J. Indones. Math. Soc.
Vol. 29, No. 01 (2023), pp. 106–115.

QUASICONTINUOUS FUNCTIONS ON STRONG FORM OF CONNECTED SPACES

Chander Mohan Bishnoi[1], Sanjay Mishra[2]

[1]**Department of Mathematics, Faculty, Lovely Professional University, Phagwara, INDIA, chandermohan.cm.b@gmail.com**

[2]**Department of Mathematics, Faculty, Lovely Professional University, Phagwara, INDIA, drsanjaymishra1@gmail.com**

Abstract. Preservation of properties under continuous functions on topological spaces is a very important tool for the classification of topological spaces. However, in some cases the quaiscontinuous functions are more useful than the continuous functions for classifying topological spaces. In this paper, we study preservation of strong forms of connectedness under quasicontinuous function that help to prove the general form of intermediate value theorem.

Key words and Phrases: Quasicontinuous, Half connected, Semi-connected and Half semi-connected.

1. INTRODUCTION

In 1899, Baire [3] used the condition of quasicontinuity to study topological spaces. Later in 1932, Kempisty [6] introduced the concept of quasicontinuous map for several real variables. The conditions for quasicontinuity of function of two variables provided by Volterra [3]. In 1976, Neubrunn [12] reformulated the Kempisty's definition of quasicontinuity for general topological spaces as: "a map $f\colon X \to Y$ is quasicontinuous at $p \in X$ if for any open sets U in X and V in Y such that $p \in U$ and $f(p) \in V$, then there exists a non-empty subset G of U such that $f(G) \subset V$. It is said to be quasicontinuous if it is quasicontinuous at any $p \in X$". All continuous maps are quasicontinuous but its converse not holds. For example $f\colon \mathbb{R} \to \mathbb{R}_l$ defined by $f(x) = x$ is quasicontinuous function but not continuous, where $\mathbb{R}$ and $\mathbb{R}_l$ are set of real numbers with usual and lower limit topology respectively. Quasicontinuity has deep connection with mathematical analysis, topology and many applications in analysis, topology, measure theory, and probability theory.

2020 Mathematics Subject Classification: 54C08; 54D05; 54C05.
Received: 17-09-2022, accepted: 16-02-2023.

Roughly speaking, a connected space is a single piece and it is defined as follows: any topological space (X, τ) is said to be connected if there does not exist any separation (i.e. it is not possible to find non-empty disjoint open sets A and B such that $X = A \cup B$). Connectedness is a topological property and has great importance in the study of topological spaces. Some of the generalized form of connectedness like semi-connectedness, α-connectedness, β-connectedness, b-connectedness, half connected, half semi-connectedness, half-α-connectedness and half β- connectedness have been introduced and studied in [16, 5, 4, 15, 15, 18]. The Cl-Cl-connectedness introduced by Modak and Noiri in [8] is a weak form of connectedness and all other are strong forms of connectedness. In this paper we study quasicontinuous functions on the mentioned forms of connected space. Throughout this paper, Qc stands for quasicontinuous and SQc stands for strongly quasicontinuous. In the second section, we mention some results that we will use in the subsequent sections. In the third section a weak form of semi-connectedness is introduced and its inter-relationship with semi-connectedness, connectedness and Cl-Cl connectedness is further studied. In last section, we discussed the preservation of some strong form of connectedness under Qc-maps and the applicability of the intermediate value theorem for Qc-maps.

2. Pre-requisites

Let X, Y and Z be topological spaces. A subset A is said to be semi-open [7] (α-open [14], b-open [2], β-open [1]) if $A \subset Cl(Int(A))$ (resp. $A \subset Int(Cl(Int(A)))$, $A \subset (Int(Cl(A))) \cup Cl(Int(A))$, $A \subset Cl(Int(Cl(A))))$. The complement of a semi-open (resp. α-open, b-open, β-open) set is said to be semi-closed (resp. α-closed, b-closed, β-closed). Semi-closure of a set A is the intersection of all semi-closed set containing A. Similar definitions for α-closure, b-closure and β-closure. Their notations are semi-closure (Scl), α-closure (αCl), b-closure (bcl), β-closure (βCl).

A map $f\colon X \to Y$ to be α-continuous or SQc-map [14] (semi-continuous [7]) the inverse image of any open set V in Y is an α-open (semi-open) set in X. For a single valued map, the concept of semi-continuity and quasicontinuity are equivalent [[11], Theorem 1.1].

Definition 2.1. *Two non-empty subsets A and B in a space X are said to be*

(1) semi-separated [16] *(resp. α-separated* [5]*, b-separated* [15]*) if $A \cap Scl(B) = \varnothing = Scl(A) \cap B$, (resp. $A \cap \alpha Cl(B) = \varnothing = \alpha Cl(A) \cap B$, $A \cap bcl(B) = \varnothing = bcl(A) \cap B$).*
(2) half semi-separated (resp. half α-separated) [18] *if $A \cap Scl(B) = \varnothing$ or $Scl(A) \cap B = \varnothing$, (resp. $A \cap \alpha Cl(B) = \varnothing$ or $\varnothing = \alpha Cl(A) \cap B$)*
(3) $Cl - Cl$ separated sets [8] *if $Cl(A) \cap Cl(B) = \varnothing$.*

Theorem 2.2. [18] *Let $f\colon X \to Y$ be a Qc-map (resp. SQc-map). Then $Scl(f^{-1}(B)) \subseteq f^{-1}(Scl(B))$ (resp. $\alpha Cl(f^{-1}(B)) \subseteq f^{-1}(\alpha Cl(B))$).*

Definition 2.3. [16] *A subset S of a space X is said to be semi-connected (resp., α-connected) if there are no two semi-separated subsets A and B (resp. α-separated) such that $S = A \cup B$.*

Theorem 2.4. [16] *Semi-connected space is connected, but not conversely.*

Definition 2.5. [18] *A set $A \subset X$ is said to be half semi- connected (resp. half α-connected) if A is not the union of two non-empty half semi-separated (resp. half α-separated) sets in X.*

Theorem 2.6. [18] *Every half semi-connected space is semi-connected, but not conversely.*

Theorem 2.7. [18] *Every semi-connected space is α-connected, but not conversely.*

Definition 2.8. [8] *A set $A \subset X$ is said to be Cl-Cl connected if A cannot be expressed as the union of two Cl-Cl separated sets in X.*

Theorem 2.9. [8] *Every connected spaces is always Cl-Cl connected, but not conversely.*

3. New Connectedness lies between semi-connectedness and Cl-Cl connectedness

In this section, we will be defining a strong form of semi-connectedness and study its relations with other forms of connectedness.

Definition 3.1 (Scl-Scl separated sets)**.** *Let A and B are two non-empty subsets of the space X are called Scl-Scl separated sets if $Scl(A) \cap Scl(B) = \varnothing$.*

Theorem 3.2. *Every Cl-Cl separated sets are always Scl-Scl separated, but not conversely.*

Proof. Let A and B are non-empty Cl-Cl separated set so $Cl(A) \cap Cl(B) = \varnothing$ Since for any subset C of X, then $Scl(C) \subset Cl(C)$. This implies that $Scl(A) \cap Scl(B) = \varnothing$. Hence A and B are Scl-Scl separated.

Converse, let $X = \{1, 2, 3\}$ with topology $\tau = \{\varnothing, \{1\}, \{1, 2\}, X\}$. Semi-open subsets of X are $\varnothing, \{1\}, \{1, 2\}, \{1, 3\}$ and X. Take $A = \{2\}$ and $B = \{3\}$ then $Scl(A) = \{2\}$ and $Scl(B) = \{3\}$. But $Cl(A) = \{2, 3\}$ and $Cl(B) = \{3\}$, then $Scl(A) \cap Scl(B) = \varnothing$ and $Cl(A) \cap Cl(B) = \{3\}$. Hence A and B are Scl-Scl separated but not Cl-Cl separated. □

Theorem 3.3. *Every Scl-Scl separated sets are always semi-separated, but not conversely.*

Proof. Given A and B are non-empty Scl-Scl separated sets, so $Scl(A) \cap Scl(B) = \varnothing$. Since for any subset C of X, then $C \subset Scl(C)$. This implies that $A \cap Scl(B) = \varnothing$ and $Scl(A) \cap B = \varnothing$. Hence A and B are semi-separated.

Converse, take $\mathbb{R}$ with usual topology. $A = \{(-1)^n \frac{1}{2^n} | n \in \mathbb{N}\}$ and $B = \{(-1)^n \frac{1}{3^n} \; | n \in \mathbb{N}\}$, then $Scl(A) = \{0\} \cup A = Cl(A)$ and $Scl(B) = \{0\} \cup B = Cl(B)$. Therefore $Scl(A) \cap Scl(B) = \{0\}$ and $Scl(A) \cap B = \varnothing = A \cap Scl(B)$. Hence A and B are semi-separated but not Scl-Scl Separated. □

A Scl-Scl separation have no relation with separation. Let $X = \{1, 2, 3\}$ with topology $\tau = \{\varnothing, \{1\}, \{1, 2\}, X\}$. Semi-open subsets of X are $\varnothing, \{1\}, \{1, 2\}, \{1, 3\}$ and X. Then set $A = \{2\}$ and $B = \{3\}$ are Scl-Scl separated but $Cl(A) \cap B = \{3\}$, hence A and B are not separated.

Consider $\mathbb{R}$ with usual topology, $A = \{(-1)^n \frac{1}{2^n} | n \in \mathbb{N}\}$ and $B = \{(-1)^n \frac{1}{3^n} | n \in \mathbb{N}\}$, then $A \cap Cl(B) = \varnothing = Cl(A) \cap B$, but $Scl(A) \cap Scl(B) = \{0\}$. Hence A and B are separated sets but not Scl-Scl separated.

Definition 3.4 (Scl-Scl connectedness)**.** *A subset A of X is said to be Scl-Scl connected if A cannot be represented as union of two Scl-Scl separated sets in X.*

Theorem 3.5. *A space X is Scl-Scl connected if and only if it is not possible to express X as union of two non-empty and disjoint semi clopen sets.*

Proof. Firstly, assume that X be Scl-Scl connected. If possible, let us assume $X = A \cup B$ such that A and B are non-empty disjoint semi clopen sets. Therefore $Scl(A) = A$ and $Scl(B) = B$, then $Scl(A) \cap Scl(B) = \varnothing$, which is contradiction to X is Scl-Scl connected. Hence it cannot be expressed as disjoint union of two non-empty semi clopen sets.

Conversely, let us assume X is not Scl-Scl connected space. Therefore there exist two non empty sets A and B such that $X = A \cup B$ and $Scl(A) \cap Scl(B) = \varnothing$. Then $X = Scl(A) \cup Scl(B)$. Both sets $U = Scl(A)$ and $V = Scl(B)$ are non-empty disjoint semi clopen sets and $X = U \cup V$, this is a contradiction. Hence X is Scl-Scl connected. □

Theorem 3.6. *Every Scl-Scl connected space is always $Cl - Cl$ connected, but not conversely.*

Proof. As X Scl-Scl connected if there are no two Scl-Scl separated subsets A and B such that $X = A \cup B$. Then by Theorem 3.2 there are no two Cl-Cl separated subsets A and B such that $X = A \cup B$. Hence X is Cl-Cl connected.

Converse, let $X = \{1, 2, 3\}$ with topology $\tau = \{\varnothing, \{1\}, \{2\}, \{1, 2\}, X\}$. Take $A = \{1\}$ and $B = \{2, 3\}$, then A and B are disjoint semi clopen sets with $X = A \cup B$. Therefore by Theorem 3.5 space X is not Scl-Scl connected space but it is connected. □

Theorem 3.7. *Every semi-connected space is always Scl-Scl connected, but not conversely.*

Proof. Let X be a semi-connected space, then there does not exists two semi-separated subsets A and B such that $X = A \cup B$. By Theorem 3.3, there are no two Scl-Scl separated subsets A and B such that $X = A \cup B$. Hence X is Scl-Scl connected. Conversely it need not be hold, by example in the proof of the Theorem 3.3. □

We found that Scl-Scl connected space has no relation with connected space. This can be verify with an example. Let $X = \{1, 2, 3\}$ with topology $\tau = \{\varnothing, \{1\}, \{2\}, \{1, 2\}, X\}$. The semi-open subsets of X are $\varnothing, \{1\}, \{2\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, X$.

Let $A = \{1\}$ and $B = \{2,3\}$ are non-empty semi clopen sets and $X = A \cup B$, then by Theorem 3.5, X is not Scl-Scl connected but it is connected. Take $\mathbb{R}$ with the usual topology is connected but not Scl-Scl connected because $\mathbb{R} = (-\infty, 1] \cup (1, \infty)$ both are disjoint and Scl-Scl separated sets. From the all results, we have the following diagram:

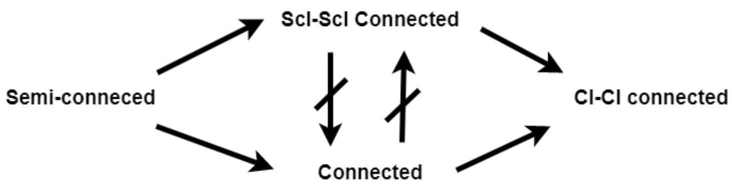

Figure 1. Relationship between different connected spaces

4. Preservation under the Qc-map

In this section, we are going to study the preservation of some strong forms of connectedness of the space under a Qc-map. At the end, we prove the intermediate value theorem for a Qc-map. All the stronger forms of Qc-map are denoted by QS-map and it is understood that any QS-map is also Qc-map but its converse need not be true, for example, continuous maps and SQc-map both are stronger then Qc-map.

Theorem 4.1. *If P property is preserved under a Qc-map, then P preserved under a QS-map.*

Proof. Given that property P is preserved under a Qc-map, that is for any Qc-map $: X \to Y$ if X has P property then $f(X)$ also has P property. To prove P is preserved under a QS-map. Let $g\colon X \to Y$ be any QS-map and X has P property. We must prove $g(X)$ has P. Since every QS-map is a Qc-map. So g is Qc-map. Therefore $g(X)$ has P property. Hence QS-map preserves P property. ☐

Corollary 4.2. *If P property is preserved under a Qc-map, then P preserved during a continuous map, but not conversely.*

Proof. Since continuous map is stronger then Qc-map. Hence it preserve property P. Conversely, let $X = \{1,2,3\}$ with topologies $\tau_1 = \{\varnothing, \{1\}, \{2\}, \{1,2\}, X\}$ and $\tau_2 = \{\varnothing, \{1\}, \{2,3\}, X\}$. Then, identity map $f : (X, \tau_1) \to (X, \tau_2)$ is a Qc-map. As (X, τ_1) is connected space but (X, τ_2) is not a connected space. ☐

Theorem 4.3. *The image of semi-connected space under a Qc-map is semi-connected.*

Proof. Given X is semi-connected and $f : X \to Y$ is a Qc-map. If possible, assume $f(X)$ is not semi-connected, so by Definition 2.3 there exist two non-void sets A and B such that $f(X) = A \cup B$ and $A \cap Scl(B) = \varnothing = Scl(A) \cap B$. Then $X = f^{-1}(A) \cup f^{-1}(B)$. Firstly, $A \cap Scl(B) = \varnothing$ implies $f^{-1}(A \cap Scl(B)) = \varnothing$ and since f is a Qc-map, so by using Theorem 2.2, we get

$$f^{-1}(A) \cap Sclf^{-1}(B) \subset f^{-1}(A \cap Scl(B)) = \varnothing$$

Similarly

$$Sclf^{-1}(A) \cap f^{-1}(B) \subset f^{-1}(Scl(A) \cap (B)) = \varnothing$$

This contradicts the semi-connectedness of X. Hence $f(X)$ must be semi-connected. □

Remark 4.4. *The image of α-connected space under a SQc-map is α-connected.*

Corollary 4.5. *The image of semi-connected space under a Qc-map is connected.*

Proof. By above Theorem 4.3 the image of semi-connected space under Qc-map is semi-connected, then by Theorem 2.4 every semi-connected is connected. □

In a similar way we can prove.

Remark 4.6. *The image of α-connected space under a SQc-map is connected.*

Corollary 4.7. *[*[18]*, Theorem* 5.4*] The image of semi-connected (resp. α-connected) space under a Qc-map (resp. SQc-map) is Cl-Cl connected.*

Proof. By above Corollary 4.5, the image of a semi-connected space under Qc-map is connected, then by Theorem 2.9, a connected space is Cl-Cl connected. □

Corollary 4.8. *The image of semi-connected space under a Qc-map is β-connected, α-connected, α_β-connected* [9]*.*

Proof. By using Theorem 4.3. □

Theorem 4.9. *The image of half semi-connected space under a Qc-map is half semi-connected.*

Proof. Given X is half semi-connected and $f : X \to Y$ is a Qc-map. Let us assume that $f(X)$ is not half semi-connected so by Definition 2.5 there exist two non-empty sets A and B such that $f(X) = A \cup B$ and $A \cap Scl(B) = \varnothing$ or $\varnothing = Scl(A) \cap B$. By hypotheses $X = f^{-1}(A) \cup f^{-1}(B)$. If $A \cap Scl(B) = \varnothing$ implies $f^{-1}(A \cap Scl(B)) = \varnothing$ and since f is a Qc-map so by using Theorem 2.2 we get

$$f^{-1}(A) \cap Sclf^{-1}(B) \subset f^{-1}(A \cap Scl(B)) = \varnothing$$

On the other hand, if $Scl(A) \cap B = \varnothing$ implies $Sclf^{-1}(A) \cap f^{-1}(B) \subset f^{-1}(Scl(A) \cap (B)) = \varnothing$. This contradicts the half semi-connectedness of X. Hence $f(X)$ must be half semi-connected. □

Remark 4.10. *The image of half α-connected space under a SQc-map is half α-connected.*

Corollary 4.11 ([18], Theorem5.4)**.** *The image of half semi-connected space under a Qc-map is Cl-Cl connected.*

Theorem 4.12. *The image of semi-connected space under a SQc-map is semi-connected.*

Proof. From Theorem 4.3, the image of semi-connected space under Qc-map is semi-connected. Since the SQc-map is stronger then Qc-map, so by Theorem 4.1, the image of semi-connected space under SQc-map is semi-connected. □

Remark 4.13. *The image of half semi-connected space under a SQc-map is half semi-connected.*

Theorem 4.14. *The image of Scl-Scl connected space under a Qc-map is Scl-Scl connected.*

Proof. Given $f\colon X \to Y$ be a Qc-map, X be Scl-Scl connected and Y be any topological space. If possible, assume that $f(X)$ is not Scl-Scl connected, there exist two non-empty disjoint subsets of $f(X)$ such that $f(X) = A \cup B$ and $Scl(A) \cap Scl(B) = \varnothing$. Then $X = f^{-1}(A) \cup f^{-1}(B)$. As $Scl(A) \cap Scl(B) = \varnothing$ implies

$$f^{-1}(Scl(A) \cap Scl(B)) = \varnothing$$

$$f^{-1}(Scl(A)) \cap f^{-1}(Scl(B)) = \varnothing$$

by Theorem 2.2 we have $Scl f^{-1}(A) \subset f^{-1}(Scl(A))$ so

$$Scl f^{-1}(A) \cap Scl f^{-1}(B) = \varnothing$$

therefore, the sets $f^{-1}(A)$ and $f^{-1}(B)$ a form Scl-Scl separation which contradicts that X is Scl-Scl connected. Hence $f(X)$ is Scl-Scl connected. □

Theorem 4.15. *The image of Scl-Scl connected space under a SQc-map is Scl-Scl connected.*

Proof. From Theorem 4.14, the image of Scl-Scl connected space under Qc-map is Scl-Scl connected. Since the SQc-map is stronger than Qc-map, so by Theorem 4.1 the image of Scl-Scl connected space under a SQc-map is Scl-Scl connected. □

Theorem 4.16. *The image of b-connected space under a Qc-map is semi-connected.*

Proof. Given X is b-connected space and $f\colon X \to Y$ is a Qc-map. Let us assume that $f(X)$ is not semi-connected, so by Definition 2.3 there exist two non-empty set A and B such that $f(X) = A \cup B$ and $A \cap Scl(B) = \varnothing = Scl(A) \cap B$. Then $X = f^{-1}(A) \cup f^{-1}(B)$. Firstly, we take $A \cap Scl(B) = \varnothing$ implies $f^{-1}(A \cap sCl(B)) = \varnothing$ and since f is a Qc-map, so by using Theorem 2.2 we get

$$f^{-1}(A) \cap Scl f^{-1}(B) \subset f^{-1}(A \cap Scl(B)) = \varnothing$$

Since we know $bcl(A) \subset Scl(A)$ then, $f^{-1}(A) \cap bcl f^{-1}(B) \subset \varnothing$. In a similar way from $Scl(A) \cap B = \varnothing$, we get $bcl f^{-1}(A) \cap f^{-1}(B) \subset \varnothing$. This shows that X is not semi-connected, a contrary to our assumption. Hence $f(X)$ must be semi-connected. □

Remark 4.17. *The image of β-connected space under a SQc-map is semi-connected.*

Theorem 4.18. *The image of b-connected (resp. β-connected* [5]*) space under a SQc-map is semi-connected.*

Proof. By Theorem 4.16 the image of b-connected space under a Qc-map is semi-connected. Since SQc-map is stronger then Qc-map. So by Theorem 4.1 the image of b-connected space under a SQc-map is semi-connected. □

Theorem 4.19. *The image of half b-connected space under a Qc-map is half semi-connected.*

Proof. Given X is half b-connected space and $f\colon X \to Y$ is a Qc-map. Let us assume that $f(X)$ is not half semi-connected so by Definition 2.5 there exist two non-empty sets A and B such that $Y = A \cup B$ and $A \cap Scl(B) = \varnothing$ or $\varnothing = Scl(A) \cap B$. By hypotheses $X = f^{-1}(A) \cup f^{-1}(B)$. If $A \cap Scl(B) = \varnothing$ implies $f^{-1}(A \cap Scl(B)) = \varnothing$ and since f is a Qc-map so by using Theorem 2.2 we get

$$f^{-1}(A) \cap Scl f^{-1}(B) \subset f^{-1}(A \cap Scl(B)) = \varnothing$$

by using $bcl(A) \subset Scl(A)$, we get $f^{-1}(A) \cap bcl f^{-1}(B) = \varnothing$. On the other hand, if $Scl(A) \cap B = \varnothing$, in similar way we get

$$bcl f^{-1}(A) \cap f^{-1}(B) = \varnothing$$

this shows that X is not half semi-connected which is a contradiction. Hence $f(X)$ must be half semi-connected. □

Remark 4.20. *The image of half β-connected space under a Qc-map is half semi-connected.*

Theorem 4.21. *The image of half b-connected (resp. half β-connected) space under a SQc-map is half semi-connected.*

Proof. From Theorem 4.19, the image of half b-connected under a Qc-map is half semi-connected. Since SQc-map is stronger then Qc-map. By Theorem 4.1, the image of half b-connected space under a SQc-map is half semi-connected. □

The main consequence of the intermediate value theorem of calculus is the study of real valued continuous functions on closed intervals $[a, b]$ of real line, where we consider the closed interval as a subset of $\mathbb{R}$. When we consider this closed interval as a topological space, then this theorem does not depend only on the continuity of function but on properties of the space also. Now the connectedness as a topological property inherited by the space comes into picture, which gives the general form of this theorem. In the next result, we see how the intermediate value theorem of calculus is generalised using topological approach as:

Theorem 4.22. *(Intermediate value theorem)*[10] *Let X be a connected space, Y be an ordered topological space and $f\colon X \to Y$ is a continuous map. If $a, b \in X$ and $r \in Y$ such that $r \in (f(a), f(b))$. Then there exist $c \in X$ such that $f(c) = r$.*

Now the question arises from the general form of the intermediate value theorem: Is it possible that the theorem also holds for a more general form of continuous map that is Qc-map? Unfortunately, we can see from the below example that it does not hold good for Qc-map.

Example 4.23. *Let $X = [0, 2], Y = \mathbb{R}$ be topological space with usual topology and $f\colon X \to Y$ is a map defined as follows:*

$$f(x) = \begin{cases} 3 & \text{if } 0 \leq x \leq 1/2 \\ 1 & \text{if } 1/2 < x \leq 1 \\ -x & \text{if } 1 < x \leq 2 \end{cases}$$

The map f is a Qc-map. Take $r = 2 \in Y$, then r lies between $f(1/2) = 3$ and $f(1) = 1$. But there is no $c \in X$ such that $f(c) = r$. Hence the intermediate value theorem is not hold when we replace continuous by a Qc-map.

Now we can find that the general intermediate value theorem is not possible for Qc-map. But in the case of semi-connected space, it works.

Theorem 4.24 (Intermediate value theorem for Qc-map)**.** *Let X be a semi-connected space, Y is an ordered topological space and $f\colon X \to Y$ is a Qc-map. If $a, b \in X$ and $r \in Y$ s.t. $r \in (f(a), f(b))$. Then there exist $c \in X$ such that $f(c) = r$.*

Proof. Given X is a semi-connected space, Y is an ordered topological space and $f\colon X \to Y$ be a Qc-map. The sets $A = f(X) \cap (-\infty, r)$ and $B = f(X) \cap (r, \infty)$ are non-empty and disjoint because $f(a) \in A$ and $f(b) \in B$. Both sets are open in $f(X)$ because both rays are open in Y. If there is no $c \in X$ such that $f(c) = r$, then $f(X)$ would be the union of A and B. Then A and B form a separation of $f(X)$, this contradicts that the image of semi-connected space under Qc-map is connected. Hence there exists a $c \in X$ such that $f(c) = r$. □

Remark 4.25. *If X be a α-connected space and f be a SQc-map, then intermediate value theorem also hold.*

5. CONCLUSION

Throughout this paper, we studied the preservations of different forms of connectedness under a quasicontinuous map.

REFERENCES

[1] Abd EI-Monsef, M.E., Mahmoud, R.A., and El-Deeb, S.N., "β-open sets and β-continuous mappings", *Bull. Fac. Sci. Assiut Univ* 1983; **12(1)**: 77-90.

[2] Andrijevi, D., "On B-open Sets", *Matematiki Vesnik* 1996; **48**: 59-64. doi:http://eudml.org/doc/252912

[3] Baire, R., "Sur les fonctions de variables reelles", *Annali di Matematica Pura ed Applicata* 1899; **3(1)**:1-123. doi:10.1007/BF02419243

[4] Ekici, E., "On separated sets and connected spaces", *Demonstratio Math* 2007; **40(1)**: 209-217. doi:10.1515/dema-2007-0122

[5] Jafari, S. and Noiri, T., "Properties of β-connected spaces", *Acta Mathematica Hungarica* 2003; **101(3)**: 227-236. doi:10.1023/B:AMHU.0000003907.90823.79

[6] Kempisty, S., "Sur les fonctions quasicontinues", *fundamenta Mathematicae* 1932; **1**: 184-197. doi:http://matwbn.icm.edu.pl/ksiazki/fm/fm19/fm19115.pdf

[7] Levine, N., "Semi-open sets and semi-continuity in topological spaces", *The American mathematical monthly* 1963; **70(1)**: 36-41. doi:10.2307/2312781

[8] Modak, S. and Noiri, T., "A weaker form of connectedness", *Communications Faculty of Sciences University of Ankara Series A1 Mathematics and Statistics* 2016; **65(1)**: 49-52.

[9] Modak, S. and Islam, Md., "More connectedness in topological spaces", *Caspian Journal of Mathematical Sciences (CJMS) peer* 2019; **8(1)**: 74-83. doi:10.22080/cjms.2018.13741.1333

[10] Munkres, J.R., " Topology", *Pearson Education* 2014.

[11] Neubrunnova, A., "On certain generalizations of the notion of continuity", *Matematick'ycasopis* 1973; **23(4)**: 374-380. doi:https://dml.cz/handle/10338.dmlcz/126571

[12] Neubrunn, T., "A generalized continuity and product spaces", *Mathematica Slovaca* 1976; **26(2)**: 97-99. doi:https://dml.cz/handle/10338.dmlcz/136112

[13] Neubrunn, T., "Quasi-continuity", *Real Analysis Exchange* 1988; **12(2)**: 259-306. doi:10.2307/44151947

[14] Noiri, T., "A function which preserves connected spaces", *asopis pro Pstovn Matematiky* 1982; **107(4)**: 393-396. doi:10.21136/CPM.1982.118148

[15] Noiri, T. and Modak, S., "Half b-connectedness in topological spaces", *Journal of the Chungcheong Mathematical Society* 2016; **29(2)**: 221-221. doi:10.14403/jcms.2016.29.2.221

[16] Pipitone, V. and Russo, G., "Spazi semiconnessi e spazi semiaperti", *Rendiconti del Circolo Matematico di Palermo* 1975; **23(4)**: 273-285. doi:10.1007/BF02843735

[17] Tyagi, B.K., Bhardwaj, M. and Singh, S., "α_β-Connectedness as a characterization of connectedness", *Journal of advanced studies in topology* 2018; **9(2)**: 119-129. doi:10.20454/jast.2019.1497

[18] Tyagi, B.K., Singh, S., et.al., "Some strong forms of connectedness in topological spaces", *Journal of advanced studies in topology* 2019; **10(1)**: 20-27. doi:10.20454/jast.2019.1497