J. Indones. Math. Soc.
Vol. 30, No. 01 (2024), pp. 110–120.

THE NON-BRAID GRAPH OF DIHEDRAL GROUP D_n

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Abstract. We introduce the non-braid graph of a group G, denoted by $\zeta(G)$, as a graph with vertex set $G \setminus B(G)$, where $B(G)$ is the braider of G, defined as the set $\{x \in G \mid (\forall y \in G) xyx = yxy\}$, and two distinct vertices x and y are joined by an edge if and only if $xyx \neq yxy$. In this paper particularly we give the independent number, the vertex chromatic number, the clique number, and the minimum vertex cover of non-braid graph of dihedral group D_n.

Keywords and Phrases: non-braid graph, dihedral group, independent number, vertex chromatic number, clique number, minimum vertex cover

1. INTRODUCTION

Recently, there have been some interesting studies on algebraic graphs since what had been done by Cayley in 1878 [2]. There are many ways to link algebraic structures to graphs and vice versa. One of those works was done by Abdollahi et.al [1] in 2006 on non-commuting graph of a group. The non-commuting graph given by Abdollahi, et all is defined as a graph with the set of all non-center elements as the vertex set, and two different vertices are connected by an edge if and only if they are not commuting. Particularly, in 2008, Talebi [6] gave some properties of non-commuting graphs of dihedral groups, including independent numbers, vertex chromatic numbers, clique numbers, and minimum vertex cover. Based on the

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2020 Mathematics Subject Classification: 05C99

Received: 11-04-2023, accepted: 10-03-2024.

work of Abdollahi et al, e construct a set center-like called braider of the group for any group. Moreover, we define a non-braid graph of any group using its braider. Motivated by the work of Talebi [6] and Hubbi, et all [9], particularly we investigate non-braid graph for dihedral group.

Throughout this paper, by graph, we mean a finite undirected simple graph. Moreover, symbols $V(\zeta)$ and $E(\zeta)$ denote the vertex set and the edge set for any graph ζ, respectively.

2. The Non-Braid Graph of Dihedral Group D_n

First, let us recall some definitions used in this section. Let G be a group. The braider of G, denoted $B(G)$ is the set $B(G) = \{x \in G | (\forall y \in G) xyx = yxy\}$. We define the non-braid graph ζ_G of G as a simple graph with $G \backslash B(G)$ as the vertex set and any two distinct vertices $x, y \in G \backslash B(G)$ are adjacent if and only if $xyx \neq yxy$. The definition of non braid graph of ring can be seen in [8]

Let us recall also that a dihedral group D_n is a group of the form

$$D_n = \langle r, s | r^n = s^2 = (sr)^2 = I \rangle$$

with I as the identity element of D_n. For this group, we have the following basic property on its braider.

Theorem 2.1. *For any natural number $n \geq 3$, $B(D_n) = \emptyset$.*

Proof. Note that if $k \equiv p_1 \pmod{n}$ then $r^k = r^{p_1}$, also if $k \equiv p_2 \pmod{2}$ then $s^k = s^{p_2}$. Furthermore, since $B(G) = \{x \in G | \ (\forall y \in G) xyx = yxy\}$ and $s^2 = r^n = I, rs = sr^{-1} \iff sr = r^{-1}s$ then we can consider the proofing with 3 cases.

(1) If $x = I$ then for all $y \in D_n$, $xyx = yxy$, then $IyI = yIy$. On the other hand, we can see if $y = r$, then implies $IyI = r = yIy = r^2$. There is a contradiction, since $r \neq r^2$.
(2) If $x = r^k$, with $1 \leq k \leq n-1$. Then by assumption, we have $xyx = r^k y r^k = y r^k y$, $\forall y \in D_n$. Let $y = I$, then $r^{2k} = r^k \iff r^k = I$. Thus, $k \equiv 0 \pmod{n}$. There is a contradiction, since $k \not\equiv 0 \pmod{n}$, for $1 \leq k \leq n-1$.
(3) If $x = sr^k$, with $1 \leq k \leq n$. Then by assumption, we have $xyx = sr^k y sr^k = y sr^k y$, $\forall y \in D_n$. Let $y = I$, then $sr^k sr^k = sr^k \iff sr^k r^{-k} s = sr^k \iff s^2 = I = sr^k$. Hence, there is a contradiction, since $sr^k \neq I$, for $1 \leq k \leq n$.

And we can conclude there is no $x \in D_n$ such that $xyx = yxy$, $\forall y \in D_n$. Hence $B(D_n) = \emptyset$. $\square$

As a direct consequence of Theorem 2.1, we obtain $D_n \backslash B(D_n) = D_n$.

Below are some examples of graph non-braid of dihedral groups.

Example 2.2. (1) *Let $D_5 = \{I, r, r^2, r^3, r^4, s, sr, sr^2, sr^3, sr^4\}$, $\zeta(D_5)$ is a complete graph.*

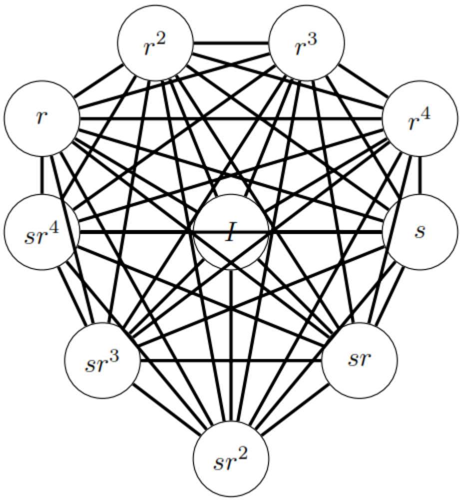

FIGURE 1. The Non-Braid Graph of Dihedral Group D_5

(2) *Let* $D_6 = \langle s, r | s^2 = r^6 = I, rs = sr^{-1} \iff sr = r^{-1}s \rangle$
Since
(i) $ssr^2s = sr^2ssr^2 = sr^4$
(ii) $ssr^4s = sr^4ssr^4 = sr^2$
(iii) $srsr^3sr = sr^3srsr^3 = sr^5$
(iv) $srsr^5sr = sr^5srsr^5 = sr^3$
(v) $sr^2sr^4sr^2 = sr^4sr^2sr^4 = s$
(vi) $sr^3sr^5sr^3 = sr^5sr^3sr^5 = sr$.
Then, we have the non-braid graph of D_6 *as given in Figure 2.*

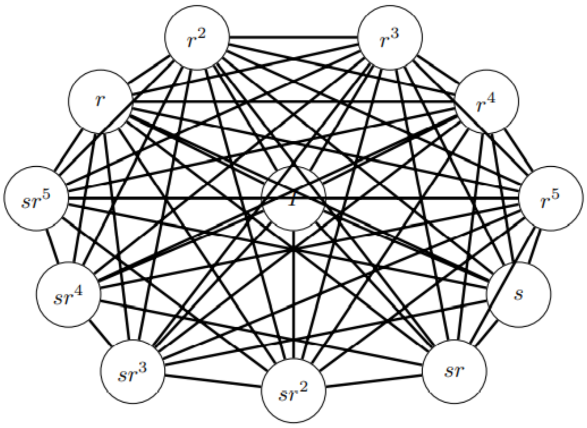

FIGURE 2. The Non-Braid Graph of Dihedral Group D_6

In the following results, we present some properties of vertex adjacency of the non-braid graph of the dihedral group.

Theorem 2.3. *Vertex I of $\zeta(D_n)$ is adjacent to every vertex in $\zeta(D_n)$.*

Proof. Let $x \neq I \in V(\zeta(D_n))$ and suppose that x and I are not adjacent. According to Theorem 2.1 we just need to check for $y = r^k$, with $1 \leq k \leq n-1$ and $y = sr^k$, with $1 \leq k \leq n$. Hence, it considers two cases as follows.

(i) Case 1: If $y = r^k$, with $1 \leq k \leq n-1$. By assumption, we have $xyx = yxy \Rightarrow Ir^kI = r^kIr^k \iff r^k = r^{2k} \iff I = r^k$. Hence, there is a contradiction since $r^k \neq I$, for $1 \leq k \leq n-1$

(ii) Case 2: If $y = sr^k$, with $1 \leq k \leq n$. By assumption, we have $xyx = yxy \Rightarrow Isr^kI = sr^kIsr^k \iff sr^k = sr^ksr^k \iff sr^k = sr^kr^{(-k)}s \iff sr^k = I$. Hence, there is a contradiction since $sr^k \neq I$, for $1 \leq k \leq n$

□

Theorem 2.4. *All vertices in $\langle r \rangle \subseteq V(\zeta(D_n))$ are adjacent to every vertex in $\zeta(D_n)$.*

Proof. Suppose there exist $x \in V(\zeta(D_n))$ and $y \in \langle r \rangle$ such that x and y not adjacent, so $xyx = yxy$. By Theorem 2.1, x and y respectively can be expressed as

$$x = s^kr^l, 0 \leq k \leq 1, 0 \leq l \leq n-1$$

$$y = r^m, 0 \leq m \leq n-1.$$

This problem can separated into 2 cases:

(i) Case 1: If $k = 0$.
Then, $xyx = r^lr^mr^l = r^{2l+m}$ and $yxy = r^mr^lr^m = r^{2m+l}$, so that $xyx = yxy \iff r^l = r^m$. On the other hand, it is known that $0 \leq l \leq n-1$ and $0 \leq m \leq n-1$. Therefore, $xyx = yxy \iff r^l = r^m \iff l = m \iff x = y$. This contradicts the statement that vertex x and y are adjacent. We conclude that $x \neq y$.

(ii) Case 2: If $k = 1$.
Then, $xyx = sr^lr^msr^l = ssr^{-l-m+l} = r^{n-m}$ and $yxy = r^msr^lr^m = sr^{-m+l+m} = sr^l$, so that $xyx = yxy \iff r^{n-m} = sr^l$. This implies a contradiction. Therefore, vertices of $\langle r \rangle \subseteq V(\zeta(D_n))$ are adjacent to every vertex of $\zeta(D_n)$.

□

Theorem 2.5. *Let $\zeta(D_n)$ be a non-braid graph of dihedral group with $n \neq 0(mod3)$. Then every vertex of $P = \{sr^i | i = 0, 1, 2, 3, \ldots, n-1\} \subseteq V(\zeta(D_n))$ is adjacent to every vertex in $V(\zeta(D_n))$.*

Proof. Suppose there exist $x \in V(\zeta(D_n)), y \in P$ such that x and y are not adjacent. Then, x and y satisfy $x = s^kr^l, 0 \leq k \leq 1, 0 \leq l \leq n-1$ and $y = sr^m$, and $xyx = yxy$. We consider two cases:

(i) Case 1: If $k = 0$
We have
$$r^l sr^m r^l = sr^m r^l sr^m \iff sr^m = r^{-l} = r^{n-l}.$$
A contradiction.

(ii) Case 2: If $k = 1$
We have
$$sr^l sr^m sr^l = sr^m sr^l sr^m \iff r^{m-l} sr^l = r^{l-m} sr^m$$
$$\iff sr^{2l-m} = sr^{2m-l} \iff r^{3(l-m)} = I$$
as $l \neq m$ and $n \neq 0 (mod 3)$. Thus $r^{3(l-m)} \neq I$, contradiction with $r^{3(l-m)} = I$.
Hence, we have a contradiction.

So far we have proven that if $\zeta(D_n)$ is the non-braid graph of dihedral group with $n \neq 0\ (mod 3)$, then each vertex in $P = \{sr^i | i = 0, 1, 2, 3, \ldots, n-1\} \subseteq V(\zeta(D_n))$ is adjacent to all vertices in $V(\zeta(D_n))$. □

Corollary 2.6. *Let $\zeta(D_n)$ be a non-braid graph of dihedral group $\zeta(D_n)$ with $n \neq 0\ (mod 3)$. Then $\zeta(D_n)$ is a complete graph.*

PROOF. We know that $|D_n| = 2n$ and by Theorem 2.3, Theorem 2.4, and Theorem 2.5, all vertices of $\zeta(D_n)$ are adjacent whenever $n \neq 0(mod 3)$. Consequently, $\zeta(D_n) = K_2 n$ is a complete graph. □

Theorem 2.7. *Let $\zeta(D_{n=3m})$ be a non-braid graph of dihedral group with $n = 0(mod 3)$. Then every two vertices in $H_i = \{sr^i, sr^{i+m}, sr^{i+2m}\}$ where $i = 0, 1, 2, \ldots, \frac{n}{3} - 1$ are not adjacent.*

PROOF. Let $x, y \in H_i$. Then x and y can be expressed as:
$$x = sr^{i+p_1 m} \text{ and } y = sr^{i+p_2 m}.$$
Furthermore,
$$\begin{aligned} xyx &= sr^{i+p_1 m} sr^{i+p_2 m} sr^{i+p_1 m} = r^{p_2 m - p_1 m} sr^{i+p_1 m} = sr^{i+2p_1 m - p_2 m} \\ &= sr^{i+2(p_1-p_2)m} = sr^{i+2(n-(p_1-p_2))m} = sr^{i+2(n+(p_2-p_1)m} = sr^{i+2(p_2-p_1)m} \\ &= sr^{i+2p_2 m - p_1 m} = r^{p_1 m - p_2 m} sr^{i+p_2 m} = s^2 r^{i-i} r^{p_1 m - p_2 m} sr^{i+p_2 m} \\ &= sr^{i+p_2 m} sr^{i+p_1 m} sr^{i+p_2 m} = yxy. \end{aligned}$$
□

Corollary 2.8. *Let $\zeta(D_{n=3m})$ be a non-braid graph of dihedral group with $n = 0(mod 3)$ and $H_i = \{sr^i, sr^{i+m}, sr^{i+2m}\}$ where $i = 0, 1, 2, \ldots, \frac{n}{3} - 1$. If $H = \{H_i | i = 0, 1, 2, \ldots, \frac{n}{3} - 1\}$ and $|H| > 1$ then every vertex in H_i is adjacent with every vertex in H_j with $i \neq j$.*

Proof. Suppose $x \in H_i$ and $y \in H_j$ with $i \neq j$ such that $xyx = yxy$. Since $x \in H_i$ and $y \in H_j$ then x and y can be expressed as:

$$x = sr^{i+p_1m} \text{ and } y = sr^{j+p_2m}.$$

Then we have

$$xyx = sr^{i+p_1m}sr^{j+p_2m}sr^{i+p_1m} = r^{(j-i)+(p_2-p_1)m}sr^{i+p_1m} = sr^{(2i-j)+(2p_1-p_2)m}$$

$$yxy = sr^{j+p_2m}sr^{i+p_1m}sr^{j+p_2m} = r^{(i-j)+(p_1-p_2)m}sr^{j+p_2m} = sr^{(2j-i)+(2p_2-p_1)m}$$

$$xyx = yxy \iff sr^{(2i-j)+(2p_1-p_2)m} = sr^{(2j-i)+(2p_2-p_1)m} \iff r^{3(i+p_1m)} = r^{3(j+p_2m)}.$$

Since $\bigcap H_i = \varnothing, i = 0, 1, 2, \ldots, \frac{n}{3} - 1$ then $r^{3(j+p_2m)}$ cannot be expressed as $r^{3(i+p_2m)}$. Since p_2 is an arbitary number $0 \leq p_2 \leq 2$ then equality $r^{3(i+p_1m)} = r^{3(j+p_2m)}$ cannot happen. This means that the supposition is failed. Hence, it is proved that each vertices in H_i and H_j are adjacent for $i \neq j$. □

In the following remaining discussion, we will give some results on the independent number, vertex chromatic number, clique number, and minimum vertex cover of non-braid graph of dihedral group

Recall that, for any graph ζ, subgraph H of ζ is called induced subgraph if $x, y \in V(H)$ and x and y are adjacent in ζ then x and y are adjacent in H. Furthermore, if $X \subseteq V(\zeta)$ then we defined the subgraph induced by X as an induced subgraph H with $V(H) = X$. A subset $X \subseteq V(\zeta)$ is called an independent set if there is no edge between any two vertices in X. Equivalently, a subset $X \subseteq V(\zeta)$ is called an independent set if the subgraph induced by X has no edges. The cardinality of a maximum independent set in a graph ζ is called the independent number of ζ and denoted $\alpha(\zeta)$.

Theorem 2.9. *Let $\zeta(D_n)$ be non-braid graph of dihedral group D_n. If $n \neq 0(mod 3)$ then $\alpha(\zeta(D_n)) = 1$.*

Proof. Let $\zeta(D_n)$ be a non-braid graph of dihedral group D_n. According to Corollary 2.6, if $n \neq 0(mod3)$, then every two vertices in $\zeta(D_n)$ are adjacent. Thus $\alpha(\zeta(D_n)) = 1$. □

Theorem 2.10. *Let $\zeta(D_n)$ be a non-braid graph of dihedral group D_n. Then $\alpha(\zeta(D_n))$ is 1 or 3.*

Proof. Let $\zeta(D_n)$ be a non-braid graph of dihedral group D_n. If $n \neq 0(mod3)$, then by Theorem 2.9 $\alpha(\zeta(D_n)) = 1$. If $n = 3m$ for some positive integer m, then by Theorem 2.7 every two vertices in $H_i = \{sr^i, sr^{i+m}, sr^{i+2m}\}, i = 0, 1, 2, \ldots, \frac{n}{3} - 1$ are not adjacent. Furthermore, according to Theorem 2.4 every vertices in $\langle r \rangle \subseteq V(\zeta(D_n))$ are adjacent to each vertices in $\zeta(D_n)$. However, by Corollary 2.8 every vertices in H_i adjacent to every vertices in H_j for $i \neq j$ and $\langle r \rangle \cup H_i = V(\zeta(D_n)), i = 0, 1, 2, \ldots, \frac{n}{3} - 1$. Thus, the maximum number of vertices that can be selected to be the induced subgraph has no edges is 3, that is $\alpha(\zeta(D_n)) = 3$. □

Recall that the vertex chromatic number of graph ζ, denoted by $\chi(\zeta)$, is the minimum positive integer n such that we can assign n colors to the vertices of ζ in such a way so that no adjacent vertices are having the same color. The following examples give us a clue about the vertex chromatic number of the non-braid graph of dihedral group.

(1) Vertex Chromatic Number of $\zeta(D_3)$
The vertex coloring on $\zeta(D_3)$ is given in Figure 3 as follows:

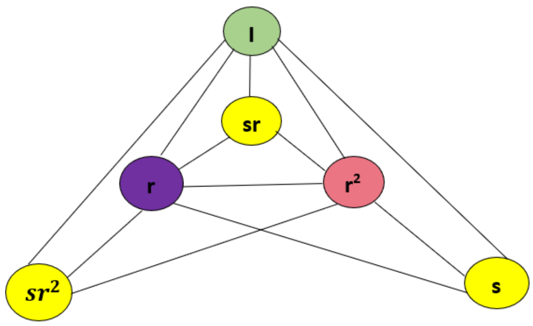

FIGURE 3. The vertex coloring on $\zeta(D_3)$

According to Figure 3, the vertex chromatic number of $\zeta(D_3)$ is 4 i.e $\chi(\zeta(D_3)) = 4$.

(2) Vertex Chromatic Number $\zeta(D_4)$
Since $\zeta(D_4)$ is a complete graph K_8, then its vertex chromatic number is 8, i.e $\chi(\zeta(D_4)) = 8$.

(3) Vertex Chromatic Number $\zeta(D_5)$
Since $\zeta(D_5)$ is a complete graph K_{10}, then the vertex chromatic number is 10, i.e $\chi(\zeta(D_5)) = 10$.

(4) Vertex Chromatic Number $\zeta(D_6)$
The vertex coloring on $\zeta(D_6)$ is given in the Figure 4 as follows.

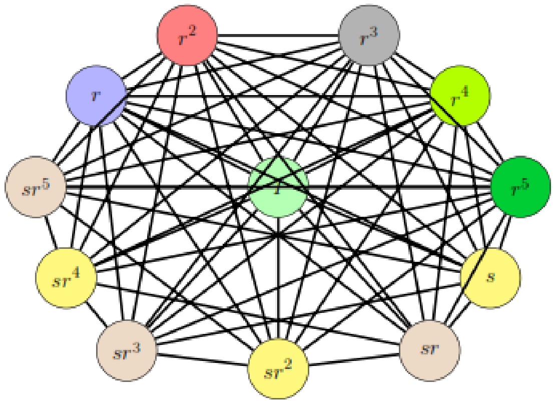

FIGURE 4. The vertex coloring on $\zeta(D_6)$

According to Figure 4, we obtain the vertex chromatic number on $\zeta(D_6)$ is 8, i.e $\chi(\zeta(D_6)) = 8$.

Below are given some vertex chromatic numbers of $\zeta(D_n)$.

TABLE 1. The Vertex Chromatic Numbers of $\zeta(D_n)$ for several n

$\zeta(D_n)$	$\chi(\zeta(D_n))$
D_3	4
D_4	8
D_5	10
D_6	8
D_7	14
D_8	16
D_9	12

From Table 1, we prove the following theorem.

Theorem 2.11. *Let $\zeta(D_n)$ be a non-braid graph of dihedral group D_n. If $n = 0(mod3)$, then the vertex chromatic number of dihedral group D_n is $\chi(\zeta(D_n)) = n + \frac{n}{3}$. If $n \neq 0(mod3)$, then the vertex chromatic number of dihedral group D_n is $\chi(\zeta(D_n)) = 2n$.*

PROOF. Let D_n be the dihedral group

$$D_n = \{I, r, r^2, \ldots, r^{n-1}, s, sr, sr^2, \ldots, sr^{n-1}\}, s^2 = I, r^n = I, rs = sr^{-1}$$

$$\iff r^{-1}s = sr.$$

(i) Let $n = 3m$ for some natural number m. Then,
$$D_{3m} = \{I, r, r^2, \ldots, r^{3m-1}, s, sr, sr^2, \ldots, sr^{3m-1}\}.$$
Let $V = \langle r \rangle = \{I, r, r^2, \ldots, r^{3m-1}\}$. According to Theorem 2.5, every vertex in V is adjacent to each vertex in $\zeta(D_n)$. Thus the minimum number of colors to color the vertices in V is n. Let :
$P_1 = \{s, sr^m, sr^{2m}\}$
$P_2 = \{sr, sr^{m+1}, sr^{2m+1}\}$
$P_2 = \{sr^2, sr^{m+2}, sr^{2m+2}\}$
$\vdots$
$P_k = \{sr^{k-1}, sr^{m+k-1}, sr^{2m+k-1}\}$.
Let color p_1 represents the vertex color of $s, sr^m, sr^{2m} \in P_1$. Color p_2 represents the vertex color of $sr, sr^{m+1}, sr^{2m+1} \in P_2, \ldots$ And lastly we have color p_k represents the vertex color of $sr^{k-1}, sr^{m+k-1}, sr^{2m+k-1} \in P_k$.
Furthermore, p_i and p_j are different color if and only if every vertex in P_i are adjacent with every vertex in P_j. On the other hand, by Corollary 2.8 vertex in P_i adjacent with vertex in P_j if $i \neq j$. This means that color p_i and p_j are different if and only if $i \neq j$. From these facts, we can conclude that the number of colors $p_1, p_2, \ldots, p_k$ is $\frac{n}{3}$.
Thus, the vertex chromatic number for $\zeta(D_{n=3m})$ is $n + \frac{n}{3}$.

(ii) Let $n \neq 0(mod3)$.
Let $D_n = \{I, r, r^2, \ldots, r^{n-1}, s, sr, sr^2, \ldots, sr^{n-1}\}$, with $n \neq 3m, m \in \mathbb{N}$. Then by Corollary 2.6 graph $\zeta(D_{n\neq 3m})$ is complete graph, hence the vertex chromatic number of $\zeta(D_{n\neq 3m})$ is $2n$.

□

A subset X of the vertex set of graph ζ is called clique if the subgraph induced by X is a complete graph. The maximum clique size in graph ζ is called the clique number of $G\zeta$, denoted by $\omega(\zeta)$. We now give the clique number of the non-braid graph of D_n in general.

Theorem 2.12. *Let $\zeta(D_n)$ be a non-braid graph of dihedral group D_n.*

(1.) *If $n = 0(mod3)$, then the clique number of $\zeta(D_n)$ is $n + \frac{n}{3}$.*
(2.) *If $n \neq 0(mod3)$, then the clique number of $\zeta(D_n)$ is $2n$.*

Proof. Let $D_n = \{I, r, r^2, \ldots, r^{n-1}, s, sr, sr^2, \ldots, sr^{n-1}\}, s^2 = I, r^n = I, rs = sr^{-1} \iff r^{-1}s = sr$.

(1.) Let $\zeta(D_n)$ be the non-braid graph of D_n, with $n = 0(mod3)$. From Theorem 2.4 we know that every vertex in $\langle r \rangle$ is adjacent to all vertices in $\zeta(D_n)$. Clearly, the cardinality of $\langle r \rangle$ is n. From Theorem 2.7 we have $H_i, i = 1, 2, \ldots, \frac{n}{3} - 1$ which all vertices in H_i are not adjacent and there are as many as $\frac{n}{3}$ H_i in $\zeta(D_n)$. Since $\langle r \rangle \cup H_i = V(\zeta(D_{3m}))$, the maximum size of clique in $\zeta(D_n)$ is $n + \frac{n}{3}$.
(2.) From Corollary 2.6 we know that $\zeta(D_n)$ is a complete graph. Hence, the maximum size of clique in $\zeta(D_n)$ is $2n$. In other word, $\omega(\zeta(D_n)) = 2n$.

□

Recall that the vertex cover of graph ζ is a subset of $V(\zeta)$ that contains at least one endpoint of every edge in ζ. The minimum size of vertex covers of ζ is denoted by $\beta(\zeta)$. Recall also the following property on vertex cover.

Lemma 2.13. [6] *For any graph ζ, $\beta(\zeta) = |V(\zeta)| - \alpha(\zeta)$.*

Now, we have the following result.

Theorem 2.14. *Let $\zeta(D_n)$ be the non-braid graph of dihedral group D_n. Then $\beta(\zeta(D_n)) = 2n - 1$ if $n \neq 0(mod3)$ and $\beta(\zeta(D_n)) = 2n - 3$ for otherwise.*

PROOF. Let $m = 0(mod3)$. By Theorem 2.9, $\alpha(\zeta(D_n))$ is equal to 1 . Using Lemma 2.13, we have $\beta(\zeta(D_n)) = |V(\zeta(D_n))| - \alpha(\zeta(D_n)) = 2n - 1$. Now, let $n = 3m$ for some natural number m. Then by Theorem 2.7, we have $H_i = \{sr^i, sr^{i+m}, sr^{i+2m}\}, i = 0, 1, 2, \ldots, \frac{n}{3} - 1$ and all vertices in H_i are not adjacent. We know that all vertices of $\langle r \rangle \subseteq V(\zeta(D_n))$ are adjacent to $V(\zeta(D_n))$. Since all vertices in H_i are adjacent to every vertex in H_j for $i \neq j$, the minimum vertex cover of $\zeta(D_n)$ is $\beta(\zeta(D_n)) = 2n - 3$. □

3. CONCLUDING REMARKS

Based on the discussion, we found that the non-braid graph of the dihedral group has the following properties:

Let $\zeta(D_n)$ be the non braid graph of dihedral group. Then

(1) For $n \neq 0(mod3)$, $\zeta(D_n)$ is a complete graph.
(2) Every vertex in $\langle r \rangle = \{r^i | i = 0, 1, 2, 3, \ldots, n-1, n\} \subseteq V(\zeta(D_n))$ is adjacent to all of vertices in $\zeta(D_n)$.
(3) For $n \neq 0(mod3)$, every vertex in $P = \{sr^i | i = 0, 1, 2, 3, \ldots, n - 1\} \subseteq V(\zeta(D_n))$ is adjacent to every vertex in $V(\zeta(D_n))$.
(4) The independent number of $\zeta(D_n)$ is 1 or 3.
(5) The chromatic number and the clique number of $\zeta(D_n)$ is $n + \frac{n}{3}$ for $n \neq 0(mod3)$ and is $2n$ for $n \neq 0(mod3)$.
(6) The minimum vertex cover of $\zeta(D_n)$ is $2n-3$ for $n \neq 0(mod3)$ and is $2n-1$ for $n \neq 0(mod3)$.

Open Problem Determine the structure of non-braid graph of any group in general.

Acknowledgement. Authors thank the referees for their valuable comment and suggestion to improve this manuscript.

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