., "Topological Indices of Zero Divisor Graph of $\mathbb{Z}_{p^n}$ dan $\mathbb{Z}_{p^n} \times \mathbb{Z}_{q^m}$"., Thesis, Institut Teknologi Bandung, 2022.

[12] Nasiri, M., Erfanian, A. and Alamsyah, I.M., " Kind of Graph Associated to a Fixed Element and a Subgroup of Group", *Southeast Asian Bulletin of Mathemathics.*, 44(6)(2020), 813-818.

[13] Wiener, H., "Structural Determination of Paraffin Boiling Points", *Journal of the American Chemical Society.*, 69(1)(1947), 17-20.

[14] Xu, K. and Das, K.C., "On Harary Index of Gaphs", *Discrete Applied Mathematics.*, 159(15)(2011), 1631-1640.

[15] Raza, Z. and Faizi, S., "Noncommuting Graph of Finitely Presented Group", *Science International (Lahore).*, 25(2013), 883-885.

[16] Samaila, D., Ibrahim, B. and Pur, M. P., "On the Conjugacy Classes, Centers, and Representation of the Groups S_n and D_n", *International Journal of Pure and Applied Sciences and Technology.*, 15(1)(2013), 87-95.

[17] Sharma, M. and Nath, R.K., "Relative g-noncommuting Graph of Finite Group", *Electronic Journal of Graph Theory and Applications .*, 10(1)(2022), 113-130.