J. Indones. Math. Soc.
Vol. 28, No. 01 (2022), pp. 8–18.

HOLLOW CYLINDER WITH THERMOELASTIC MODELLING BY REDUCED DIFFERENTIAL TRANSFORM

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Abstract. The term thermal stresses are related to mechanics of materials. The thermal stress is formed due to any change in temperature of a material. The large change in temperature concludes to higher the thermal stresses. Also, there is an effect of thermal expansion coefficient on thermal stresses. The thermal expansion coefficient is different for different materials. In the present paper, the design of a mathematical model concerning the thermal stresses in hollow cylinder subject to the heat conduction with initial and boundary conditions have developed. The basic aim of this work is related to calculations of thermal stresses and thermoelastic displacement in the hollow cylinder by using the reduced differential transform method. The analytical solution is satisfied with the aim of special cases for the copper material properties. The numerical results are illustrated graphically by using mathematical software SCILAB.

Key words and Phrases: Thermal stresses, Radial displacement, Heat conduction, Reduced differential transform, SCILAB.

1. Introduction

Thermoelasticity is concern with the study of theory of elasticity and heat conduction. In the recent years lot of researchers have worked on the thermal stresses and thermoelastic displacements in various solids. The researchers are doing their research on thermoelasticity because of its wide applications in engineering and physics field. The different methods of calculations provide the brief results of the thermoelastic phenomenon. The results obtained in thermoelastic models are depend on the initial and boundary conditions of heat conduction problems.

2020 Mathematics Subject Classification: 35Qxx, 35K05
Received: 03-08-2021, accepted: 03-01-2022.

The different authors have been developed the various model along with the mediums and heat sources, by which the effect may be shown on thermal stresses and thermoelastic displacements of the various solids.

In 1993, Ozisik [11] study the homogeneous and non-homogeneous temperature distribution of circular solids.He has developed the heat conduction equations and their solution in various coordinate system. Further Noda [9] derived the stress functions in the combination of the complementary function and particular integral. He has studied the thermal stresses and various properties regarding the thermoelasticity in various solids. The stress-strain relations has used and determine the thermal stresses analytically with steady state heat conduction as well as transient temperature distribution. Tikhe [20] has studied an inverse heat conduction problem in a thin circular plate and determine the thermal deflection. Nowacki W. [10] successfully investigated the steady-state thermal stresses of thick circular plate through axisymmetric heat distribution on upper, lower, and circular edges. Sherief, H. H. and Anwar, M. N. (1994) [13] studied the two dimensional generalized thermoelasticity problem for an infinitely long Cylinder. Roychoudhary S. K. [12] studied the quasi-static thermal stresses in a thin circular plate due to transient temperature applied along the circumference of a circle over the upper face and a note on quasi-static thermal deflection of a thin clamped circular plate due to ramptype heating of a concentric circular region of the upper face.

In 2016, Mallick A., Ranjan R., and Sarkar P. K. [8] calculated an approximate analytic solution on the heat transfer effect on thermal stresses in an annular hyperbolic fin. Boley B. A. and Weiner [4] discussed heat transfer theory and discussed various methods for solving boundary value problems on heat conduction and gives the practical approach for analyzing thermal stresses on various strength of materials. Keskin Y. and Oturanc G. [6, 7] studied an alternate technique, called the Reduced Differential Transform Method (RDTM), which is very simple, powerful, efficient technique for finding exact solutions. In his paper, he studied the RDTM for partial differential equation and Gas dynamic equation. Also, discussed the analytic solution of linear and nonlinear wave equation. Further, he concluded that, it is an iterative procedure based on the use of the Taylor Series solution of differential equations. Taha B. A. [17] in 2011 discussed the use of RDTM for evaluating Partial Differential Equations with Variable Coefficients and found exact solutions. Taghavi A., Babaei A. and Mohammadpour A. [16] in 2015 discussed the application of RDTM for solving nonlinear Reaction-Diffusion-Convection Problems. Al. Amr M. O. [2] in 2014 studied the new approach towards RDTM and Vineet Shrivastava [14] in 2017 considered the RDTM to find the solutions of two and three dimensional second order hyperbolic telegraph equation. N.Bildik, A.Konuralp [3] uses the variational iteration method, differential transform method and Adomian decomposition method for solving different types of nonlinear partial differential equations.

Yoitiro Takeuti [18] has studied thermoelastic model of circular disc with instantaneous line heat source. He has determined the thermal stresses in circular disc with instantaneous heat source on the concentric arc and along its radius. A.

M. Abd-Alla [1] has considered an infinite circular cylinder and determined the thermal stresses by using the Fourier transform method. The thermal stresses in a transversely isotropic medium with a penny-shaped crack has been studied by Y. M. Tasi [19] . A. H. Elsheikha [5] has considered the model for determination of thermal stresses and deflection in thick circular plate with axisymmetric heat source. He has been applied the Greens function method to obtain the results. Very recently C. S. Sutar[15] has determined bending in rectangular plate with the aid of thermal stresses. He explained the effect of thermal stresses on bending of plate by considering the copper material. This paper concern with the determination of thermal stresses, radial displacement of the hollow cylinder using stress function occupying the region $a \leq r \leq b$,$0 \leq \theta \leq 2\pi$ with consideration by inner and outer radius. The initial and boundary conditions has been considered alongwith the angular coordinate and by using the reduced differential transform method, the solution is obtained. The results are discussed through graphically which are drawn by using the mathematical software SCILAB.

Nomenclature:

α - Thermal expansion coefficient,
E - Young's Modulus,
χ -Stress function,
u - Thermal Displacement,
T - Temparature field,
χ_c -Complementary solution,
χ_p -Particular solution,
T_0 -Initial temparature,
ν -Poission's ratio,
$\varepsilon_{\theta\theta}$ -Plane Strain,
Γ -Change in temparature.

1.1. **Reduced differential transform method(RDTM).** If s(x, y, z, t) is analytic and continuosly differentiable with repect to x, y, z, t then the reduced differential transform function of s(x, y, z, t) is defined as[2]

$$S_k\left(x,y,z,t\right)=\frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}s\left(x,y,z,t\right)\right]_{t=t_0}. \tag{1}$$

And the inverse differential reduced transformed function of $S_k\left(x,y,z,t\right)$ is given by

$$s\left(x,y,z,t\right)=\sum_{k=0}^{\infty}S_k\left(x,y,z,t\right)\left(t-t_0\right)^k. \tag{2}$$

2. Formulation of Problem

Consider the one dimensional long hollow circular cylinder with inner radius a and outer radius b with $0 \leq \theta \leq 2\pi$, the steady state heat conduction governing equation is given by [9],

$$\nabla^2 T = 0, \tag{3}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2},$$

with $T_0 = i_0$, $\left(\frac{\partial T}{\partial \theta}\right)_{\theta=0} = i_1$.

The thermal stress function is given by the equation [9]

$$\chi = \chi_c + \chi_p, \tag{4}$$

where the χ_c is complementary function and χ_p is the particular solution of χ. The χ_c satisfies the following equation

$$\nabla^4 \chi_c = 0. \tag{5}$$

The χ_p satisfies the following equation

$$\nabla^2 \chi_p = -\alpha E \Gamma, \tag{6}$$

where $\Gamma = T - T_0$.

The thermal stresses are given by the equations[9]

$$\sigma_{rr} = \frac{1}{r^2}\frac{\partial^2 \chi}{\partial \theta^2} + \frac{1}{r}\frac{\partial \chi}{\partial r}, \tag{7}$$

$$\sigma_{\theta\theta} = \frac{\partial^2 \chi}{\partial r^2}, \tag{8}$$

$$\sigma_{r\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \chi}{\partial \theta}\right). \tag{9}$$

The stress strain relation is given by [9]

$$\varepsilon_{\theta\theta} = \frac{1}{E}\left(\sigma_{\theta\theta} - \nu \sigma_{rr}\right) + \alpha \Gamma.$$

Also, the strain-displacement relation [9] is given by

$$\varepsilon_{\theta\theta} = \frac{u}{r}.$$

Therefore the radial displacement is given by the equatin

$$u = \frac{r}{E}\left(\sigma_{\theta\theta} - \nu \sigma_{rr}\right) + r\alpha\Gamma. \tag{10}$$

3. Solution of the problem

Applying RDTM [by equation(1)] to equation-(3) and using intial and boundary conditions we get,

$$T_k = i_k, \quad for \ \ k = 2, 3, 4, ..., \tag{11}$$

where $i_k = \dfrac{1}{k(k-1)}\left\{-r^2(i_{k-2})_{rr} - r(i_{k-2})_r\right\}$.
Taking inverse RDT [by equation(2)] to equation (11) we get,

$$T = i_0 + i_1\theta + \sum_{k=2}^{\infty} i_k\theta^k. \tag{12}$$

To obtain the theoretical solution for χ_c from the equation (5), consider the initial and boundary conditions as,

$$(\chi_c)_0 = g_0, \ \ (\chi_c)_1 = g_1, \ \ (\chi_c)_2 = g_2, \ \ (\chi_c)_3 = g_3,$$

and by using RDTM we get,

$$\chi_c = g_0 + g_1\theta + g_2\theta^2 + g_3\theta^3 - \frac{1}{24}g_4\theta^4 - \frac{1}{120}g_5\theta^5 - \sum_{k=6}^{\infty}\frac{1}{k(k-1)(k-2)(k-3)}g_k\theta^k, \tag{13}$$

where

$$F_k = ((\chi_c)_k)_{rr} + \frac{1}{r}((\chi_c)_k)_r + \frac{1}{r^2}(k+1)(k+2)(\chi_c)_{k+2},$$

$$g_4 = r^4(F_0)_{rr} + r^3(F_0)_r + 2r^2(g_2)_{rr} + 2r(g_2)_r,$$

$$g_5 = r^4(F_1)_{rr} + r^3(F_1)_r + 6r^2(g_3)_{rr} + 6r(g_3)_r,$$

$$g_k = r^4(F_{k-4})_{rr} + r^3(F_{k-4})_r - \frac{1}{(k-4)(k-5)}[r^2(g_{k-2})_{rr} + r(g_{k-2})_r].$$

Also applying RDTM for the equation (6) and using equation (12) we get,

$$\chi_p = -\frac{\alpha E r^2}{2}h_0\theta^2 - \frac{\alpha E r^2}{6}h_1\theta^3 - \frac{1}{12}s_1\theta^4 - \frac{1}{20}s_2\theta^5 - \sum_{k=6}^{\infty}\frac{1}{k(k-1)}s_{k-3}\theta^k \tag{14}$$

where

$$h_k = \left[\frac{1}{k!}\frac{\partial^k h}{\partial\theta^k}\right]_{\theta=0}, \quad h = T - T_0,$$

$$s_1 = \alpha E r^2 h_2 - \frac{\alpha E r^2}{2}\left[(r^2h_0)_{rr} + (rh_0)_r\right],$$

$$s_2 = \alpha E r^2 h_3 - \frac{\alpha E r^2}{6}\left[(r^2h_1)_{rr} + (rh_1)_r\right],$$

$$s_k = \alpha E r^2 h_{k+1} - \frac{1}{k(k+1)}\left[r^2(s_{k-2})_{rr} + r(s_{k-2})_r\right], \quad k = 3, 4, ...$$

with initial and boundary condition $(\chi_p)_0 = 0, \quad (\chi_p)_1 = 0.$

From equation(4),(13),(14) we get,

$$\chi = g_0 + g_1\theta + g_2\theta^2 + g_3\theta^3 - \frac{1}{24}g_4\theta^4 - \frac{1}{120}g_5\theta^5 - \sum_{k=6}^{\infty}\frac{1}{k(k-1)(k-2)(k-3)}g_k\theta^k -$$

$$\frac{\alpha E r^2}{2}h_0\theta^2 - \frac{\alpha E r^2}{6}h_1\theta^3 - \frac{1}{12}s_1\theta^4 - \frac{1}{20}s_2\theta^5 - \sum_{k=6}^{\infty}\frac{1}{k(k-1)}s_{k-3}\theta^k. \tag{15}$$

Therefore, the thermal stresses and radial displacement are given by

$$\sigma_{rr} = \frac{1}{r^2}\left[g_0 + g_1\theta + g_2\theta^2 + g_3\theta^3 - \frac{1}{24}g_4\theta^4 - \frac{1}{120}g_5\theta^5\right]_{\theta\theta}$$

$$-\frac{1}{r^2}\left\{\sum_{k=6}^{\infty}\frac{1}{k(k-1)(k-2)(k-3)}\left[k(k-1)g_k\theta^{k-2} + 2k\theta^{k-1}(g_k)_\theta + \theta^k(g_k)_{\theta\theta}\right]\right\}$$

$$-\alpha E h_0 - \alpha E h_1\theta - \frac{1}{r^2}\left[\frac{1}{12}s_1\theta^4 - \frac{1}{20}s_2\theta^5 - \sum_{k=6}^{\infty}\frac{1}{k(k-1)}s_{k-3}\theta^k\right]_{\theta\theta} + \frac{\eta}{r}, \tag{16}$$

where

$$\eta = \left\{g_0 + g_1\theta + g_2\theta^2 + g_3\theta^3 - \frac{1}{24}g_4\theta^4 - \frac{1}{120}g_5\theta^5 - \sum_{k=6}^{\infty}\frac{1}{k(k-1)(k-2)(k-3)}g_k\theta^k\right\}_r$$

$$-\frac{\alpha E\theta^2}{2}(2rh_0 + r^2(h_0)_r) - \left\{\frac{\alpha E r^2}{6}h_1\theta^3 + \frac{1}{12}s_1\theta^4 + \frac{1}{20}s_2\theta^5 - \sum_{k=6}^{\infty}\frac{1}{k(k-1)}s_{k-3}\theta^k\right\}_r.$$

$$\sigma_{\theta\theta} = (g_0)_{rr} + (g_1)_{rr}\theta + (g_2)_{rr}\theta^2 + (g_3)_{rr}\theta^3 - \frac{1}{24}(g_4)_{rr}\theta^4 - \frac{1}{120}(g_5)_{rr}\theta^5$$

$$+\sum_{k=6}^{\infty}\frac{1}{k(k-1)(k-2)(k-3)}(g_k)_{rr}\theta^k - \frac{\alpha E}{2}(r^2(h_0)_{rr} + 4r(h_0)_r + 2h_0)\theta^2 - \frac{\alpha E}{6}(r^2(h_1)_{rr} + 4r(h_1)_r + 2h_1)\theta^3$$

$$-\frac{1}{12}(s_1)_{rr}\theta^4 - \frac{1}{20}(s_2)_{rr}\theta^5 - \sum_{k=6}^{\infty}\frac{1}{k(k-1)}(s_{k-3})_{rr}\theta^k. \tag{17}$$

$$\sigma_{r\theta} = \frac{1}{r^2}(\zeta - \xi) - \frac{1}{r}(\zeta_r - \xi_r), \tag{18}$$

where

$$\zeta = (g_0)_\theta + g_1 + (g_1)_\theta\theta + (g_2)_\theta\theta^2 + 2\theta g_2 + 3g_3\theta^2 + (g_3)_\theta\theta^3 - \frac{1}{24}[4g_4\theta^3 + (g_4)_\theta\theta^4]$$

$$-\frac{1}{120}[5g_5\theta^4 + (g_5)_\theta\theta^5] - \sum_{k=6}^{\infty}\frac{1}{k(k-1)(k-2)(k-3)}[kg_k\theta^{k-1} + (g_k)_\theta\theta^k]$$

and

$$\xi = \frac{\alpha E r^2}{2}[2h_0\theta + (h_0)_\theta\theta^2] + \frac{\alpha E r^2}{6}[3h_1\theta^2 + (h_1)_\theta\theta^3] + \frac{1}{12}[(s_1)_\theta\theta^4 + 4s_1\theta^3] + \frac{1}{20}[(s_2)_\theta\theta^5 + 5s_2\theta^4]$$

$$+ \sum_{k=6}^{\infty} \frac{1}{k(k-1)}[(s_{k-3})_\theta\theta^k + ks_{k-3}\theta^{k-1}].$$

The radial displacement is given by

$$u = \frac{r}{E}\left\{(g_0)_{rr} + (g_1)_{rr}\theta + (g_2)_{rr}\theta^2 + (g_3)_{rr}\theta^3 - \frac{1}{24}(g_4)_{rr}\theta^4 - \frac{1}{120}(g_5)_{rr}\theta^5\right\}$$

$$-\frac{r}{E}\left\{\sum_{k=6}^{\infty} \frac{1}{k(k-1)(k-2)(k-3)}(g_k)_{rr}\theta^k - \frac{\alpha E}{2}(r^2(h_0)_{rr} + 4r(h_0)_r + 2h_0)\theta^2\right\}$$

$$-\frac{\alpha r}{6}(r^2(h_1)_{rr} + 4r(h_1)_r + 2h_1)\theta^3$$

$$-\frac{r}{E}\left\{\frac{1}{12}(s_1)_{rr}\theta^4 - \frac{1}{20}(s_2)_{rr}\theta^5 - \sum_{k=6}^{\infty} \frac{1}{k(k-1)}(s_{k-3})_{rr}\theta^k\right\}$$

$$-\frac{r\nu}{E}\left\{\frac{1}{r^2}\left[g_0 + g_1\theta + g_2\theta^2 + g_3\theta^3 - \frac{1}{24}g_4\theta^4 - \frac{1}{120}g_5\theta^5\right]_{\theta\theta}\right\}$$

$$+\frac{r\nu}{E}\left\{\frac{1}{r^2}\left\{\sum_{k=6}^{\infty} \frac{1}{k(k-1)(k-2)(k-3)}\left[k(k-1)g_k\theta^{k-2} + 2k\theta^{k-1}(g_k)_\theta + \theta^k(g_k)_{\theta\theta}\right]\right\}\right\}$$

$$+r\alpha[i_0 + i_1\theta + \sum_{k=2}^{\infty} i_k\theta^k]. \tag{19}$$

4. Discussion and Conclusions

Consider the hollow cylinder of copper material with inner radius 6mts and outer radius 8mts.

Also $i_0 = r^2, \quad i_1 = 2r^2,$

$g_0 = r^2, \quad g_1 = -r^2, \quad g_2 = 0 = g_3,$

$\alpha = 16.5 \times 10^{-6}(^0\mathrm{C})^{-1}$, $E = 1.2 \times 10^{11}(N/m^2)$, $\nu = 0.34$.

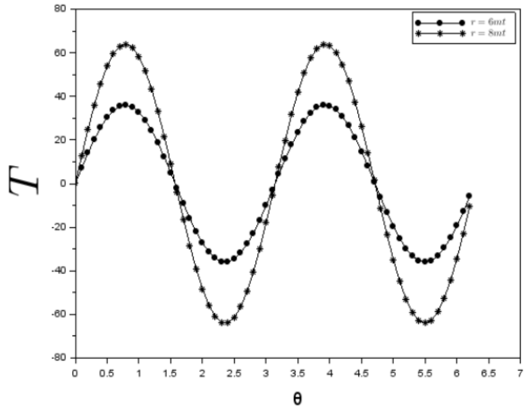

FIGURE 1. T Vs θ

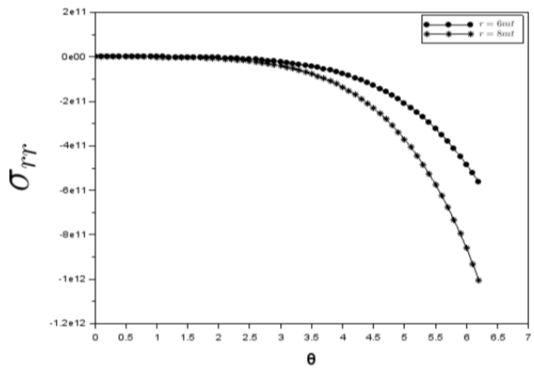

FIGURE 2. σ_{rr} Vs θ

Figure 1 shows the result of temperature change at the inner and outer boundary of hollow cylinder. The inner temperature of the cylinder is less than that of at surface of the cylinder with repect to change in θ. But at some stage the temperature distribution changes in vice-versa. This process continues in sequentially manner.

The figure 2 gives the change in thermal stresses σ_{rr} with respect to change in θ. The nature of thermal stresses remains same at inner and outer surfae of the cylinder. The thermal stresses upto some value of θ are constant and decreases suddenly as increase in θ. Also figure 3 discribes the same nature of $\sigma_{\theta\theta}$ with respect to θ as by figure 2 but its change in numerical values of the thermal stresses $\sigma_{\theta\theta}$.

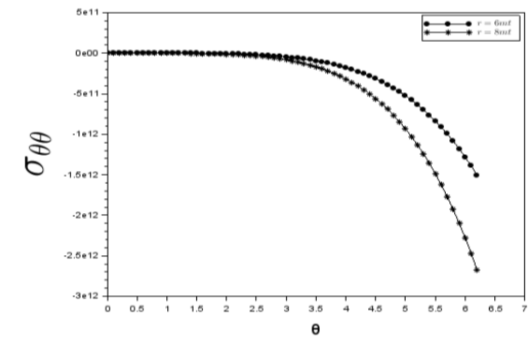

Figure 3. $\sigma_{\theta\theta}$ Vs θ

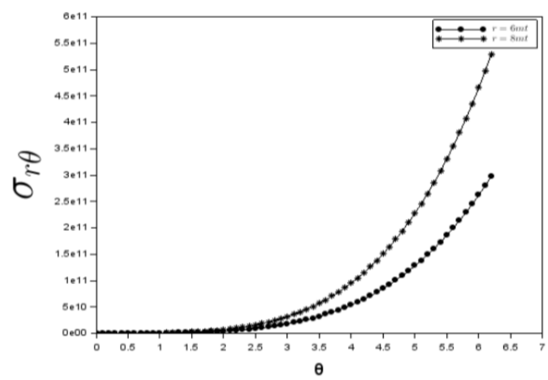

Figure 4. $\sigma_{r\theta}$ Vs θ

In Figure 4 the change in the thermal stress $\sigma_{r\theta}$ with respect to θ has shown. The nature discribes that upto $\theta = 2$ the thermal stress remains constant but as increases the value of θ the $\sigma_{r\theta}$ suddenly increases.

Figure 5 gives the nature of radial displacement u with the value of θ. The radial displacement at inner and outer surface of cylinder is constant for some values of θ but after that it will decreases with increase in θ. The nature of radial displacement is depends on the values of thermal stresses $\sigma_{\theta\theta}$ and σ_{rr}. Also we can conclude that there is little change in displacement at inner and outer surface of the cylinder whenever there is change in the values of θ.

In the proposed paper we have discussed the steady state heat conduction for hollow circular cylinder with inner and outer radius a and b respectively.We have determined the thermal stresses and radial displacement with the effect of temperature of the cylinder at inner and outer surface. By means of thermal stress

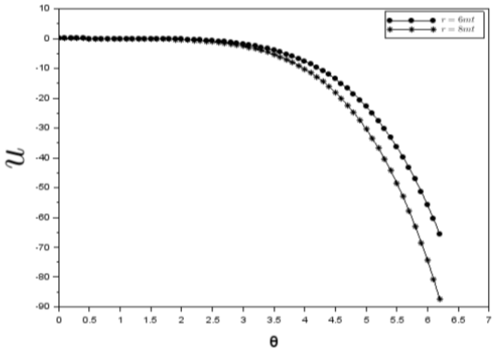

FIGURE 5. u Vs θ

fuction and with heat flux boundary condition at θ, the mathematical model has solved by using the reduced differential transform method.It observed that the temperature distribution and thermal stress obtained are in the Taylor's series form. The radial displacement is depend upon the radius of the cylinder and θ which is also in the Taylor's series form. The nature of thermal stresses at inner and outer boundary of the cylinder remains same but the numerical values are different at both the boundaries. That means we conclude that the thermal stresses and displacement of cylinder are different at inner and outer boundary surface of the cylinder.

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