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ABUSE OF PSYCHOACTIVE DRUGS AND ITS PREVENTION IN A HOST COMMUNITY: MATHEMATICAL ANALYSIS

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Abstract. In this paper, psychoactive drug abuse and prevention in a host community is illustrated by a mathematical model, studying the human population into four groups: those who are at risk of being initiated into psychoactive drug abuse, those who are currently abusing psychoactive drugs, those who are undergoing treatment for psychoactive drug abuse, and those who give up drug abuse through willingness or therapy. The positivity and invariant region of the model are investigated. The basic reproduction number R_drg was also obtained. The numerical simulations were performed using the computer software MATLAB. The dynamics of variables and the sensitivity of parameters are displayed graphically to demonstrate that treatment and willingness to stop drug abuse are effective ways to reduce the threat of psychoactive drug abuse in a host community.

Key words: stability; numerical simulation; drug abuse; reproductive number.

1. Background to the study

In just the last five years, approximately 15% of Nigeria's adult population, approximately 14.3 million people, reported using psychoactive drugs at a "considerable level." This is a significantly higher rate than the global average for adults in

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2016 of 5.6%. A psychoactive drug is considered a substance of a chemical nature that modifies brain activity and causes changes in mood, behavior, consciousness, perception, or cognition [1]. Humans employ psychoactive chemicals for a variety of objectives in order to accomplish particular goals. Many psychoactive drugs, including those with approved uses in psychiatry and medicine, are utilized for their ability to change mood and perception. The usage of psychoactive drugs dates back to prehistoric times. Psychoactive chemicals (mainly plants) have been used for at least 10,000 years, according to archeological findings, and historical proof of their cultural use throughout the previous 5,000 years exists [2]. For instance, chewing coca leaves has been a part of Peruvian culture for over 8000 years [3, 4]. These drugs can be taken therapeutically, recreationally, to enhance performance intentionally, or to change one's state of awareness. Misuse, addiction, and reliance on psychoactive drugs have sparked legal actions and ethical discussions. Regulating the production, distribution, and prescription drug use by the government aims to lower the use of harmful pharmaceuticals. Here are a few instances of psychotropic drugs [5].

The history of drug abuse in Nigeria was during the civil war, the unexpected oil boom that followed with its sharp rise in the Gross National Product (GNP), the rapid socioeconomic changes and urbanization that also resulted in the breakdown of the family social network, and the rise in drug availability are just a few of the factors that have shaped Nigeria's history of drug abuse. [6, 7]. More accurate data and better research have shown that drug use has more serious and pervasive negative health effects than previously believed. According to a recent World Drug Report of the United Nations Office on Drugs and Crime (UNODC), most 35 million people worldwide are thought to have drug use disorders and need treatment services [8]. For adults, drug usage may result from a divorce or job loss; for teenagers, peer pressure and changing schools may be the cause. Early adolescence—the transition from middle to high school—can be a catalyst for drug use for kids because of the additional social and academic demands placed on them. Again, this is the time when they are initially exposed to a variety of substances that can be abused, such as alcohol and cigarettes.

This work presents a mathematical model that describes the social threat of psychoactive drug misuse among various groups of the human host population. We organized this paper as follows: The next section contains a model formulation of psychoactive substance misuse. Section 3 provides the psychoactive abuse reproduction number. Section 4 depicts the model's steady-state solutions. Section 5 discusses the steady-state solutions for psychoactive substance misuse. Section 6 presents numerical simulations and a discussion of the results, while Section 7 concludes the work.

2. Mathematical formulation of psychoactive drug abuse

There are four classes in the human population studied and denoted $P(t)$ at any given time t.The human population's governing equation can be described as

follows:

$$P(t) = S(t) + U(t) + T(t) + Q(t) \tag{1}$$

where $S(t)$ denotes the individuals at risk of being initiated into psychoactive drug abuse, $U(t)$ denotes who are psychoactive drug abuse, $T(t)$ denotes those who are undergoing treatment for psychoactive drug abuse and $Q(t)$ denotes those who give up drug abuse through willingness or treatment.

We adopt the following assumptions:

i. Initiation to psychoactive drug abuse happens through successful social interactions between those who abuse psychoactive drugs and those who are at risk of doing so.
ii. Through birth, people are added to the susceptible population
iii. Due to rehabilitation, treated drug abusers cannot initiate susceptible individuals.
iv. Recovery from psychoactive drug abuse is permanent.
v. There is uniform mingling amongst members of various population classes.
vi. Mortality due to other factors is applicable to all classes of the population.
vi. All users have the same initiation ability.

We describe the dynamics of the psychoactive drug abuse population as a system of four ordinary differential equations, we obtain the following model:

$$\begin{cases} \dfrac{dS}{dt} &= \Pi - (\mu + \lambda)S, \\ \dfrac{dU}{dt} &= \lambda S - (\mu + \phi + \alpha + \gamma)U, \\ \dfrac{dT}{dt} &= \alpha U - (\mu + \sigma)T, \\ \dfrac{dQ}{dt} &= \sigma T + \gamma U - \mu Q, \end{cases} \tag{2}$$

and the initial variables are all positif.

$$S(0) \geq 0, U(0) \geq 0, T(0) \geq 0, Q(0) \geq 0,$$

with

$$\lambda = \beta \frac{U(0)}{P(0)}.$$

It is expected that every model parameter is positive. The parameter Π is the recruitment of susceptible through birth, β is the initiation rate between a drug abuser and individuals who are at risk of becoming drug abusers, and σ is the transition rate of treated individuals to recovered class, γ is the rate of psychoactive drug abusers willingly can give up drug abuse, μ is the rate of normal death, α is the rate of psychoactive drug abuser can be treated, and ϕ is the mortality rate due to excessive use of psychoactive drugs. Figure 2 shows the schematic behavior of how the variables in the model (2) interact.

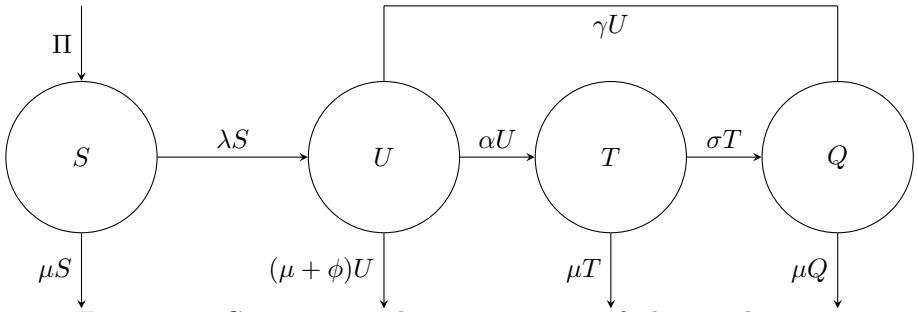

FIGURE 1. Compartmental representation of the psychoactive drug abuse among classes of human population

2.1. **Analysis of the model equations for psychoactive drug abuse.**

The following result gives the positivity of solutions for the systems of equations given by (2) as follows.

Theorem 2.1. *Consider the systems of equations in (2) subject to the positive initial conditions then the solutions are all positive.*

Proof. Suppose that $\tau = sup\,\{t\in\mathbb{R}^+\colon S(t)\geq 0,\, U(t)\geq 0,\, T(t)\geq 0,\, Q(t)\geq 0\} \in [0,t]$. Thus $\tau>0$ and from the first equation of systems (2), we have that

$$\frac{dS}{dt} = \Pi - (\mu+\lambda)S.$$

Using the integrating factor method, this becomes

$$\frac{d}{dt}\left[S(t)\exp\left\{\mu t + \int_0^\tau \lambda(s)ds\right\}\right] = \Pi\exp\left[\mu t + \int_0^\tau \lambda(s)ds\right].$$

Solving further,

$$S(\tau)\exp\left[\mu\tau + \int_0^\tau \lambda(s)ds\right] - S(0) = \int_0^\tau \Pi\exp\left[\mu\tau + \int_0^\tau \lambda(\omega)d\omega\right].$$

So that

$$S(\tau) \geq S(0)\exp\left[-\left(\mu\tau + \int_0^\tau \lambda(s)ds\right)\right]$$

$$+\exp\left[-\left(\mu\tau + \int_0^\tau \lambda(s)ds\right)\right]\left[\int_0^\tau \Pi\exp\left(\mu\tau + \int_0^\tau \lambda(\omega)d\omega\right)d\tau\right] \geq 0.$$

From the second equation of (2), we get :

$$\frac{dU}{dt} \geq -(\mu+\phi+\alpha+\gamma)U,$$

$$U(t) \geq U(0)\exp(\mu+\phi+\alpha+\gamma)t \geq 0.$$

Likewise, it can be demonstrated that for any $t \geq 0$, $T(t) \geq 0$ and $Q(t) \geq 0$. The proof is now complete. □

The next result summarizes the invariant region of the model (2).

Theorem 2.2. *The feasible area* Γ *defined by:*

$$\Gamma = \left\{ (S,U,T,Q) \in \mathbb{R}^4_+ \mid 0 \leq P \leq \frac{\Pi}{\mu} \right\}$$

For the positive initial conditions $S(0) \geq 0$, $U(0) \geq 0$, $T(0) \geq 0$, $Q(0) \geq 0$. *The feasible area is positively invariant with respect to* (2) *is restricted.*

Proof. From (1), differentiating both sides of the above equation, one obtains

$$\frac{dP}{dt} = \frac{dS}{dt} + \frac{dU}{dt} + \frac{dT}{dt} + \frac{dQ}{dt}. \tag{3}$$

Using (2) and (3),

$$\frac{dP}{dt} = \Pi - \mu(S+U+T+Q) - \phi U.$$

In the absence of mortality due to excessive psychoactive drug abuse, the last equation becomes

$$\frac{dP}{dt} + \mu P \leq \Pi.$$

Integrating both sides

$$P(t)e^{-\mu t} \leq \int \Pi e^{\mu t} dt \leq \Pi \frac{e^{\mu t}}{\mu} + C.$$

Simplifying further also yields

$$P \leq \frac{\Pi}{\mu} + Ce^{-\mu t}.$$

Where C is a constant.
Using the initial condition $P(0) = P_0$, we try to choose the optimal constant, thus

$$C = P_0 - \frac{\Pi}{\mu}\ .$$

Therefore

$$P(t) \leq \frac{\Pi}{\mu}\left(1 - e^{-\mu t}\right) + P_0 e^{-\mu t}. \tag{4}$$

From (4),

$$0 \leq \lim_{t\to\infty} \inf P(t) \leq \lim_{t\to\infty} \sup P(t) \leq \frac{\Pi}{\mu}.$$

So that

$$P(t) \leq \frac{\Pi}{\mu} \quad \forall, t \geq 0.$$

Hence $\lim_{t\to\infty} \sup \frac{\Pi}{\mu}$ as $t \to \infty$ in (4), $P(t) \to \frac{\Pi}{\mu}$ such that $0 \leq P(t) \leq \frac{\Pi}{\mu}$. Then, the feasible domain of the model with positive initial conditions is restricted. Therefore,

$$\Gamma = \left\{ (S,\, U,\, T,\, Q)\,, \in \quad \mathbb{R}^4_+ \middle|\, 0 \leq P \leq \frac{\Pi}{\mu} \right\}$$

This completes the proof. □

The following remarks were deduced:

(a) Equation (2) is mathematically well-posed and realistic in the sense of social psychoactive drug abuse menace.
(b) If $\phi \neq 0$, that is, absence of psychoactive drug abuse, it is assumed that for a low level of mortality due to drug abuse ($\phi \approx 0$) if the system is slightly perturbed, that is, small fraction of drug abusers are introduced into the system, it goes back to its steady state value $\frac{\Pi}{\mu}$ in a short time.

3. The reproduction number of the psychoactive drug abuse

To determine the reproduction number of psychoactive drug misuse, R_{drg}, we compute the next generation matrix. As it is used in many papers, for example, [10, 11, 12]. The computation of R_{drg} is linearized around the psychoactive drug - free equilibrium $D_0\left(S_0, U_0, T_0, Q_0\right) = \left(\frac{\Pi}{\mu}, 0, 0, 0\right)$ to give the following matrices

$$F_1 = \begin{pmatrix} \beta S & 0 \\ 0 & 0 \end{pmatrix}$$

$$F_2 = \begin{pmatrix} \mu + \phi + \alpha + \gamma & 0 \\ -\alpha & \mu + \sigma \end{pmatrix}$$

The spectral radius of the following generation matrix $F_1 F_2^{-1}$ is therefore R_{drg}. Hence

$$R_{drg}\left(F_1 F_2^{-1}\right) = \frac{\Pi\beta}{\mu(\mu + \phi + \alpha + \gamma)}. \tag{5}$$

The following remarks are deduced as follows :

(a) In the context of this work, R_{drg} is characterized as the mean number of secondary cases of psychoactive drug abusers generated when a single psychoactive drug abuser is introduced into the naive population of individuals who are at the risk of being drug abusers. β is defined as the initiation rate per drug abuser; each abuser is going to spend an average $\frac{1}{\alpha+\gamma}$ time units in the psychoactive drug abusers class. Later, the class will be reduced by natural mortality due to other factors and mortality due to excessive drug abuse.
(b) If $R_{drg} < 1$, the social menace of psychoactive drug abuse vanishes away from the system. Therefore, controls of education awareness, rehabilitation, governmental policies against drug abuse are effective strategies for keeping the R_{drg} threshold below 1.
(c) If $R_{drg} > 1$, then the problem of psychoactive drug abuse persists in the system.

4. Steady state solutions of model

The steady-state solutions of the model (2) are obtained to determine what will likely happen in the short or long term if psychoactive drug abuse is eliminated or become persistent, leading to a full-blown social menace.

4.1. **Local asymptotic stability analysis of the psychoactive drug abuse-free steady-state solutions of the model.**

To check for the local asymptotic stability of the psychoactive drug abuse-free steady-state solutions of the model (2), the Jacobian variational matrix of the system of equations must be obtained; see [12]. Also, according to [13], the P. drug abuse - free steady states are said to be locally asymptotically stable if the eigenvalues of the Jacobian matrix of model (2) evaluated at the steady state solutions have negative real parts. The following results give the local asymptotic stability analysis of the psychoactive drug-free steady-state solutions as follows;

Theorem 4.1. *The psychoactive drug abuse-free steady-state solutions are locally asymptotically stable if* $R_{drg} < 1$ *and unstable if* $R_{drg} > 1$.

Proof. The Jacobian matrix of system (2) evaluated at the psychoactive drug abuse-free steady-state solutions is given by

$$J = \begin{pmatrix} -\mu & 0 & 0 & 0 \\ 0 & \lambda S - (\mu + \phi + \alpha + \gamma) & 0 & 0 \\ 0 & \alpha & -(\mu + \sigma) & 0 \\ 0 & \gamma & \sigma & -\mu \end{pmatrix}$$

It is observed that the eigenvalues with negative real parts are $-\mu, -(\mu + \sigma)$ and $-\mu$ except $\lambda S - (\mu + \phi + \alpha + \gamma)$, it will be negative when $R_{drg} < 1$.
This implies that $\lambda S > (\mu + \phi + \alpha + \gamma)$, so that $\left(\frac{\lambda S}{\mu+\phi+\alpha+\gamma}\right) - 1 > 0$, so that $(R_{drg} - 1) > 0$.
Therefore, the psychoactive drug abuse-free steady-state solutions are locally asymptotically stable.
This completes the proof. □

4.2. **Global stability of the psychoactive drug abuse-free steady-state solutions.**

Theorem 4.2. *The psychoactive drug abuse-free steady state solution is global asymptotic stable if* $R_{drg} < 1$.

Proof. A Lyapunov function is defined to be

$$\frac{dV_l}{dt} = \frac{dU}{dt}.$$

Along model system (2) gives

$$\frac{dV_l}{dt} = [\lambda S - (\mu + \phi + \alpha + \gamma)U]\,.$$

Therefore

$$\frac{dV_l}{dt} = \frac{1}{(\mu + \phi + \alpha + \gamma)}[R_{drg} - 1]U.$$

For $R_{drg} < 1$, when $U = 0$ as a steady state solution

$$\frac{dV_l}{dt} = 0.$$

Also, $R_{drg} = 1$, then $S = 1$. Therefore, $R_{drg} < 1$ shows that the psychoactive drug abuse-free steady state is globally asymptotic stable. □

5. Steady state of the psychoactive drug abuse model

The drug present steady state of system (2) satisfies

$$\begin{cases} \Pi - (\mu + \lambda)S^a &= 0, \\ \lambda S^a - (\mu + \phi + \alpha + \gamma)U^a &= 0, \\ \alpha U^a - (\mu + \sigma)T^a &= 0, \\ \sigma T^a + \gamma U^a - \mu Q^a &= 0. \end{cases} \tag{6}$$

From (6)

$$T^a = \Phi_1 U^a, \quad \Phi_1 = \frac{\alpha}{\mu + \sigma}.$$

Substituting T^a into the last equation of (6) to obtain

$$Q^a = \Phi_2 U^a, \quad \Phi_2 = \frac{\sigma}{\mu}\Phi_1 + \frac{\gamma}{\mu}.$$

From (6)

$$U^a \left(\frac{\beta S}{N} - (\mu + \phi + \alpha + \gamma) \right) = 0.$$

So, we have $U^a = 0$ corresponding to the drug-free equilibrium point
Or

$$S^a = \frac{\mu + \phi + \alpha + \gamma}{\beta}.$$

Then

$$U^a = \frac{\Pi}{\mu + \phi + \alpha + \gamma} - \frac{\mu}{\beta}.$$

This implies that

$$U^a = \frac{1}{\beta}\,(R_0 - 1)\,.$$

Thus, E^a the endemic equilibrium is given by $E^a = (S^a, U^a, T^a, Q^a)$ where:

$$S^a = \frac{1}{R_0}, U^a = \frac{1}{\beta}(R_0 - 1), T^a = \frac{\alpha}{\beta(\mu+\sigma)}(R_0 - 1), Q^a = \frac{1}{\beta}\left(\frac{\sigma\alpha}{\mu(\mu+\sigma)} + \frac{\gamma}{\mu}\right)(R_0 - 1).$$

Theorem 5.1. *The psychoactive drug abuse-free steady state solution is globally asymptotic stable if $R_{drg} > 1$.*

Proof. Since the steady state solutions were given by (S^a, U^a, T^a, Q^a).
A Lyapunov function is defined by

$$(S^a, U^a, T^a, Q^a) = \frac{(S - S^a)^2}{2S^a} + \left(U - U^a - U^a \ln \frac{U}{U^a}\right) + \frac{\alpha}{(\mu+\sigma)}\left(T - T^a - T^a \ln \frac{T}{T^a}\right)$$

The Lyapunov function V_l is positive definite and continuous for all S^a, U^a, T^a, Q^a. The function V_l takes the value $V_l(S, U, T) = 0$ at the psychoactive drug abuse present steady state solutions S^a, U^a, T^a. Differentiating $V_l(S, U, T)$ along the solutions after some simplifications yields

$$\frac{(S - S^a)}{S^a}\left[\frac{\beta S^a I^a}{P} + \mu S^a - \frac{\beta SI}{P} - \mu S\right] + \frac{(U - U^a)}{U^a}\left[\frac{\beta S^a I^a}{PU^a} - \frac{SI}{U}\right]$$

$$\frac{\alpha}{(\mu+\sigma)}(T - T^a)\left(\frac{T^a}{U^a} - \frac{T}{U}\right)$$

$$\frac{(S - S^a)}{S^a}\frac{\beta}{P}\left[S^a(I^a - I) + \mu(S^a - S) + I(S - S^a)\right] + \frac{(U - U^a)}{U^a}\left[\frac{\beta}{P}\left[\frac{S^a(I - I^a)}{U^a} + \frac{I(S - S^a)}{U}\right]\right]$$

$$\frac{\alpha}{(\mu+\sigma)}\frac{(T - T^a)}{T^a}\left(\frac{T^a}{U^a} - \frac{T}{U}\right) \leq 0$$

$\dot{V}_l \leq 0$ for $S, U, T, Q > 0$. The equality $\dot{V}_l(S, U, T, Q) = 0$ holds only when $S = S^a$, $U = U^a$, $T = T^a$.
Therefore, The psychoactive drug abuse-present steady state is the largest invariant set in

$$\left\{(S, U, T, Q) \in \Gamma : \dot{V}_l(S, U, T, Q) = 0\right\}$$

The psychoactive drug abuse-present steady state is globally asymptotically stable. □

6. Numerical simulations and discussion of results

In this section, the simulations are done via implicit Euler discrete numerical method in MATLAB script with the set of parameters given in Table 1. Also, the following initial starts were adopted in the course of the simulations as $S = 500, U = 400, T = 300$, and $Q = 200$.
The parameters were considered in realistic ranges from the literature concerning drug abuse in Nigeria.

Parameter	Description	Value	Reference
Π	Birth rate	0.0346	[14]
μ	Death due to natural factors	0.0091	[14]
β	Drug abuse initiation rate	7.7	[15]
ϕ	Mortality due to excessive drug abuse	2.11	[16]
α	Treatment rate for drug abusers	0.4	[16]
γ	Rate of willingness to quit drug abuse	0.0002	Estimated
σ	Transmission rate	0.0341	Estimated

TABLE 1. Parameters, descriptions and values of model (2)

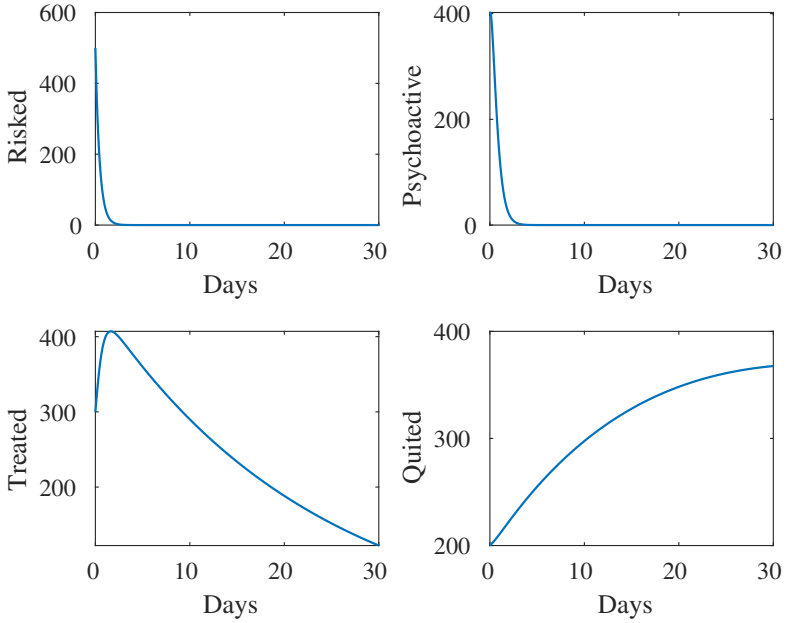

FIGURE 2. Numerical simulation results of the risked psychoactive drug abuse, psychoactive drug abuse, treated psychoactive drug abuse, and quite psychoactive drug abuse.

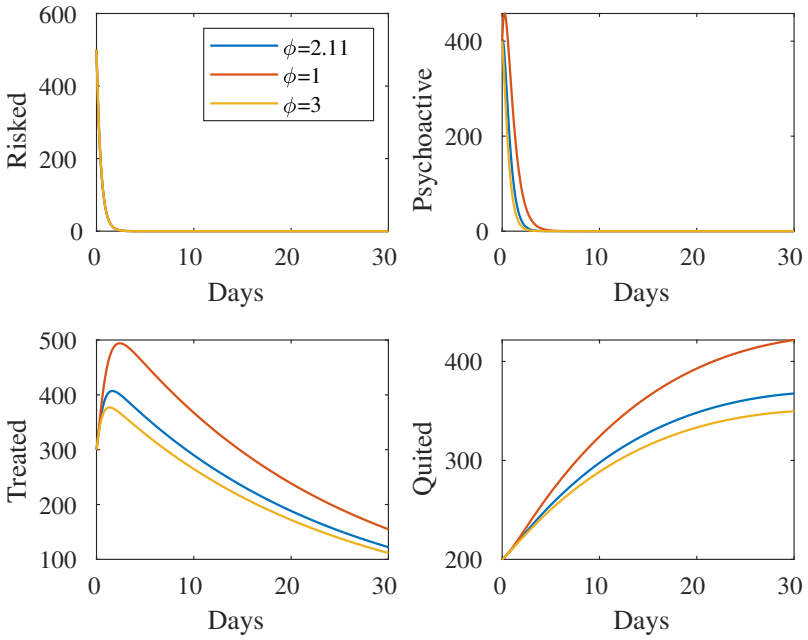

FIGURE 3. Numerical simulation results of the risked psychoactive drug abuse, psychoactive drug abuse, treated psychoactive drug abuse, and quite psychoactive drug abuse for different values of ϕ.

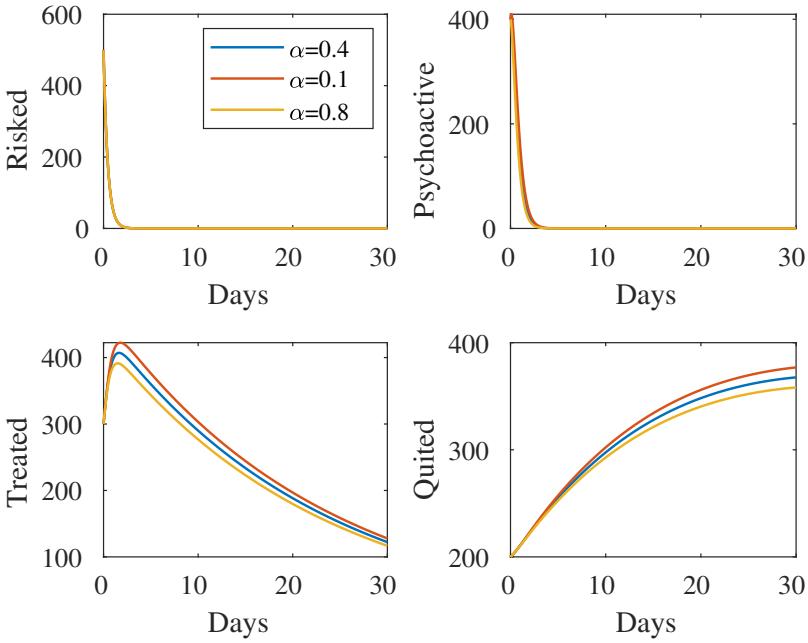

FIGURE 4. Numerical simulation results of the risked psychoactive drug abuse, psychoactive drug abuse, treated psychoactive drug abuse, and quite psychoactive drug abuse for different values of α.

6.1. **Discussion of results.**

Figure 2 describes the behaviour of the susceptible class, as time increases, this class decreases. The sharp decline depicts that between $1 - 30$ days, there would be a quick outflow from this class due to likely initiation into psychoactive drug abuse or death due to natural factors. The figure depicts a sharp increase in the cases of psycho-drug abusers, between $200 - 600$ individuals within a few days. Still, a gradual decline occurs and flattens out after a few days due to the availability of treatment, willingness to quit, death due to psychoactive drug abuse, and death that occurs due to natural factors. It also describes the behavior of the class of treated individuals. As time increases with the administration of treatment, a gradual decline is experienced to show that effective treatment reduces drug abuse in the human host community. The figure indicates the behavior of the recovered class. The steady increase in the curve between $0 - 10$ days depicts the wellness of drug abusers. A slight decrease observed after the 10 first days may occur due to the natural death rate.

Figure 3 describes the impact of the variation of ϕ(1 - 3). As ϕ increases, at the first days, more drug psychoactive drug abusers will die as a result of excessive use of drugs and failure to be available for treatment and other rehabilitation policies. Figure 4 depicts the impact of the rate at which psychoactive drug abusers are treated by varying α(0.1-0.8). As α increases, more individuals are treated, which is effective and essential to stop the psychoactive drug abuse menace.

7. Conclusion

We formulated a mathematical model describing the psychoactive drug abuse initiation among the classes of human individuals. The model positivity and well-posedness were analyzed. The steady-state solution of the model was obtained to show that if $R_{drg} < 1$, the psychoactive drug abuse free steady state is locally asymptotically stable, and if $R_{drg} > 1$, the psychoactive drug abuse present steady state is globally asymptotically stable. Realistic parameter values obtained from the Nigerian drug abuse cases adopted in the course of simulations revealed that concerted efforts of health policymakers through orientation, treatment, rehabilitation, and strong governmental laws prohibiting drug abuse are effective in curtailing the social menace of drug abuse.

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